TEMPERATURE DISTRIBUTION IN A HOLLOW CYLINDRICAL MANOMETER COLUMN

by

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I. INTRODUCTION

The use of liquid metals in nuclear reactor technology is becoming increasingly important. One problem encountered in the use of liquid metals is the accurate measurement of pressure in experimental equipment. One method for accurate measurement of low pressures involves the use of a manometer column. This method of measurement involves two problems in the determination of the pressure. The first is the determination of the height or level of the liquid metal and the second is the calculation of the pressure when the column height is known.

The level of the liquid metal in a manometer column can be determined by use of a differential transformer. By letting the manometer column act as the core of the transformers, a maximum output voltage will be attained when one transformer has the empty column as a core and the other transformer has a core consisting of the manometer column filled with the liquid metal. With the two transformers placed close together the maximum voltage occurs when the level is between the transformers.

The temperature is an important consideration in the second problem since most metals are molten only above room temperature and the density of most liquid metals changes significantly with temperature. Therefore, the density or temperature of the column must be known to determine the pressure.

The purpose of this investigation is to establish and check the validity of a mathematical model used to solve for the temperature distribution in a manometer column containing sodium. If the mathematical model can be shown to be valid over the desired operating range, the temperature distribution of the sodium in the column can be computed mathematically for any desired combination of loop and column temperatures. The height of the column necessary before the temperature gradient along the column becomes essentially zero can also be determined.

II. REVIEW OF LITERATURE

A review of the literature was first conducted to see whether or not a general solution for the temperature distribution in a finite hollow cylinder had been published. Carslaw and Jaeger (1) have conducted the most extensive work on the temperature distributions in various geometries and with various boundary conditions. Some of the boundary conditions used by Carslaw and Jaeger for finite hollow cylinders with time independent solutions are

1. Finite hollow cylinder $a < r < b$, $o < z < L$. $r = a$ maintained at $f(z)$, and all other faces at zero temperature.

2. Finite hollow cylinder $a < r < b$, $o < z < L$. Flux of heat into the solid at $r = a$, a prescribed function $f(z)$, and all other surfaces kept at zero temperature.

3. Finite hollow cylinder $a < r < b$, $o < z < L$. The surface $z = o$ kept at $f(r)$, all other surfaces are kept at zero temperature.

4. Finite hollow cylinder $a < r < b$, $o < z < L$. $r = a$ is kept at temperature $f(r)$. Radiation at all other surfaces into medium at zero temperature.

5. Finite hollow cylinder $a < r < b$, $o < z < L$. No heat flow across $r = a$, zero temperature at $z = o$ and $z = L$, and temperature $f(z)$ at $r = b$.

6. Finite hollow cylinder $a < r < b$, $o < z < L$. Heat production at constant rate A_o per unit time per unit volume, no flow over $r = a$, and zero temperature at other surfaces.

7. Finite hollow cylinder $a < r < b$, $o < z < L$. Heat production at constant rate A_o per unit time per unit volume, zero temperature at $z = o$,

$z = L$, and $r = b$, and water cooling over $r = a$.
None of these systems of boundary conditions fit those needed for the present problem.

Schneider (4) and Carslaw and Jaeger (1) both give the steps necessary in the solution of a problem of this type. Their results were used as a guide for applying the desired boundary conditions to obtain the solution for the temperature distribution in the manometer column.

III. APPARATUS

The manometer column under consideration is to be used to measure the pressure of molten sodium in a horizontal 3/4 inch O.D. schedule 40 Inconel 600 pipe. The column was constructed of 3/8 inch O.D. by 0.035 inch wall Inconel 600 tubing welded perpendicularly to the pipe.

Since sodium has a melting point of 97.8^{o}C (208^{o}F), it was necessary to incorporate a heat source in the manometer column in order to keep the sodium molten. The heat source for the manometer column consisted of a single strand of #24 chromel A wire along the axis of the manometer column. The heater wire was insulated with 1/8 inch alundum beads and this assembly was placed in a 5/32 inch O.D. by 0.014 inch wall Inconel 600 tube to shield it from the sodium. The end of the 5/32 inch tube was fused closed around one end of the heater wire. The 5/32 inch tube served as electrical ground and completed the circuit for the heater. With this system placed concentrically inside of the manometer tube, the space left for the sodium in the manometer column was an annulus or hollow cylinder. Figure 1 is a photograph of the cross section of the manometer tube with the heater assembly.

The 3/4 inch pipe was used to simulate a portion of a sodium loop. For this apparatus a special heater in the pipe was necessary to obtain simulated loop temperatures. A concentric heater was placed in the pipe to supply this heat. Figure 2 is a sketch of the manometer column with the pipe and heaters.

The 3/4 inch pipe was attached to a reservoir of sodium which could be forced into the manometer column by helium pressure. Two pressure gages,

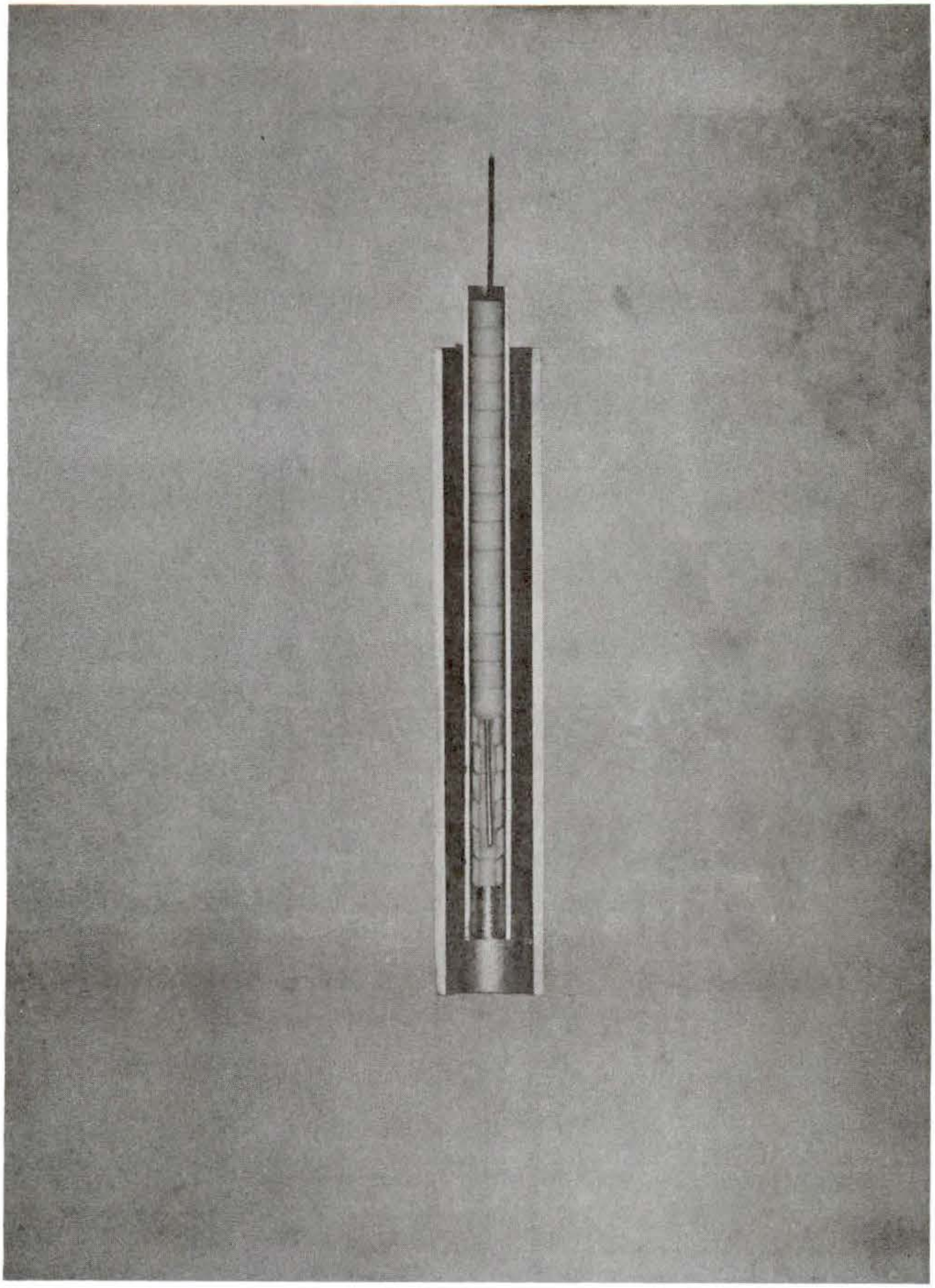

Figure 1. Cross sectional view of manometer column including heater assembly

one attached to the reservoir and the other to the top of the manometer column, were used to give an indication of the sodium height in the column.

For the purpose of measuring the temperature at several points along the manometer column, several chromel-alumel thermocouples were attached to the surface of the inconel tube. The thermocouples used were made from standard #24 chromel-alumel thermocouple wire with a guaranteed accuracy of $\pm$ 4^{o}F from 0 to 530^{o}F and $\pm$ 3/4% from 530 to 2300^{o}F. The junction was formed by melting the two wires into a bead. This bead was spot-welded to the surface of the inconel tube and the bead was then ground down to minimize the surface defect. Figure 3 is a photograph of a 3/8 inch tube with thermocouples attached similarly to those on the manometer column.

The temperatures from the thermocouples were recorded on a twelve-point Brown recorder with a 0-500^{o}C scale. Since 24 thermocouples were used on the system, each recorder point recorded alternately two specific thermocouples. The calibration of the recorder was checked with a potentiometer both by calibrating the recorder and by reading the emf of the thermocouples periodically on the potentiometer.

Simpson voltmeters and ammeters were placed in the circuit to measure the power inputs to each heater. These meters were calibrated with a Westinghouse type TA power analyzer.

Figure 4 is a photograph of the apparatus used including the auxiliary equipment.

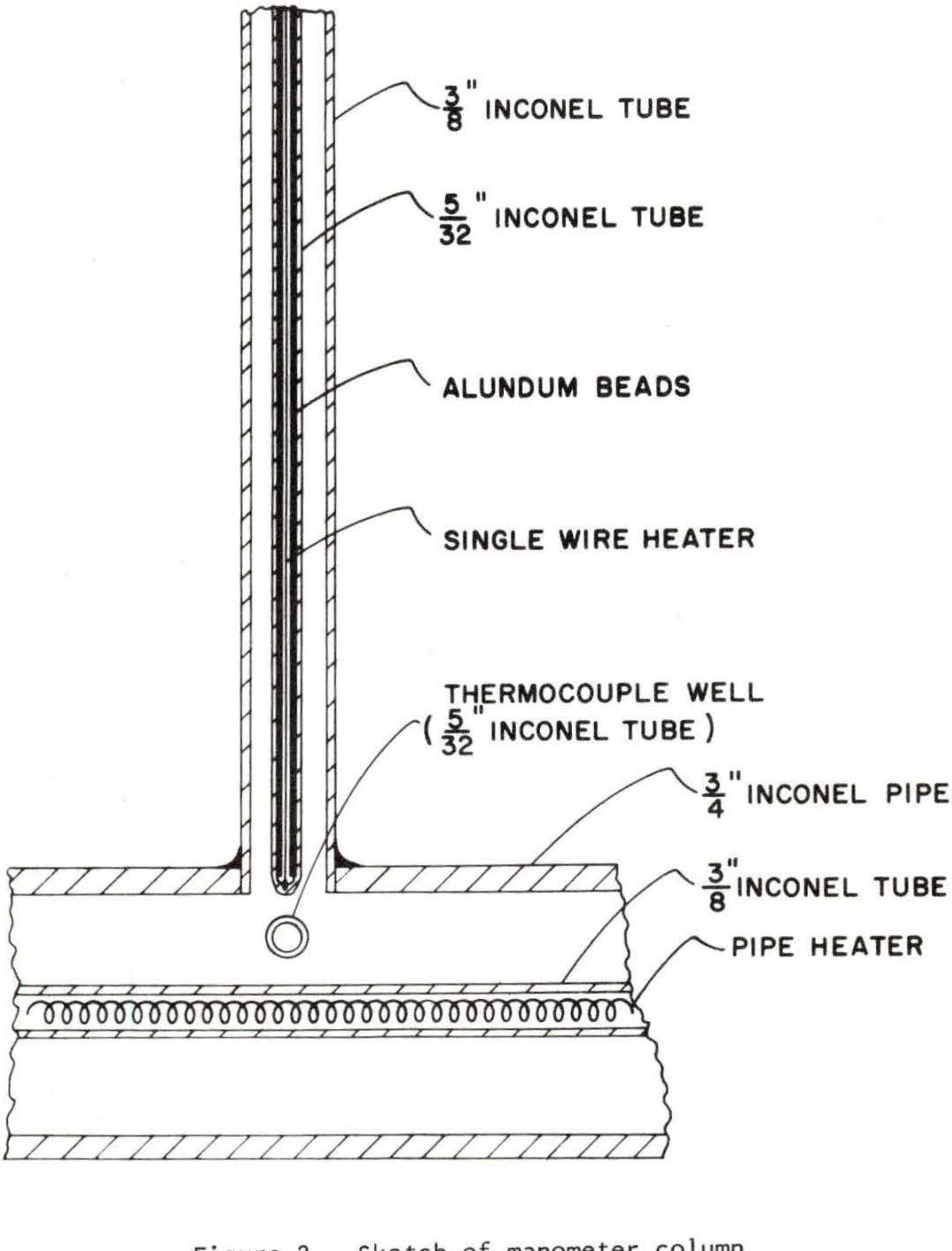

Figure 2. Sketch of manometer column and loop pipe with heaters

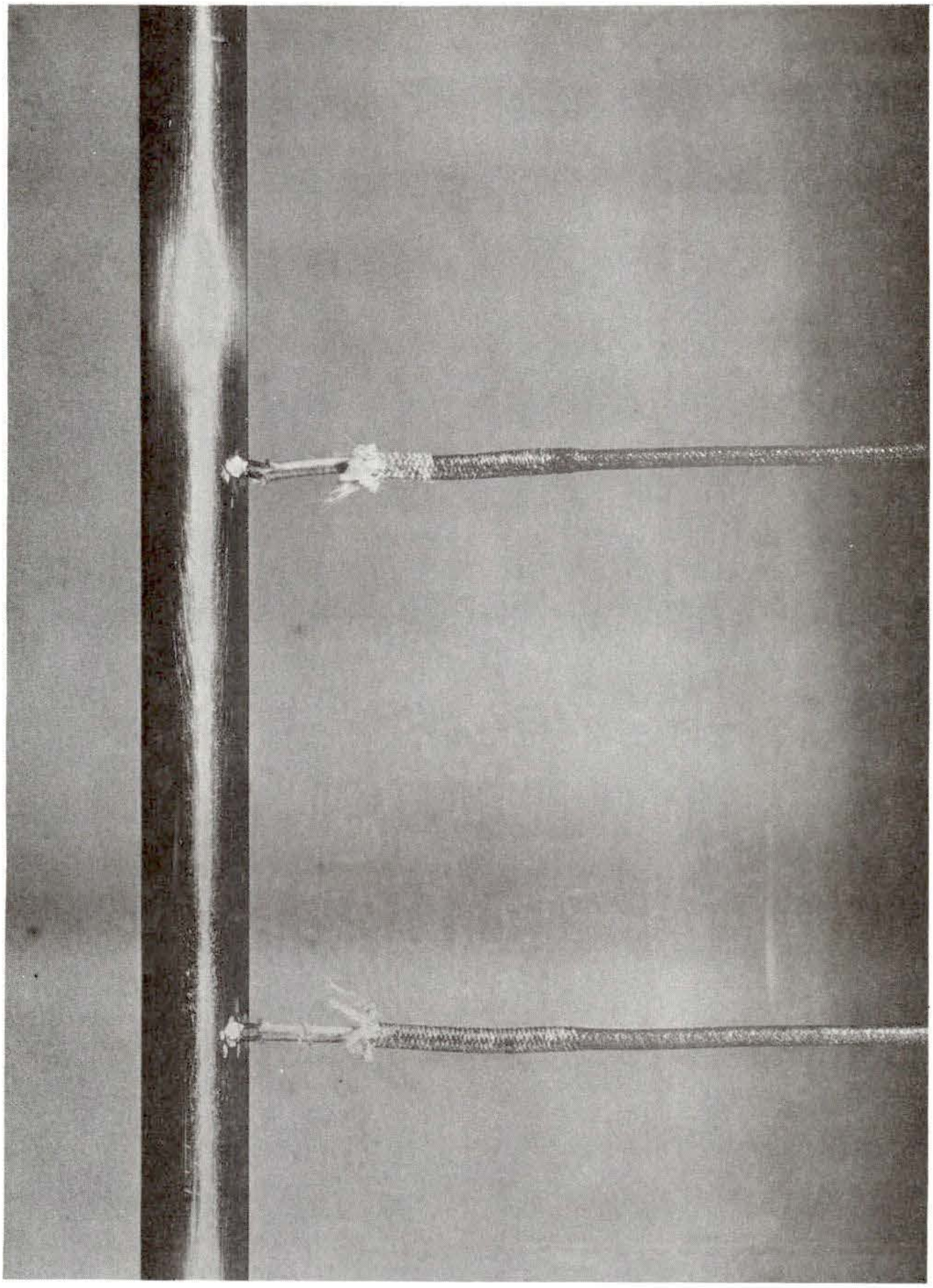

Figure 3. Section of manometer tube showing thermocouples attached

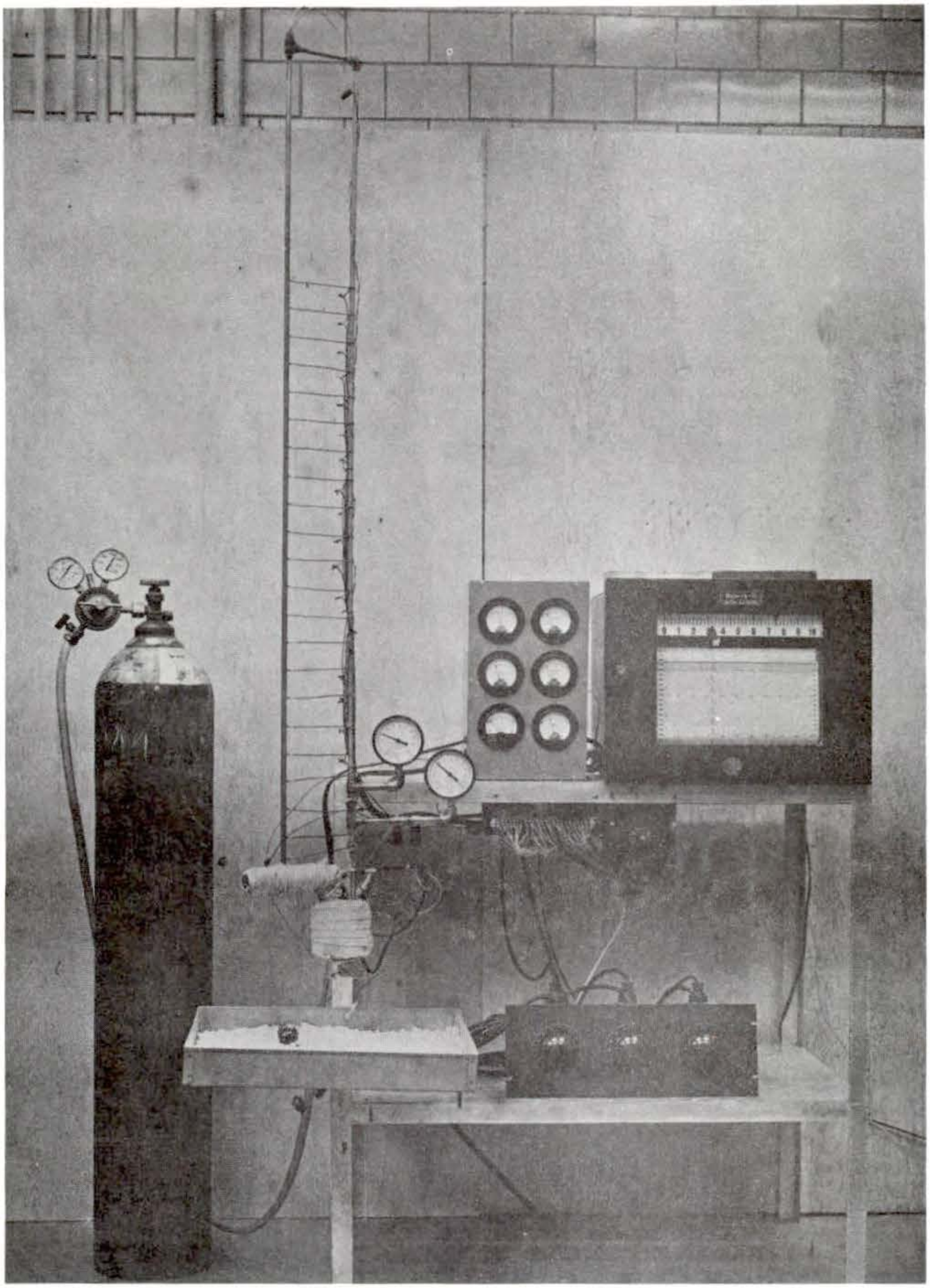

Figure 4. Picture of experimental apparatus
with auxiliary equipment

IV. GENERAL THEORY

If the heat transfer in the manometer column is assumed to be due solely to conduction, the temperature distribution in the column can be determined from the general heat conduction equation which is given by Schneider (4) as

$$\nabla^2 u + \frac{q'''}{K} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \tag{1}$$

For a heat conducting medium in which there are no heat sources or heat sinks, the term $\frac{q'''}{K}$ becomes zero. For a steady state or time independent solution, the term $\frac{1}{\alpha}\frac{\partial u}{\partial t}$ also vanishes leaving the heat conduction equation in the form of Laplace's equation

$$\nabla^2 u = 0 \tag{2}$$

Laplace's equation may be expressed in cylindrical coordinates for the case under consideration which is given as

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{3}$$

where r is the radial variable and z is the longitudinal variable.

The solution of the Laplace equation is obtained by the technique of separation of variables. The solution of the temperature as a function of position r and z has the form

$$u(r,z) = R(r)\,Z(z) \tag{4}$$

Two differential equations, one with the variable r and the other with the variable z, are obtained and are solved independently.

$$\frac{\partial^2 Z(z)}{\partial z^2} + \beta^2 Z(z) = 0 \tag{5}$$

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r}\frac{\partial R(r)}{\partial r} - \beta^2 R(r) = 0 \tag{6}$$

Equation 5 has the form of the wave equation which has a general solution

$$Z(z) = A\cos\beta z + B\sin\beta z \tag{7}$$

Equation 6 has the form of the modified Bessel's equation which has a solution of the form

$$R(r) = C\,I_0(\beta r) + D\,K_0(\beta r) \tag{8}$$

The complete general solution for the temperature distribution in a finite hollow cylinder is obtained by substituting equations 7 and 8 into equation 4.

$$u(r,z) = \left[A\cos\beta z + B\sin\beta z\right]\left[C\,I_o(\beta r) + D\,K_o(\beta r)\right] \tag{9}$$

The particular solution of this equation is determined by applying the existing boundary conditions and obtaining values for the constants A, B, C, D, and β . The following section discusses the assumptions and boundary conditions used in the solution of these constants.

V. THEORETICAL ANALYSIS

The accuracy with which the temperature of the sodium must be known was determined by the accuracy with which the level of the sodium can be measured. The liquid level measuring device for the sodium had a standard deviation of about ± 0.1 cm. Therefore, it will be sufficient to maintain the error in the measurement of pressure which is due to the uncertainty in the value of the density of the sodium to a similar magnitude of ± 0.1 cm of sodium.

The density of sodium as a function of temperature is given by Sittig (5) as

$$\rho = 0.9490 - 2.23 \times 10^{-4} u - 1.75 \times 10^{-8} u^2 \qquad (10)$$

This equation is stated by Sittig to be valid to about ± 0.20% for temperatures up to 640°C. The rate of change of density with temperature is obtained from the derivative of equation 10.

$$\frac{\partial \rho}{\partial u} = -2.23 \times 10^{-4} - 3.50 \times 10^{-8} u \qquad (11)$$

Since the density of sodium varies nearly linearly with temperature in the range being considered, i.e., 100-500°C, the average rate of change of density with temperature in this range is approximately that at 350°C. At the temperature of 350°C the rate of change of density is -2.35×10^{-4} gm/cm^3-°C and the density is 0.8688 gm/cm^3. An error of 0.1 cm. in the height of a sodium column at 350°C is equivalent to an error in pressure of 0.08688 gm/cm^2. Therefore, the product of the sodium height, the uncertainty in the temperature, and $\frac{\partial \rho}{\partial u}$ must be less than 0.08688 gm/cm^2 in order that the error be less than 0.1 cm. This may be expressed as

$$h \, \Delta u \, (2.35 \times 10^{-4}) < 0.08688 \tag{12}$$

or

$$h \, \Delta u < 370 \tag{13}$$

Since the maximum sodium height which can be measured is about 100 cm., the uncertainty in the temperature must be less than 3.7^{o}C to insure that the error due to the uncertainty in the density is less than the error due to the measurement of the sodium level.

The transfer of heat in the vertical direction of the manometer column is due mainly to the sodium. The heat transfer up the column by the Inconel tubes and the alundum insulators is small and is taken into account by using an equivalent area for the sodium. The equivalent sodium area is determined by determining the ratio of the total heat conducted by the complete manometer column and that conducted only by the sodium. The one dimensional heat conduction equation is given by Schneider (4) as

$$q = - KA \frac{\partial u}{\partial z} \tag{14}$$

If the temperature gradient is the same for all materials, the heat flow will be proportional to the product of the thermal conductivity and the area of the particular material. The materials being considered are the sodium, Inconel 600, and the alundum beads. Table 1 gives values of the thermal conductivities and the areas of the various materials which make up the cross section of the manometer column. The values of the thermal conductivities of the materials in the table are for a temperature

Table 1. Properties of materials at 350°C

Material	Area cm^2	Thermal Conductivity $cal/cm^2\text{-}sec\text{-}^{\circ}C/cm$
Sodium	0.3481	0.1760[a]
Inconel 600	0.2815	0.04705[b]
Alundum	0.04048	0.0370[c]

[a]Sittig (5).

[b]Lucks and Deem (3).

[c]Lee and Kingery (2).

of 350°C which is approximately the average of the temperatures being considered. The heat flow in the vertical direction of the sodium is given by

$$\frac{q}{\frac{\partial u}{\partial z}} = -K_{Na}A_{Na} \tag{15}$$

The total heat flow in the vertical direction of the manometer column is given by

$$\frac{q}{\dfrac{\partial u}{\partial z}} = -K_{Na}A_{Na} - K_{In}A_{In} - K_A A_A \qquad (16)$$

The value obtained from equation 15 for the heat flow only in the sodium is 0.06022 cal/sec-^{o}C/cm and the similar value for the complete manometer column obtained from equation 16 is 0.7496 cal/sec-^{o}C/cm. The equivalent area of sodium necessary to transfer the same amount of heat as the complete column is found by multiplying the ratio of the heats by the actual sodium area. The equivalent area has a value of 0.4277 cm^2 which corresponds to an annulus with an inside radius of 0.1981 cm and an outside radius of 0.4188 cm. The error introduced in the heat flow by using this equivalent area of sodium is a maximum of about 3.5% at both 100 and 500^{o}C.

The basic assumption has been made that the transfer of heat by the sodium is due entirely by conduction. This assumption should not introduce much error since the size of the annulus is quite small which would tend to eliminate any convection.

Using the previous assumptions, the following boundary conditions can be used to determine the values of the constants in equation 9 which is the general solution of the heat conduction equation.

1.) The plane at the base of the manometer column (z = o) is an infinite heat source with a temperature which corresponds to the temperature of the sodium in a loop.

2.) The manometer column is high enough so that there is essentially no heat loss from the top of the manometer column ($z = L$).

3.) The single wire heater in the manometer column is a constant heat source along the column at $r = P$.

4.) The heat flux out of the sodium at $r = R$ is equal to the heat loss to the atmosphere which is a function of the temperature of the column.

In boundary condition 4 the relationship between the heat loss to the atmosphere and the temperature of the manometer column was determined empirically. The results are shown in Table 2 and the curve of heat loss from the manometer tube per centimeter of length vs. temperature is shown in Figure 5.

Table 2. Heat input to manometer column

Tube wall temperature ^{o}C	Heat input to column cal/cm-sec
120.5	0.0874
130.5	0.0997
148.5	0.128
164.0	0.153
177.0	0.173
196.5	0.207
209.5	0.230
223.0	0.262
234.5	0.285
247.0	0.313
265.0	0.347
282.0	0.380
301.0	0.427
324.5	0.490
347.5	0.546

Table 2. (Continued)

Tube wall temperature ^{o}C	Heat input to column cal/cm-sec
357.5	0.577
375.5	0.638
398.0	0.699

The equation for the curve can be expressed as a polynomial. The degree of the polynomial which fits the experimental data is determined by the method of differences as described by Sokolnikoff and Redheffer (6). The heat loss from the manometer tube at $50^{o}C$ intervals is obtained from the experimental curve and tabulated in Table 3. The first, second, third, and fourth forward differences are also shown in Table 3. Since the smallest value of the sum of the terms divided by the number of terms is for $\Delta^3 q''$, the polynomial to be used is a second degree polynomial.

$$q'' = b_1 + b_2 u + b_3 u^2 \qquad (17)$$

The constants in equation 17 were determined by applying a least squares fit to the data in Table 2. The least squares fit is obtained by requiring that S be a minimum where S is defined as

$$S = \sum_{i=1}^{n} \left[f(u_i) - q_i'' \right]^2 \qquad (18)$$

Substituting equation 17 for $f(u_i)$ gives

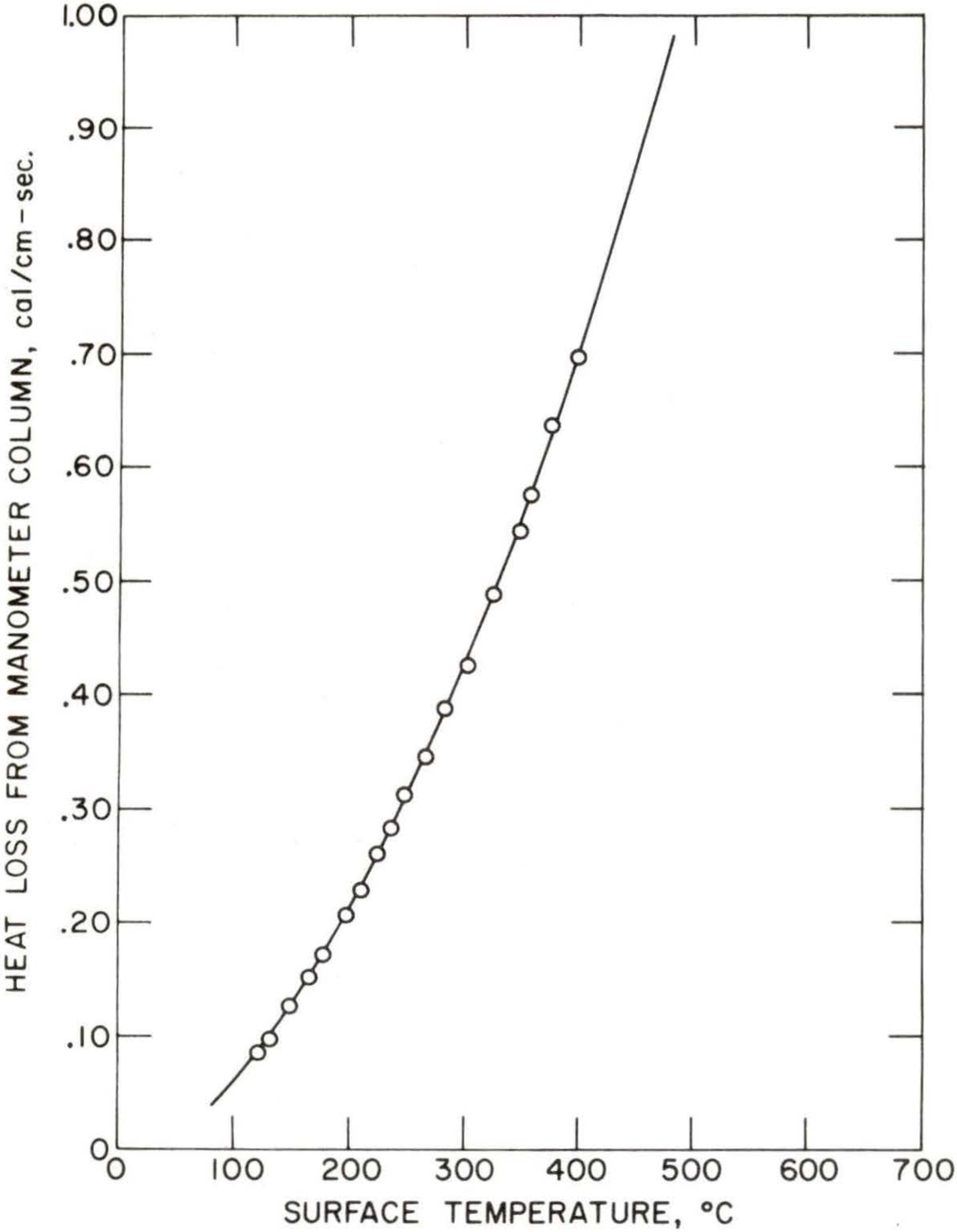

Figure 5. Empirical relation for heat loss from manometer column

$$S = \sum_{i=1}^{n}\left[b_1 + b_2 u_i + b_3 u_i - q_i''\right]^2 \qquad (19)$$

The minimum value of S is obtained by setting the partial derivatives

Table 3. Forward differences for heat loss

Temperature U °C	Heat loss q'' cal/cm-sec	$\Delta q''$	$\Delta^2 q''$	$\Delta^3 q''$	$\Delta^4 q''$
100	0.0610				
		0.070			
150	0.131		0.014		
		0.084		0.004	
200	0.215		0.018		-0.010
		0.102		-0.006	
250	0.317		0.012		0.007
		0.114		0.001	
300	0.431		0.013		0.008
		0.127		0.009	
350	0.558		0.022		
		0.149			
400	0.707				
Sum	2.420	0.646	0.079	0.020	0.025
Average	0.346	0.108	0.016	0.005	0.008

of S with respect to b_1, b_2, and b_3 equal to zero. The following three equations are obtained and solved simultaneously for b_1, b_2, and b_3 using values from Table 2.

$$b_1 n + b_2 \sum_{i=1}^{n} u_i + b_3 \sum_{i=1}^{n} u_i^2 = \sum_{i=1}^{n} q_i'' \qquad (20)$$

$$b_1\sum_{i=1}^{n} u_i + b_2\sum_{i=1}^{n} u_i^2 + b_3\sum_{i=1}^{n} u_i^3 = \sum_{i=1}^{n} u_i q_i'' \tag{21}$$

$$b_1\sum_{i=1}^{n} u_i^2 + b_2\sum_{i=1}^{n} u_i^3 + b_3\sum_{i=1}^{n} u_i^4 = \sum_{i=1}^{n} u_i^2 q_i'' \tag{22}$$

Using values obtained from these equations, equation 17 becomes

$$q'' = -0.0272 + 5.84 \times 10^{-4} u + 3.12 \times 10^{-6} u^2 \tag{23}$$

The complete solution of the general heat conduction equation can now be obtained using the assumptions and boundary conditions previously discussed. The general heat conduction equation for a medium with no heat sources or sinks and for steady state conditions is given by Laplace's equation

$$\nabla^2 u = 0 \tag{2}$$

The first two boundary conditions are

$$u(r,0) = T_L \tag{24}$$

$$\left(\frac{\partial u}{\partial z}\right)_{z=L} = 0 \tag{25}$$

For simplification in solving equation 2 for these boundary conditions, the following substitution is made

$$w(r,z) = u(r,z) - T_L \tag{26}$$

The boundary conditions in equations 4 and 5 applied to the variable w(r,z) give

$$w(r,0) = 0 \tag{27}$$

$$\left(\frac{\partial w(r,z)}{\partial z}\right)_{z=L} = 0 \tag{28}$$

The heat conduction equation still has the form of Laplace's equation.

$$\nabla^2 w(r,z) = 0 \tag{29}$$

The Laplacian operator in the form of cylindrical coordinates is given by

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} = 0 \tag{30}$$

By applying the technique of separation of variables, the solution of this equation will have the form

$$w(r,z) = R(r)\,Z(z) \tag{31}$$

Solving for the first and second partial derivatives of equation 21 with respect to both r and z, substituting these into equation 30 and dividing by R(r)Z(z) gives an equation in which each term is a function of only one variable.

$$\frac{1}{R(r)}\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{rR(r)}\frac{\partial R(r)}{\partial r} + \frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2} = 0 \qquad (32)$$

Since a variation of z has no effect on the first two terms of this equation and a variation of r has no effect on the third term, both the first two terms and the third term must be constant. Since their sum is equal to zero, the value of the third term must be the negative of the value of the first term. The boundary conditions specified by equations 27 and 28 require that this constant be real and positive as used in the following equation.

$$\frac{1}{R(r)}\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{rR(r)}\frac{\partial R(r)}{\partial r} = -\frac{1}{Z(z)}\frac{\partial^2 Z(z)}{\partial z^2} = \beta^2 \qquad (33)$$

The two separate differential equations obtained from equation 33 are

$$\frac{\partial^2 Z(z)}{\partial z^2} + \beta^2 Z(z) = 0 \qquad (34)$$

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r}\frac{\partial R(r)}{\partial r} - \beta^2 R(r) = 0 \qquad (35)$$

Equation 35 has the form of the wave equation which has a solution of the form

$$Z(z) = A' \cos \beta z + B' \sin \beta z \tag{36}$$

Applying the boundary conditions given by equations 27 and 28 one obtains

$$Z(0) = 0 \quad \therefore \quad A' = 0 \tag{37}$$

$$\left(\frac{\partial Z(z)}{\partial z}\right)_{z=L} = 0 \quad \therefore \quad B'\beta \cos \beta L = 0 \tag{38}$$

Avoiding the trivial solution of $B^1 = 0$ one is left with

$$\cos \beta L = 0 \tag{39}$$

which gives

$$\beta = \frac{(2n-1)\pi}{2L} \qquad n = 1, 2, 3, \cdots \tag{40}$$

The solution of the wave equation for this case is an infinite series

$$Z(z) = \sum_{n=1}^{\infty} B_n' \sin \beta_n z \tag{41}$$

Equation 35 has the form of the modified Bessel's equation. The general solution for this equation is given by Schneider (4) as

$$R(r) = A'' I_o(\beta r) + B'' K_o(\beta r) \tag{42}$$

The complete solution is obtained by substituting equations 41 and 42 into equation 31. The constants are combined leaving only two unknown constants for the remaining two boundary conditions.

$$w(r,z) = \sum_{n=1}^{\infty} \left[A_n I_o(\beta_n r) + B_n K_o(\beta_n r) \right] \sin \beta_n z \tag{43}$$

The boundary condition at the inside surface of the annulus, (i.e. $r = P$) is that the heat flux thru the wall is constant at any point along the column. This is given by the one dimensional heat conduction equation

$$q = - K_N A \frac{\partial u}{\partial r} \tag{44}$$

in which $q(P,z)$ = constant which is equal to the heat input of the manometer heater per unit length. The thermal conductivity of sodium is given by Sittig (5) as a linear function of temperature

$$K_N = a_1 + a_2 u = 0.2166 - 1.16 \times 10^{-4} u \tag{45}$$

Since the substitution of $u = w + T_L$ was made to simplify the solution, the thermal conductivity of sodium in terms of w is

$$K_N = a_1 + a_2 T_L + a_2 w \tag{46}$$

The heat flux into the sodium at P per unit length of column is given by

$$q' = \frac{q}{L} = -2\pi K_N P \frac{\partial w}{\partial r} \tag{47}$$

The partial derivative of equation 43 with respect to r is

$$\frac{\partial w(r,z)}{\partial r} = \sum_{m=1}^{\infty} \left[A_m \beta_m I_1(\beta_m r) - B_m \beta_m K_1(\beta_m r) \right] \sin \beta_m z \tag{48}$$

For simplification in writing the equations in the remainder of the derivation the following forms will be used

$$\begin{aligned} F_o(g) &= A_f I_o(\beta_f g) + B_f K_o(\beta_f g) \\ F_1(g) &= A_f \beta_f I_1(\beta_f g) - B_f \beta_f K_1(\beta_f g) \end{aligned} \tag{49}$$

where F, f may represent H, h; M, m; or N, n and g may represent r, R, or P. The following substitution will also be made

$$C_{hmn} = \frac{1}{2h-2m+2n-1} + \frac{1}{2h+2m-2n-1} + \frac{1}{2h-2m-2n+1} + \frac{1}{2h+2m+2n-3} \tag{50}$$

Using these simplifications, substituting equations 43, 46, and 48 into equation 47, and setting $r = P$ gives

$$\frac{q'}{2\pi P} + \left[a_1 + a_2 T_L\right] \sum_{m=1}^{\infty} M_1(P) \sin \beta_m z$$
$$+ a_2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} N_o(P) M_1(P) \sin\beta_n z \sin\beta_m z = 0 \qquad (51)$$

This equation can be simplified by obtaining an orthogonal set of equations from the infinite series in the second term. If this equation is multiplied by the factor sin $_h z$ and the resulting equation integrated over the length of the column from z = o to z = L, the following set of h equations are obtained where h = 1, 2, 3, -----

$$\frac{q'L}{\pi^2 P} \frac{1}{2h-1} + \left[a_1 + a_2 T_L\right]\left[\frac{L}{2}\right] H_1(P)$$
$$+ \frac{a_2 L}{2\pi} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} N_o(P) M_1(P) C_{hmn} = 0 \qquad (52)$$

The boundary condition at r = R is given by $q'' = f(u)$ where f(u) is the function of the heat loss of the column and is given by equations 17 and 23. Using equation 47 and making the proper substitutions gives

$$b_1 + b_2 T_L + b_3 T_L^2 + \left[b_2 + 2 b_3 T_L\right] w + b_3 w^2$$
$$= - 2\pi R\left[a_1 + a_2 T_L + a_2 w\right] \frac{\partial w}{\partial r} \qquad (53)$$

Substitution of equations 43 and 48 into this equation gives the complete form of equation 53.

$$\left[b_1 + b_2T_L + b_3T_L^2\right] + 2\pi R\left[a_1 + a_2T_L\right]\sum_{m=1}^{\infty} M_1(R)\sin\beta_m z$$

$$+ \left[b_2 + 2b_3T_L\right]\sum_{n=1}^{\infty} N_0(R)\sin\beta_n z + b_3\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} N_0(R)M_0(R)\sin\beta_n z\,\sin\beta_m z \quad (54)$$

$$+ 2\pi R a_2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} N_0(R)M_1(R)\sin\beta_n z\,\sin\beta_m z = 0$$

As was done for equation 51, this equation is multiplied by the factor $\sin\beta_h z$ and integrated over the interval of z from 0 to L. This gives another set of equations which contain a double infinite series.

$$\left[b_1 + b_2T_L + b_3T_L^2\right]\left[\frac{2L}{\pi(2h-1)}\right] + \left[b_2 + 2b_3T_L\right]\left[\frac{L}{2}\right]H_0(R) + \pi RL\left[a_1 + a_2T_L\right]H_1(R)$$

$$+ \frac{b_3L}{2\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} N_0(R)M_0(R)C_{hmn} + a_2RL\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} N_0(R)M_1(R)C_{hmn} = 0 \quad (55)$$

Equations 52 and 55 were solved simultaneously to determine A_h and B_h. Since these two equations contain double infinite series, the solution for the values of A_h and B_h were obtained on a computer by an iterative process. In the two final equations for A_h and B_h, i.e., equations 52 and 55, the contribution of the series is small compared to the other terms in the equations. A fair approximation of A_h and B_h can be obtained by omitting the series terms and solving the two equations simultaneously. The approximation for A_h and B_h can be improved by using the previously determined values of A_h and B_h in the series terms and then solving the remainder of the equation for A_h and B_h again. This process can be repeated until the desired accuracy of A_h and B_h is obtained.

The following set of values were used for the constants in equations 52 and 55 to determine the values of A_h and B_h.

$L = 100$ cm

$P = 0.1981$ cm

$R = 0.4188$ cm

$a_1 = 0.2166$

$a_2 = -1.16 \times 10^{-4}$

$b_1 = -0.0272$

$b_2 = 5.84 \times 10^{-4}$

$b_3 = 3.12 \times 10^{-6}$

A separate set of values for A_h and B_h were obtained for each value of simulated loop temperature, T_L, and each value of heat loss from the manometer column, q''. The four sets of boundary values listed in Table 4 were used for the calculations.

Table 4. Boundary values

Set No.	T_L ^oC	q'' cal/cm-sec
1	200	0.09455
2	300	0.09455
3	400	0.1306
4	500	0.1306

Only the first twenty values of A_h and B_h were solved and these are shown in Tables 6 and 7.

The values from Tables 6 and 7 were used in equation 43 to obtain the series dolution for w(r,z). The actual theoretical temperature distribution, $u(r,z)$, was then obtained by substituting equation 43 into equation 26. The values of the temperature obtained from this equation at the outer radius $r = R = 0.4188$ cm at various points along the manometer column are given in Table 8.

VI. EXPERIMENTAL PROCEDURE

The temperature recorder used to obtain the temperatures of the various points along the column was calibrated with a potentiometer. Periodically throughout the runs the temperatures of the recorder were checked against those read by the potentiometer. The ammeters and voltmeters used for measuring the power input to the heaters were checked against a power analyzer before and after the run.

After the sodium was molten, it was forced into the manometer column from the reservoir by pressurized helium. A pressure difference of about 1 3/4 psi between the reservoir and the top of the manometer column produced a column of sodium approximately 120 cm.

The first part of the experiment consisted of determining the relationship between the heat loss from the manometer column and the temperature of the column. This was achieved by setting the heaters to obtain a constant temperature along the manometer column and determining the power input to the manometer column by reading the voltage and current to the manometer heater. The temperatures of the column were obtained from the thermocouple recorder by averaging the temperatures over five recorder cycles. The recorder printed the temperature at 24 points in a period of six minutes. This allowed enough time to assure that the temperature had reached steady state conditions. These measurements were made at column temperatures ranging from 100 to 400°C and the results are shown in Table 2 and Figure 5.

The principal part of the experiment consisted of measuring the temperature distribution along the manometer column when the temperature of the sodium in the pipe at the bottom of the manometer column was at a higher temperature than that in the column. This was done by setting the current to the manometer heater to the value obtained from Figure 5 which would give the desired manometer column temperature. The reservoir and pipe heaters were then turned up to give a desired higher temperature. The temperatures at the points along the manometer column were obtained in a manner similar to that used in the first part of the experiment. The data obtained from these measurements are shown in Table 5.

VII. DISCUSSION AND RESULTS

The results of the theoretical calculations are tabulated in Table 9 and are compared with the experimental data in Figures 6, 7, 8, and 9. Since the experimental measurements were taken on the outside of the manometer tube and the theoretical values were calculated for the outer surface of the sodium, the temperature drop across the tube wall is not taken into consideration in the comparison of the experimental and theoretical data. This temperature difference can be shown to be small in the temperature range being considered by calculating the temperature drop across the tube wall at some mean temperature using the one dimensional heat conduction equation.

$$\frac{q}{L} = -2\pi K \frac{\partial u}{\partial r} \tag{54}$$

For a mean temperature of $400^{o}C$ which is in the upper part of the temperature range being considered, and using the following set of values for the calculations, the temperature drop across the 35 mil tube wall is less than one-half of one degree centigrade.

$q/L = 0.705$ cal/cm-sec

$K = 0.0490$ cal/cm^2-sec-^{o}C/cm

$r_o = 0.476$ cm

$r_i = 0.387$ cm

The four plots of temperature vs. position along the manometer column show relatively good agreement between the experimental points and the theoretical curves over most of the range. The greatest discrepancies

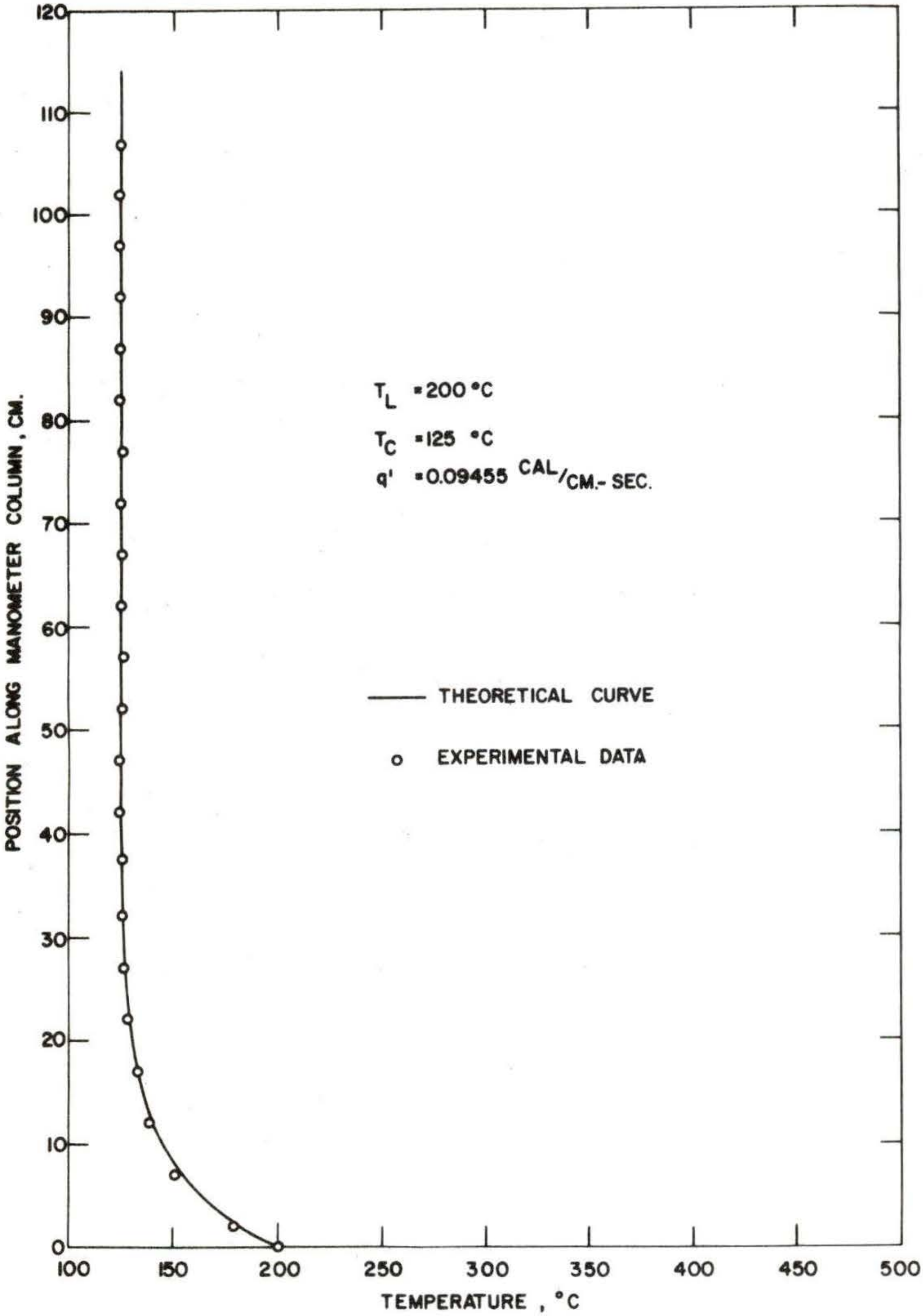

Figure 6. Temperature gradient along manometer column for Setting No. 1

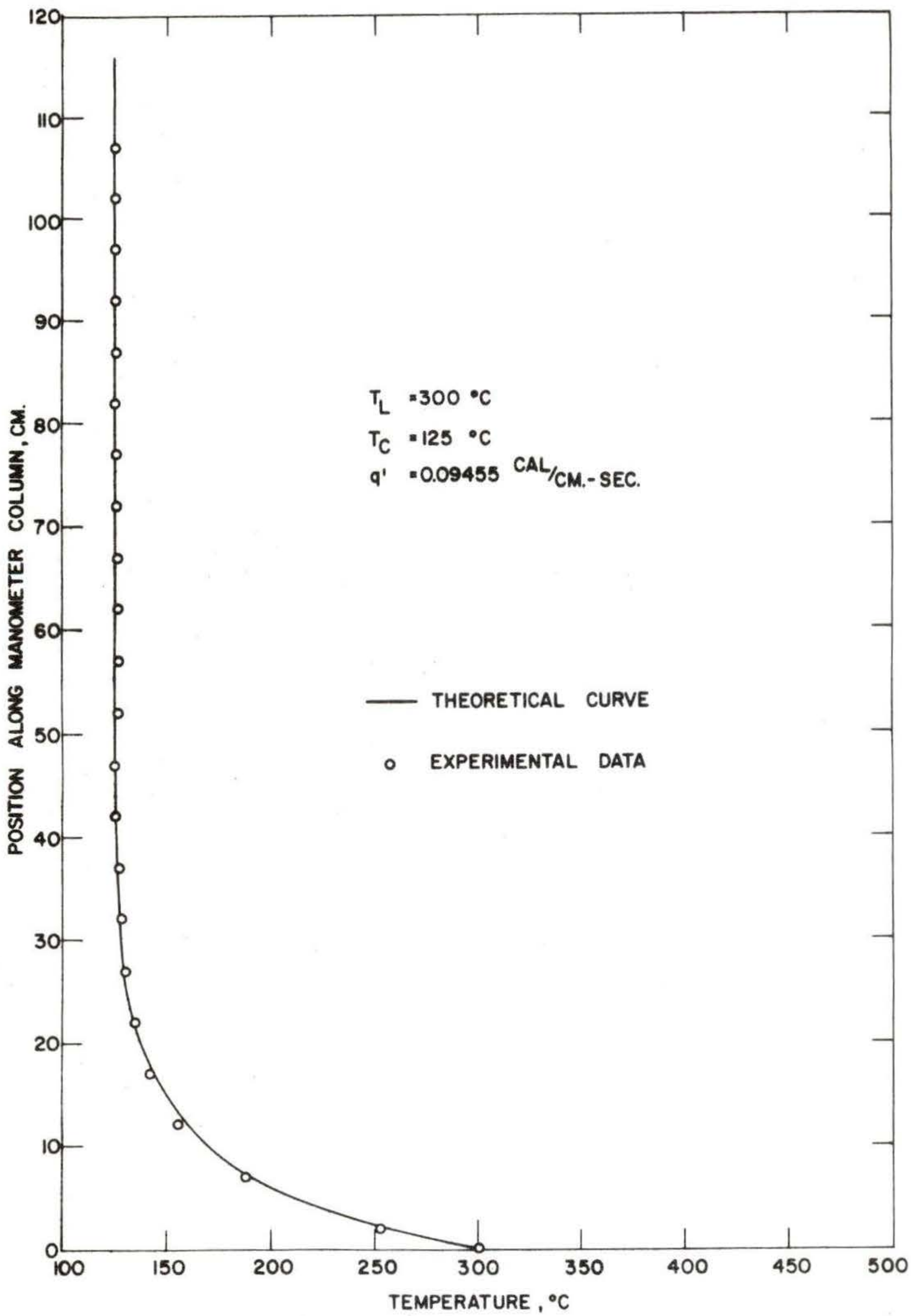

Figure 7. Temperature gradient along manometer column for Setting No. 2

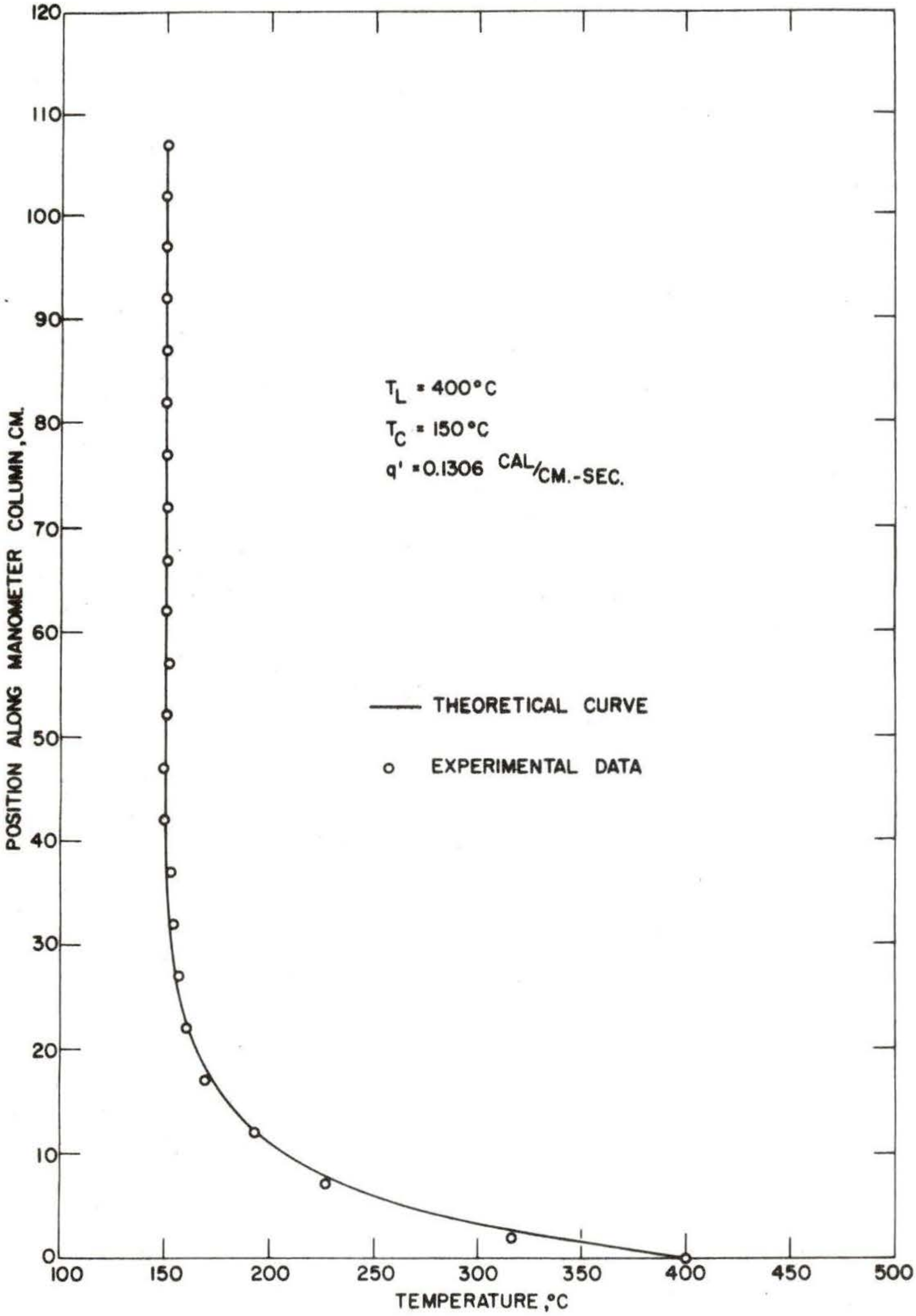

Figure 8. Temperature gradient along manometer column for Setting No. 3

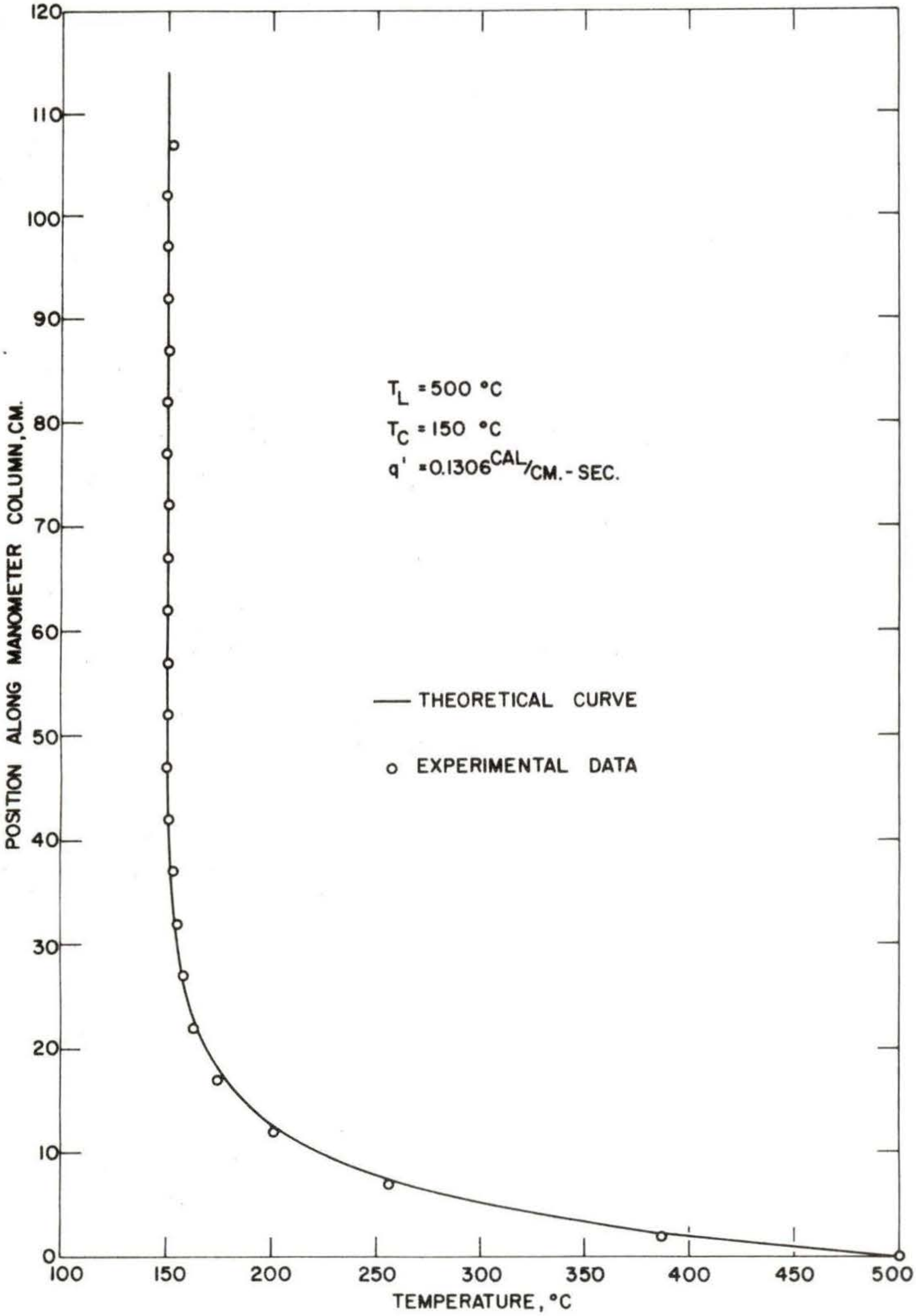

Figure 9. Temperature gradient along manometer column for Setting No. 4

appear at the low end of the column at the elevation two, seven, and twelve centimeters. The experimental temperatures are all lower than the theoretical values at these points. The difference at the two centimeter elevation is quite large. Since the other measurements agree, it is believed that this measurement was too close to the cross pipe to give an accurate temperature indication. The average permissible error in temperature was previously calculated to be about $3.7^{o}C$. The agreement between the theoretical and experimental values is within this allowable error over most of the column length and the average error would be within the allowable amount.

VIII. CONCLUSIONS AND RECOMMENDATIONS

The mathematical model used for the calculation of the temperature distribution in the manometer column gave results in good agreement with the experimental data. The two approximations, reducing the infinite series to series of twenty terms and using the iterative method for calculating A_h and B_h, were apparently satisfactory. The last iteration changed the values of A_h and B_h by less than 0.05% in all cases except for A_1 of run 4. The change was only slightly greater in this case.

It can be concluded, therefore, from the results of this investigation that the theoretical procedure used here may be used to determine the temperature distribution within the stated accuracy for loop temperatures from 200 to 500°C and for column temperatures of 125 to 150°C.

In this investigation the assumption was made that the level of the sodium in the manometer column was always considerably above the level at which the temperature gradient in the z direction became zero. Further studies could be made on the theoretical solution of the problem in which the sodium level is below the point at which the temperature gradient along the column becomes zero. This problem would involve a change in the boundary condition at z = L which in this case was

$$\left(\frac{\partial u}{\partial z}\right)_{z=L} = 0 \qquad (55)$$

The new problem would consider heat flow across the boundary at z = L.

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XI. APPENDIX

Table 5. Experimental data

Thermocouple	Position Along Column cm	Temperature, ^{o}C $T_L = 200$	$T_L = 300$	$T_L = 400$	$T_L = 500$
1	2.0	178.8	252.6	316.0	386.6
2	7.0	151.2	188.4	226.4	256.0
3	12.0	138.8	155.5	192.4	200.8
4	17.0	132.8	142.2	169.4	174.0
5	22.0	128.2	134.6	160.2	162.8
6	27.0	126.5	130.2	156.2	157.8
7	32.0	125.8	127.8	153.6	154.8
8	37.0	125.8	127.0	152.2	153.0
9	42.0	124.0	125.2	150.4	151.0
10	47.0	124.0	124.6	149.6	149.5
11	52.0	125.8	126.2	150.4	150.4
12	57.0	126.6	126.8	151.6	151.2
13	62.0	125.5	125.8	150.2	150.2
14	67.0	125.6	126.0	150.2	150.2
15	72.0	125.0	125.6	150.0	150.0
16	77.0	125.5	125.6	150.0	150.2
17	82.0	124.2	124.6	149.4	149.8
18	87.0	124.6	125.6	150.2	151.0
19	92.0	124.2	125.0	149.8	150.0
20	97.0	124.0	124.8	149.4	150.0
21	102.0	124.0	124.6	149.4	150.0
22	107.0	124.4	125.2	150.4	152.0

Table 6. Values of A_h

h	T_L = 200	T_L = 300	T_L = 400	T_L = 500
1	-94.46	-220.2	-315.1	-440.6
2	-28.18	-66.18	-96.10	-135.1
3	-14.02	-33.34	-49.49	-70.23
4	-8.000	-19.33	-29.36	-42.13
5	-4.927	-12.11	-18.78	-27.28
6	-3.211	-8.026	-12.68	-18.63
7	-2.191	-5.563	-8.925	-13.26
8	-1.553	-3.998	-6.501	-9.760
9	-1.136	-2.962	-4.870	-7.381
10	-0.8538	-2.250	-3.736	-5.709
11	-0.6556	-1.746	-2.924	-4.501
12	-0.5149	-1.379	-2.327	-3.606
13	-0.4107	-1.107	-1.880	-2.930
14	-0.3326	-0.9012	-1.538	-2.409
15	-0.2728	-0.7422	-1.273	-2.001
16	-0.2264	-0.6177	-1.063	-1.678
17	-0.1897	-0.5188	-0.8956	-1.417
18	-0.1605	-0.4392	-0.7600	-1.205
19	-0.1368	-0.3745	-0.6491	-1.030
20	-0.1175	-0.3213	-0.5575	-0.8856

Table 7. Values of B_h

h	T_L = 200	T_L = 300	T_L = 400	T_L = 500
1	0.09442	0.09376	0.1314	0.1309
2	.03054	.02904	.04068	.03920
3	.01750	.01540	.02145	.01925
4	.01190	.009500	.01302	.01035
5	.008862	.006339	.008437	.005485
6	.006990	.004455	.005678	.002571
7	.005740	.003252	.003904	.0007237
8	.004855	.002445	.002710	- .0004883
9	.004200	.001881	.001879	- .001301
10	.003697	.001475	.001284	- .001851
11	.003300	.001175	.0008509	- .002224
12	.002979	.0009485	.0005297	- .002474
13	.002715	.0007748	.0002886	- .002637
14	.002494	.0006391	.0001056	- .002737
15	.002305	.0005315	.00003443	- .002791
16	.002143	.0004449	- .0001425	- .002812
17	.002001	.0003742	- .0002264	- .002807
18	.001876	.0003155	- .0002920	- .002783
19	.001764	.0002660	- .0003436	- .002745
20	.001664	.0002239	- .0003839	- .002696

Table 8. Theoretical results

Position Along Column cm	Temperature, °C T_L = 200	T_L = 300	T_L = 400	T_L = 500
1.0	191.4	278.7	367.2	451.9
2.0	183.2	258.5	335.9	406.4
3.0	175.7	240.1	307.9	365.7
4.0	169.0	224.0	283.7	331.0
5.0	163.4	210.5	263.8	302.9
7.5	152.9	186.5	229.8	256.2
10.0	145.7	170.4	208.0	227.4
12.5	139.9	157.4	190.4	203.6
15.0	135.6	147.8	177.4	185.8
20.0	131.0	137.8	165.2	170.4
25.0	128.0	131.2	156.7	158.8
30.0	126.8	128.8	154.4	156.2
35.0	125.9	126.6	151.5	152.2
40.0	125.6	126.3	151.5	152.5
50.0	125.3	125.5	150.7	151.5
60.0	125.2	125.3	150.5	151.2
70.0	125.2	125.2	150.4	151.1
80.0	125.2	125.2	150.4	151.1
90.0	125.1	125.2	150.4	151.0
100.0	125.1	125.2	150.4	151.0