

An emissions-constrained dispatch algorithm

by

Emmanouil Vlassios Obessis

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE

Department: Electrical Engineering and Computer Engineering
Major: Electrical Engineering

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa

1992

TABLE OF CONTENTS

	LIST OF TABLES	v
	LIST OF FIGURES	vi
	ACKNOWLEDGMENTS	viii
1.	INTRODUCTION	1
	1.1 The Overall Problem	1
	1.2 Justification of this Work	3
	1.3 Scope of this Work	4
2.	BACKGROUND MATERIAL	6
	2.1 The 1990 CAA Amendments	6
	2.1.1 Title IV	8
	2.1.2 Other titles	10
	2.2 Emission Allowances	11
	2.2.1 Legislative provisions and concessions	11
	2.2.2 Emission allowances trading	14
	2.3 Compliance Strategies	15
3.	EMISSIONS-CONSTRAINED DISPATCHING TECHNIQUES	24
	3.1 Introduction	24
	3.2 Literature Review	26
	3.3 General Comments	30
	3.4 The Proposed Solution Approach	32

4.	SOLUTION OF THE EMISSIONS-CONSTRAINED DISPATCH PROBLEM	34
4.1	Introduction	34
4.2	Modeling of the Generating Units	35
4.2.1	Input-Output characteristic	35
4.2.2	Emissions modeling	38
4.3	Mathematical Formulation of the Problem	39
4.4	Definition of the Lagrangian Multipliers	42
4.5	Economic, Minimum Emission and Ecological Dispatching Cases	42
4.6	The Incremental Emissions per Incremental Cost Solution	44
4.7	Overall Solution Approach	47
4.8	Similarities of the Proposed Approach with Other Methods	49
4.9	Special Cases	52
4.10	Other Modeled Issues	53
4.10.1	Dispatching of the jointly-owned units	53
4.10.2	Startup procedure	55
4.10.3	Loss representation	56
4.10.3.1	Power losses	56
4.10.3.2	Emission losses	60
4.11	Implementation of the Algorithm	61
4.12	Advantages of the Proposed Algorithm	63
5.	NUMERICAL RESULTS	65
5.1	Introduction	65

5.2	Unconstrained Dispatches	71
5.3	Emissions-Constrained Dispatches	73
5.4	Validation of Results	81
6.	SUGGESTIONS FOR FUTURE WORK, CONCLUSIONS AND SUMMARY	85
6.1	Suggestions for Future Work	85
6.2	Conclusions and Summary	86
	BIBLIOGRAPHY	88

LIST OF TABLES

Table 2.1	Geographical distribution of the affected powerplants	9
Table 5.1	Economic modeling data	66
Table 5.2	Emissions modeling data	68
Table 5.3	Startup data	69
Table 5.4	Results for 168 hour examples: total minimization dispatches, power pool	71
Table 5.5	Typical weekly startup costs and startup fuel requirements	75
Table 5.6	Results for AP&L 24 hour example: a) 1 NO _x constraint; b) 2 NO _x constraints	76
Table 5.7	Results for AP&L 24 hour example: a) 1 SO ₂ constraint; b) 2 SO ₂ constraints	78
Table 5.8	Results for NEU 24 hour example: 2 constraints	79
Table 5.9	Results for power pool 168 hour example: 2 constraints	83

LIST OF FIGURES

Figure 1.1	Cost of pollution abatement and control in the U.S.	2
Figure 2.1	Utility emission trends	7
Figure 2.2	Estimate of NO _x emissions by source	12
Figure 2.3	Relative costs of SO ₂ emission control strategies	16
Figure 2.4	Cost effectiveness of SO ₂ emission control strategies	19
Figure 2.5	Capital cost range for NO _x emission control approaches	22
Figure 2.6	Estimate of what utilities will do to meet new clean air regulations	23
Figure 4.1	Interaction of the algorithms	36
Figure 4.2	Flowchart of the general solution	48
Figure 4.3	Economic dispatch with updated penalty factors	59
Figure 4.4	Block diagram of the solution implementation	62
Figure 5.1	Typical weekly load curves: (A) AP&L (B) NEU	70
Figure 5.2	Effect of various dispatching techniques on daily power pool SO ₂ emissions	72
Figure 5.3	Effect of various dispatching techniques on daily power pool NO _x emissions	72
Figure 5.4	Effect of loss representations on daily power pool costs	74
Figure 5.5	Effect of various NO _x limits on daily NEU NO _x emissions	76
Figure 5.6	NEU daily operating cost subject to various SO ₂ emission constraints	82

Figure 5.7 NEU daily operating cost subject to various NO_x emission constraints

ACKNOWLEDGMENTS

I would like to thank my major professor, Dr. John Lamont, for his guidance, support, and help during this research. Comments and suggestions from Dr. R. Alexander and Dr. G. Sheble', additional members of my committee, are also very much appreciated. Thanks to the fellow graduate students in the power area for the many laughs and the helpful hints in times of difficulty. Finally, I would like to thank my parents, to whom this work is dedicated; without their continuous encouragement and support, this work would have never been completed.

1. INTRODUCTION

1.1 The Overall Problem

The main objective of the electric power industry has traditionally been to generate and supply power, in order to satisfy the load, in the most economic, safe and reliable way. This situation has been altered in recent years. Social and economical trends have caused an increasing loading of the existing transmission system. Additional constraints have been imposed on utility operations and issues such as voltage stability, transient stability, and security assessment, have been of increasing concern. Addressing these constraints and concerns has increased the complexity of power system operation.

The whole situation recently became even more complex because of the increasing public concern regarding atmospheric pollution and the extent to which electric utilities contribute to it. Figure 1.1 shows the costs involved in the pollution abatement and control, during the period 1972-1987, in the U.S. As early as the 1970's, federal environmental standards were set, constraining electric companies to maximum allowable emissions rates. More recently, amendments were enacted to the 1970 Clean Air Act (CAA), which set even more stringent limitations. Because of its dependence on fossil fuels, the electric power

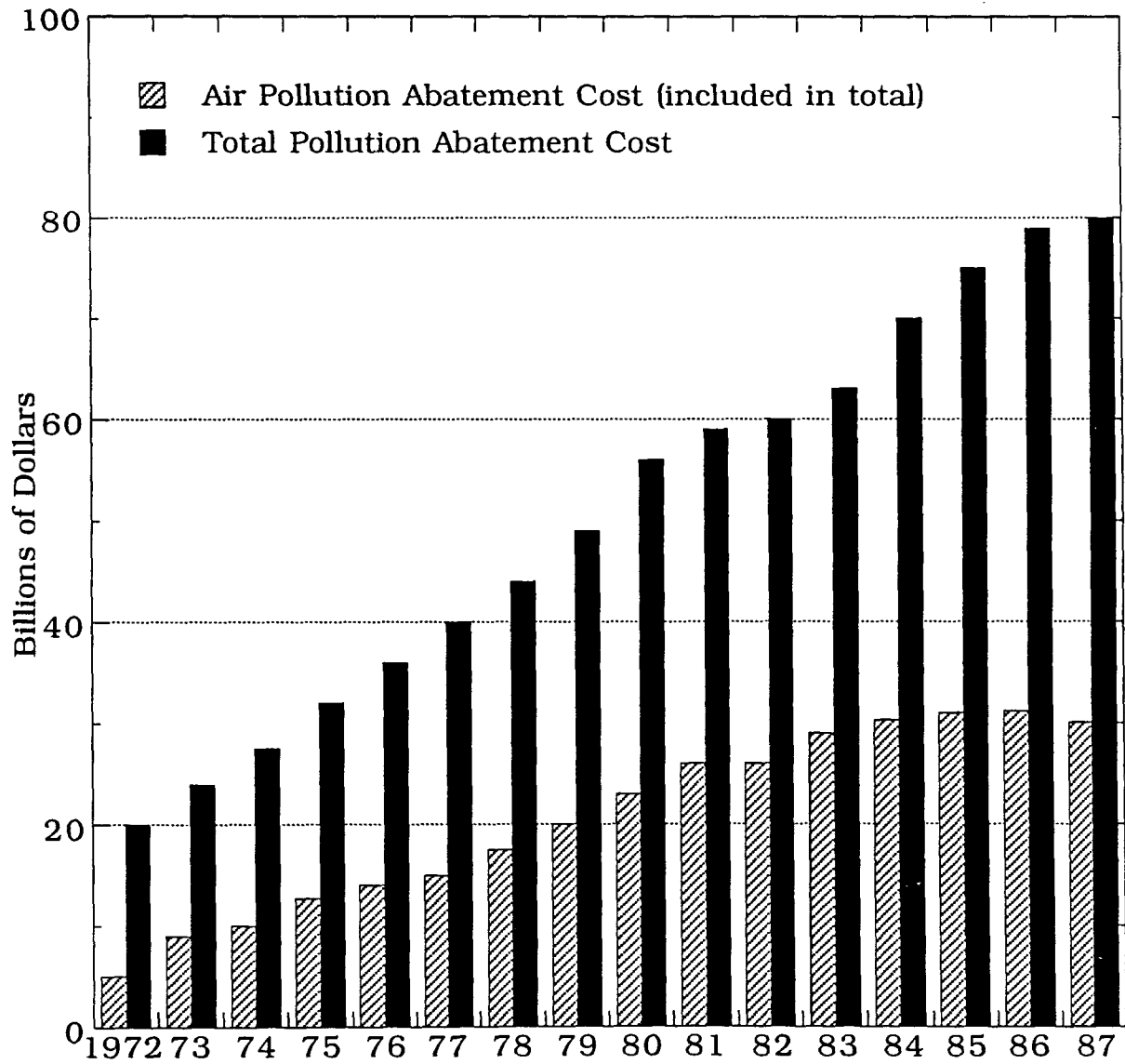


Figure 1.1 Cost of pollution abatement and control in the U.S. [1]

industry is greatly affected by the new regulations. Utilities now must deal with the environmental issues by incorporating additional constraints into the planning and operation of the interconnected system.

To further increase the problem's complexity, the new legislation is sometimes vague and several questions are still pending. It is anticipated that it will be many months before the amendments' implementation is fully defined and understood by all parties involved. The Clean Air Act Amendments (CAAA) offer the utilities wider flexibility in choosing their strategies to meet the new emission limits. Yet to date, key parameters, such as emission allowances trading prices, are still unknown, thus causing a high degree of uncertainty. Considering the uncertainty, the very tight time frame for compliance, and the big capital expenses involved, one realizes the challenges the utilities have to overcome. One also understands the industry's growing concerns regarding the issue.

1.2 Justification of this Work

It is likely that implementation of the amended CAA will cost the power industry billions of dollars. The exact figure can not be precisely predicted, but it is estimated that utilities will have to spend \$5--\$7 billion annually to achieve compliance. If federal government introduces limits on air toxic substances, an additional annual \$8 billion is

anticipated. Therefore, the utilities must decide on their solution strategies as soon as possible, to survive in an already extremely competitive market. To find the best solutions will require active involvement of both the decision makers and operating personnel.

Several compliance strategies--scrubbers, fuel switching, furnace modifications, and participation in the allowances market, just to name a few--are available for investigation and implementation. Proper evaluation of the possible options would require a significant period; this process is clearly applicable only for the long-term compliance solutions. Immediate action must be taken and the appropriate techniques must be adopted for short-term planning, as the time restrictions set by the legislation are tight.

1.3 Scope of this Work

As mentioned above, modified dispatching techniques appear to provide one tool to reduce emissions. This research focuses on a new, emissions-constrained economic dispatch. A new dispatching algorithm using a matrix formulation is presented and explained. In this algorithm, an iterative scheme, based on individual unit emission shadow prices, is employed to curtail generators in order to meet preset emission limits. Formulated as the ratio of incremental emissions over incremental cost, emission shadow prices may be otherwise defined as emission sensitivities with respect to cost. Individual unit emission rates are

taken into account. The software includes a modified search technique, inclusion of jointly-owned units (JOU's) and different transmission loss representations. Incorporated into a unit commitment program, the software may provide a more complete summary of operating costs. Since economics are one of the dominant factors in interconnected system's operation, operating cost minimization is still one of the primary objectives. Cost sensitivity with respect to system parameters, such as emission limits, was also analyzed.

In the next chapter, the necessary background material is presented. Information on the 1990 CAA Amendments and emission allowances are given. Available compliance options are also presented and discussed. Chapter 3 reviews and comments on the emissions-constrained dispatching (ECD) algorithms presented to date. In Chapter 4, the general formulation and solution of ECD is presented. Discussions follow on specific items and developments, such as the proposed incremental emissions per incremental cost method, different representations of the system losses and scheduling of JOU's. In Chapter 5, test results are provided and discussed. The test cases were run for different system configurations and for various periods. Finally, Chapter 6 contains suggestions for future work, final conclusions and a brief summary.

2. BACKGROUND MATERIAL

2.1 The 1990 CAA Amendments

Ending a legislative debate that lasted nearly a decade, the 1990 CAA Amendments (CAAA) [2,3] were signed into law on November 15, 1990. The new legislation introduces a novel approach to the way emissions are regulated. Previously, the traditional command-and-control approach virtually dictated that certain techniques be used to reduce emissions. The new bill, though tightening the federal air pollution standards, offers the electric power industry a wider flexibility in choosing their compliance strategies. This is achieved by imposing limitations on the cumulative annual tons emitted. Emission allowances (EA's) will be distributed annually by the Environmental Protection Agency (EPA). An EA provides the right to emit one ton of SO₂.

The amendments contain eleven titles, five of which affect the power utilities. Title IV--acid rain control--mandates the nitrogen oxides (NO_x) and sulfur dioxide (SO₂) reductions; it is this title that directly affects the electric power companies and has raised extensive discussions and interpretations. Figure 2.1 shows current and future utility emission (NO_x and SO₂) trends, as projected by EPRI. Four other

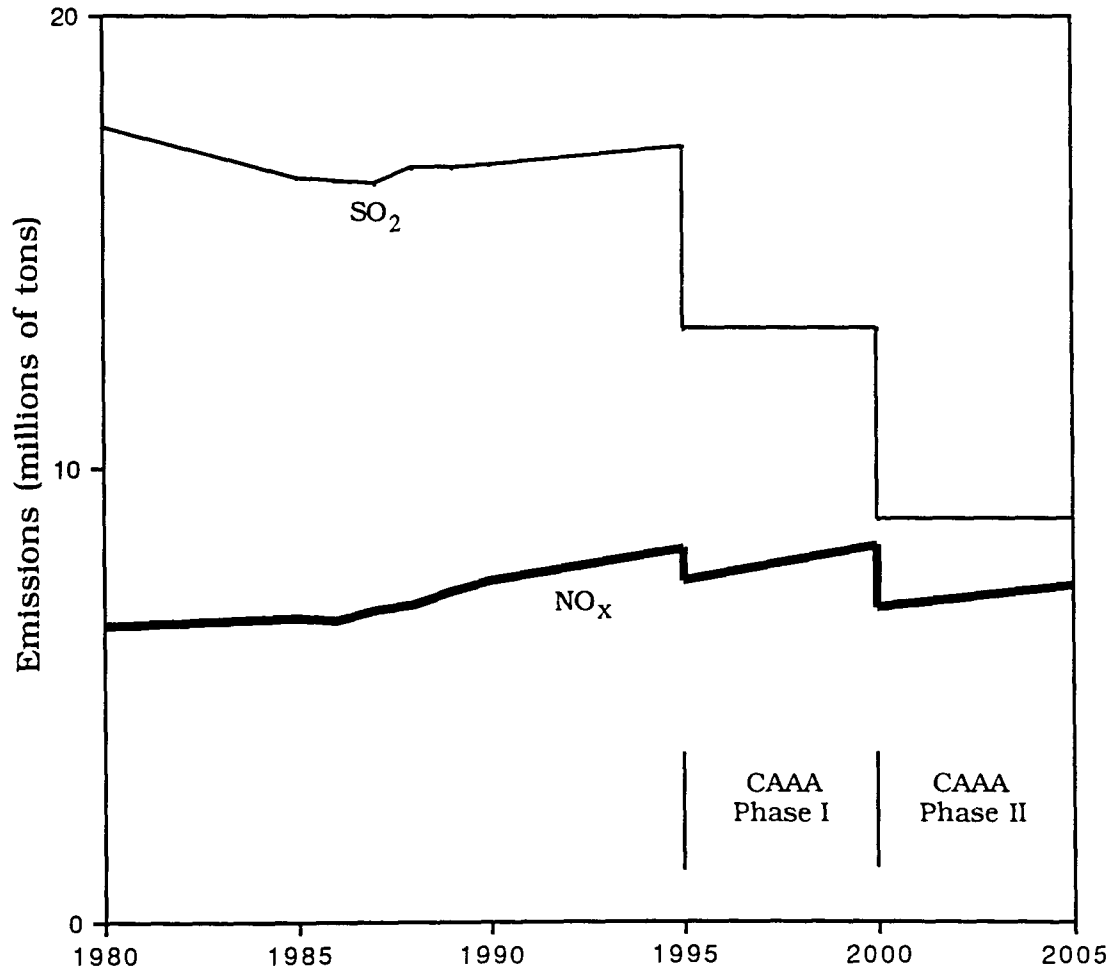


Figure 2.1 Utility emission trends [4]

titles (I--non attainment, III--air toxics regulation, V--permitting and VI--enforcement) are also anticipated to impact the power industry.

2.1.1 Title IV

This title defines a two-phase activity that, by the year 2000, will produce an SO₂ emissions reduction of 10 million tons, from the 1980 levels. After that time, a nationwide cap of 8.9 million tons per year will be effective (see Figure 2.1).

Approximately 1/3 of the SO₂ total reduction is required by January 1, 1995, when Phase I is completed. By that time, 110 plants (261 units), explicitly identified in the amendments, must reduce their emissions to an annual average rate of 2.5 lb/MBtu of input energy. Table 2.1 shows the geographical distribution of the affected plants. To avoid unpleasant surprises, the legislation mandates that the identified units must maintain their fuel input at levels equaling or exceeding their average fuel use during the period 1985-1987. The only exception is when another Phase I non-affected unit is declared as a replacement. In such case, the replacement (compensating) unit is then subject to the provisions of the law. Companies that install scrubbers on identified Phase I units to lower their emissions, will be granted a January 1, 1997, extension.

The remaining reduction will occur during Phase II that terminates on January 1, 2000. By that time, all 25 MW or greater units must reduce their SO₂ emissions to 1.2 lb/MBtu. At that time, each utility will be required to possess sufficient amount of allowances for the

Table 2.1 Geographical distribution of the affected powerplants [1]

State	powerplants affected	State	powerplants affected
Alabama	2	Mississippi	1
Florida	2	Missouri	8
Georgia	5	New Hampshire	1
Illinois	8	New Jersey	1
Indiana	15	New York	5
Iowa	6	Ohio	15
Kansas	1	Pennsylvania	9
Kentucky	10	Tennessee	4
Maryland	3	West Virginia	6
Michigan	1	Wisconsin	6
Minnesota	1		

SO₂ amounts it emits. EA's are discussed in a subsequent section of this chapter because they represent a very interesting and important issue. Several temporary exceptions are provided in the amendments. Not until the year 2010 will the new law become fully effective.

The same 261 units identified in the CAAA will have to reduce their NO_x emissions by 2 million tons during Phase I. Emissions from wall-fired units are limited to 0.5 lb/MBtu. NO_x emissions from tangentially-fired units are similarly limited to 0.45 lb/MBtu. Limits for the other types of burners will be established by 1997.

Several issues that require final EPA decisions are still pending. By 1994, the EPA must establish revised New Source Performance Standards (NSPS) for NO_x emissions. Another important issue is the possible trading between NO_x and SO₂ emissions. Although not provided by the law, an EPA study on consequences from NO_x-SO₂ trading must be sent to Congress by early 1994.

2.1.2 Other titles

Complying with Title IV provisions alone, is the wrong approach. As previously mentioned, four more titles will have an impact on the electric utilities.

The EPA will present a study on NO_x and ozone. Under Title I NO_x may be treated as a non-attainment pollutant, since the EPA considers it a precursor to ozone formation. In non-attainment areas, new units will be required to adopt lowest achievable emission rate

(LAER) technology. Furthermore, additional NO_x emissions reduction may be mandated; if this happens several units may be forced into early retirement. Figure 2.2 shows an estimate of NO_x emissions by source, in the United States, in 1982.

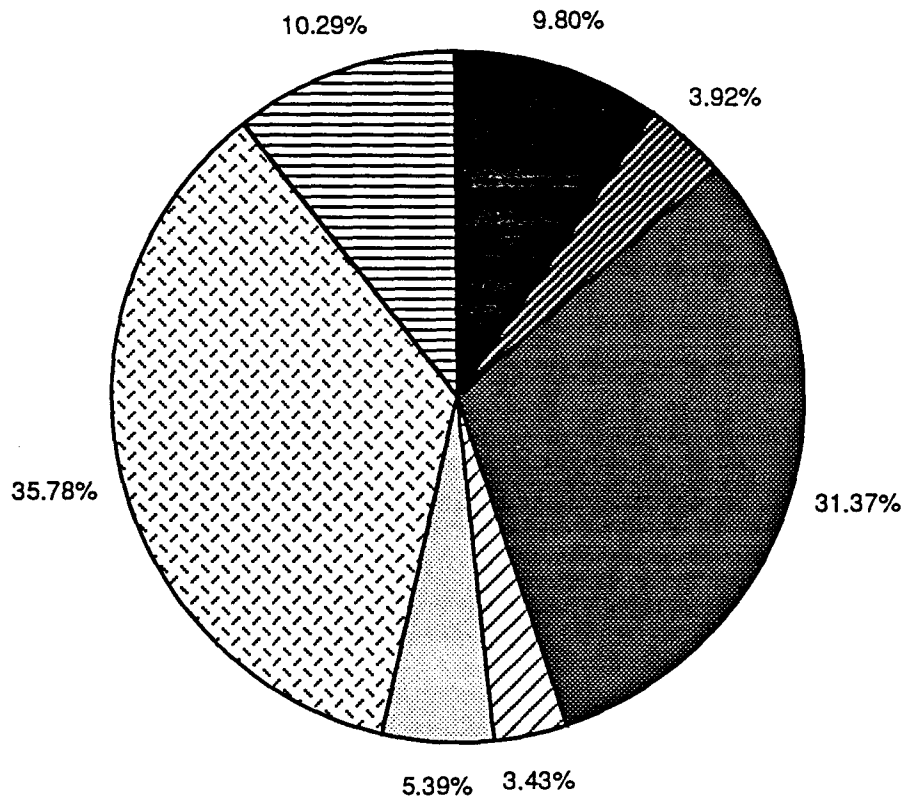
Title III regulates the air toxics. These are 189 specific substances that present a threat to the public health. Thirty seven of them, i.e. cadmium, arsenic, nickel, mercury, lead, were detected in power plant stack gas. Presently, utilities are not directly regulated under this title. The EPA will determine appropriate air toxics controls based on studies to be completed within three years. The mercury study will exceptionally require four years [6].

Titles V and VI provide means to force compliance and expand the EPA's authority to issue penalties and citations in case of violations. Specifically, plant supervisors are subject to imprisonment in case the plant violates the new regulations.

2.2 Emission Allowances

2.2.1 Legislative provisions and concessions

One EA gives the right to emit one ton of SO₂. EA's will start being distributed in 1993 when a small number will be offered in an annual auction. The EPA will distribute annually, allowances to utilities, starting in 1995. The exact number given to each utility is specified by the amendments. Allowances may be treated as a financial asset; therefore



Mobile Sources

☐ Highway Vehicles ☐ Aircraft, Trains and Other

Stationary Source Fuel Combustion

■ Coal ▨ Fuel Oil ■ Natural Gas

▨ Industrial Processes

☐ Other Miscellaneous Sources

Figure 2.2 Estimate of NO_x emissions by source [5]

they can be banked, traded or sold. EA's change the planning process, since emission reduction becomes a continuous process rather than an one-time decision. A utility may decide to overcomply, saving its EA's for sale when market conditions are favorable. In fact, the law provides bonus allowances for those sources that overcomply. With the nationwide cap on SO₂ emissions beginning the year 2000, companies must have sufficient allowances to cover the emissions of any new generation. New plants, except those already under construction, will receive no allowances. Thus, generation expansion will become more complicated, especially for those companies already operating near their maximum capacities. Other provisions are:

- High-sulfur coal producing states, including Ohio, Illinois and Indiana, are expected to be affected the most by the new law. These states will have access to a 200,000 EA's reserve. Similarly, 50,000 bonus EA's are reserved for 10 other midwestern states.
- Units whose scrubbers have efficiencies greater than 90% during Phase I earn extra allowances from a 3.5 million EA's reserve.
- Another provision withholds 2.8% of the annual EA's allocation for an EPA administered public auction. This is to ensure that each utility has access to EA's. Beginning in the year 2000, 50,000 EA's will be sold directly each year.
- Another 300,000 EA's are reserved for renewable energy programs. Units repowered during Phase II extend their deadline by a maximum of four years, and obtain non-transferable EA's during the extension.

2.2.2 Emission allowances trading

Another innovative issue is the provision for EA's trading. Markets are already being formed, where EA's will be sold, purchased or traded. The situation is not completely unprecedented as emissions trading began in 1976 in areas where air quality standards were violated. NO_x, SO₂ and particulates have been traded and the trading has definitely saved money for the utilities that had to meet ambient air quality standards (AAQS). Unknown is, how a free market, as outlined in the law, can function in the electric power industry's extremely regulated environment. There are concerns that state regulatory agencies may oppose the free EA's trading, thus prohibiting a nationwide market formation. Some state commissioners have already publicized their intentions to limit EA's trading within the state borders, arguing that allowance trading affects the future economic growth of the state [4]. Free trading supporters, on the other hand, claim that such an approach would defeat the law's intention. The National Association of Regulatory Utility Commissioners (NARUC) intends to develop formal directions that will serve as guidelines for the state regulatory bodies.

Details regarding the allowances market are still unknown. Wisconsin Power and Light sold the first emission allowances to the Tennessee Valley Authority (TVA) for an estimated \$250--\$300 per ton. It is likely that most utilities will participate in the emissions trading [7]. Commodity markets, like the Chicago Board of Trade, are already expanding their operation to include allowance trading. EA's and their

trading will be an integral part of compliance strategy decisions. This will add a marketing aspect to the ECD problem. Figure 2.3 shows how the relative costs of several compliance strategies are affected by the EA prices. Initially, utilities will proceed conservatively because of no past experience. Nevertheless, the market is anticipated to develop quickly as independent power producers (IPP's) also participate. Trade participation, though risky, has the potential of significant savings and profits for utilities prepared to "play the game."

2.3 Compliance Strategies

Two general types of compliance strategies exist; they are:

1) Management-level methods such as dispatching algorithms incorporating "environmental" constraints, energy conservation, utility-to-utility transactions, and active involvement in the EA's market.

2) Plant-level methods that modify the actual plant operating conditions. Several of these techniques are described in the remainder of this section [9,10].

Fuel switching from high- to low-sulfur coal appears to be an attractive compliance strategy, although it is considered as the conservative approach. Requiring low capital investment, it reduces emission output immediately. However, the method is not without its shortcomings. Several pieces of power plant equipment--especially the Electrostatic Precipitators (ESP)--must be modified. Furthermore, ash

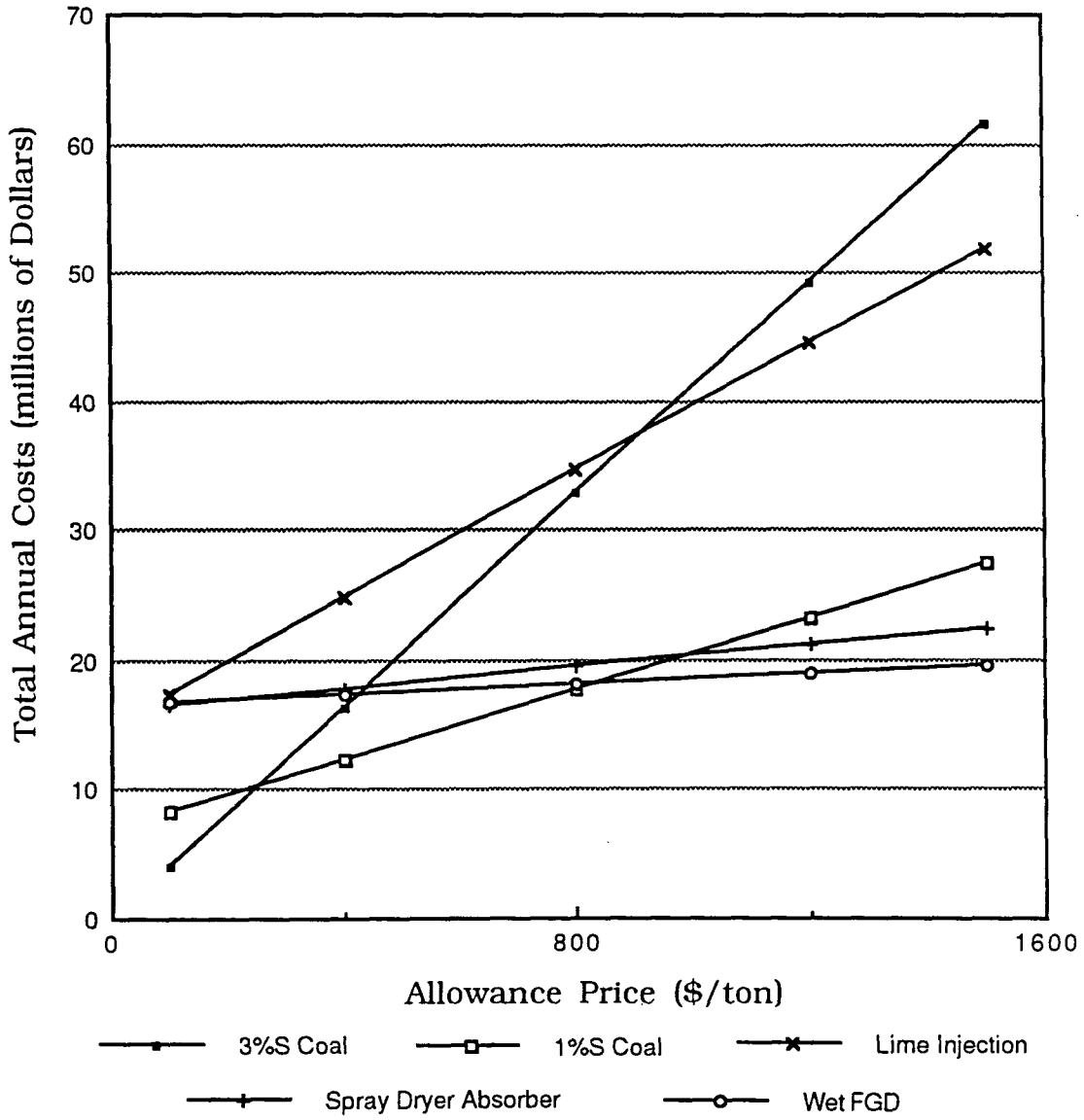


Figure 2.3 Relative costs of SO₂ emission control strategies [8]

content is usually higher in the low-sulfur coals. This fact will result in changes of the slagging, corrosion, and erosion characteristics, which will alter maintenance requirements and boiler performance. In several states, switching to low-sulfur fuel must be carefully evaluated, keeping in mind the total economic impact on both the area and the state.

Fuel blending consists of mixing high- and low-sulfur coals. To meet the new standards, at least 60--70% must be low-sulfur fuel. Additional fuel cleaning, after the preliminary processing at the minemouth, presently is not considered a competitive strategy. There are indications, however, that the necessary technology will be developed soon so as to offer another option. Switching to or cofiring natural gas is another attractive solution, but is met with scepticism because it is anticipated that gas prices would increase. Nevertheless, it is expected to play a significant role, especially during Phase II.

Flue gas desulfurization (FGD) systems (scrubbers) are the most commonly used SO₂ reduction method. No other technique can match the scrubbers' efficiency and utilities already have years of experience with them. Scrubbers are efficient (up to 99% SO₂ removal), reliable (95%), and require low power consumption (less than 1.5% of plant output). Additionally, they do not present adverse impacts on boiler or electrostatic precipitator operation. Although scrubber installation involves very high capital costs, further developments are underway that are expected to make wet scrubbing the predominant solution. A practical problem, however, involves the limited space available in

several existing units. Spray dryers and dry scrubbing are two other methods that have recently reported very high SO₂ removal percentages and very favorable economics. Use of these techniques, particularly overseas, is frequent; yet, there are several technical issues to be solved. Other low cost and low efficiency options for SO₂ removal involve the many sorbent injection processes reported to date.

All the methods mentioned in the previous paragraph, inject compounds, e.g. lime, that react with the SO₂ to produce solid waste. An associated concern is waste material processing and utilization. Utilities' preference for FGD systems can not be considered the sole solution. If the mandatory 10 million tons of SO₂ emissions reduction were achieved using only scrubbers, at least 25 million tons of solid waste would be produced annually. Since gypsum is the main waste component, developing ways of reusing this material is the main focus. Unfortunately, the wallboard industry that could absorb big quantities of gypsum, is in a severe depression. Research has been also undertaken to transform other byproducts into useful chemicals, e.g. magnesium hydroxide in the case of Mg-enhanced lime scrubbers. Figure 2.4 shows the cost effectiveness of several SO₂ emissions control strategies.

The most favorable method for NO_x compliance is the low NO_x burners (LNB's) or appropriate furnace modifications. These modifications may include flue gas recirculation (FGR), low excess air firing (LEAF), overfire air (OFA), fuel reburning, steam injection, etc. Compared to uncontrolled firing, these methods may result in reducing

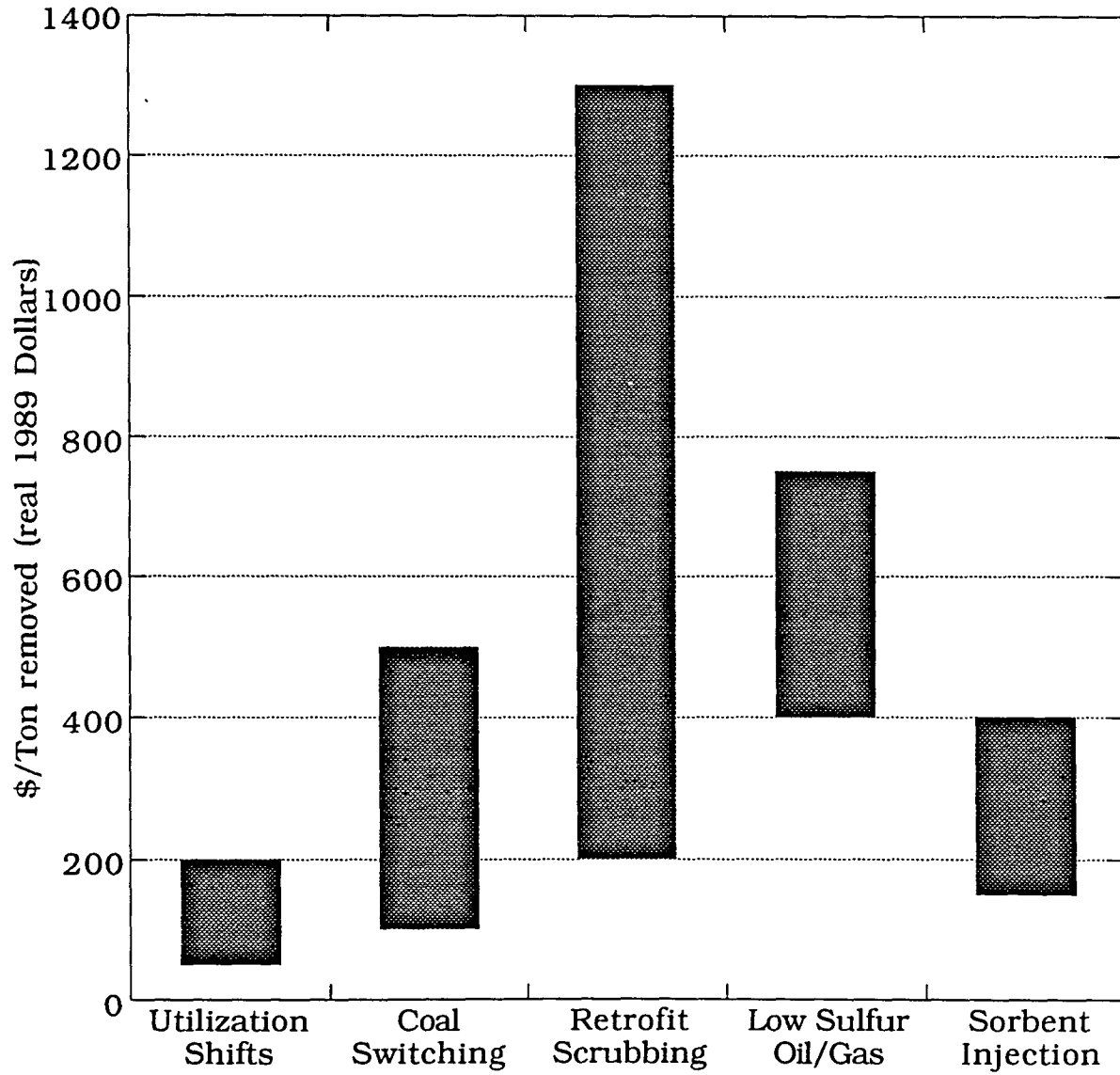


Figure 2.4 Cost effectiveness of SO₂ emission control strategies [9]

NO_x emissions up to 60%. All these approaches attempt to reduce O₂ concentrations in critical NO_x-formation zones or the amounts of fuel burned at high combustion temperatures. These methods modify the means, conditions or rates of fuel and air introduction. A unit retrofitted with LNB's will require greater attention to its operation because of increased complexity.

Thermal or selective non catalytic reduction (SNCR) processes are another way to meet the NO_x regulations. These methods involve the injection of nitrogen-rich compounds to transform NO_x into water and nitrogen. Increased CO emissions, undesirable byproduct formation, controlling unreacted ammonia (NH₃), and fly ash contamination with ammonia, are related issues that raise concerns. Recent research has reported NO_x removal of up to 80% with acceptable byproduct levels. The use of both SNCR processes and combustion modifications provides very high removal levels. Selective catalytic reduction (SCR) is another available option in which a catalyst is used to increase NO_x reduction reactions rates. It removes as much as 90% of the NO_x; it may be used in boilers of commercially available designs without modification. At least one utility has already used SCR. However, SCR has its own drawbacks. The processes are considered expensive because of the high catalyst replacement costs. Several technical deficiencies still exist. Furthermore, sulfur from high-sulfur coal greatly affects the catalyst and the processes are not well behaved. If SCR and SNCR techniques are to be used, extensive care is required to prevent ammonia concentrations

from becoming a serious polluting problem. Figure 2.5 presents the capital cost range of several NO_x emissions control strategies; efficiencies of the individual approaches are also shown.

Other more technically complicated methods are combined SO₂/NO_x removal processes and regenerable FGD (scrubber) systems. Utilities are already considering retiring units and/or repowering. The latter option has already received increased attention within the power industry and resultant high efficiency values have been reported.

It is clear that large number of alternatives exist. All the above methods should not be evaluated independently of each other. Combined use of multiple techniques may, not only provide the best results, but reduce the drawbacks of the individual methods. Figure 2.6 shows an estimate of which of the available options will utilities most likely follow, in order to comply with the new clean air requirements.

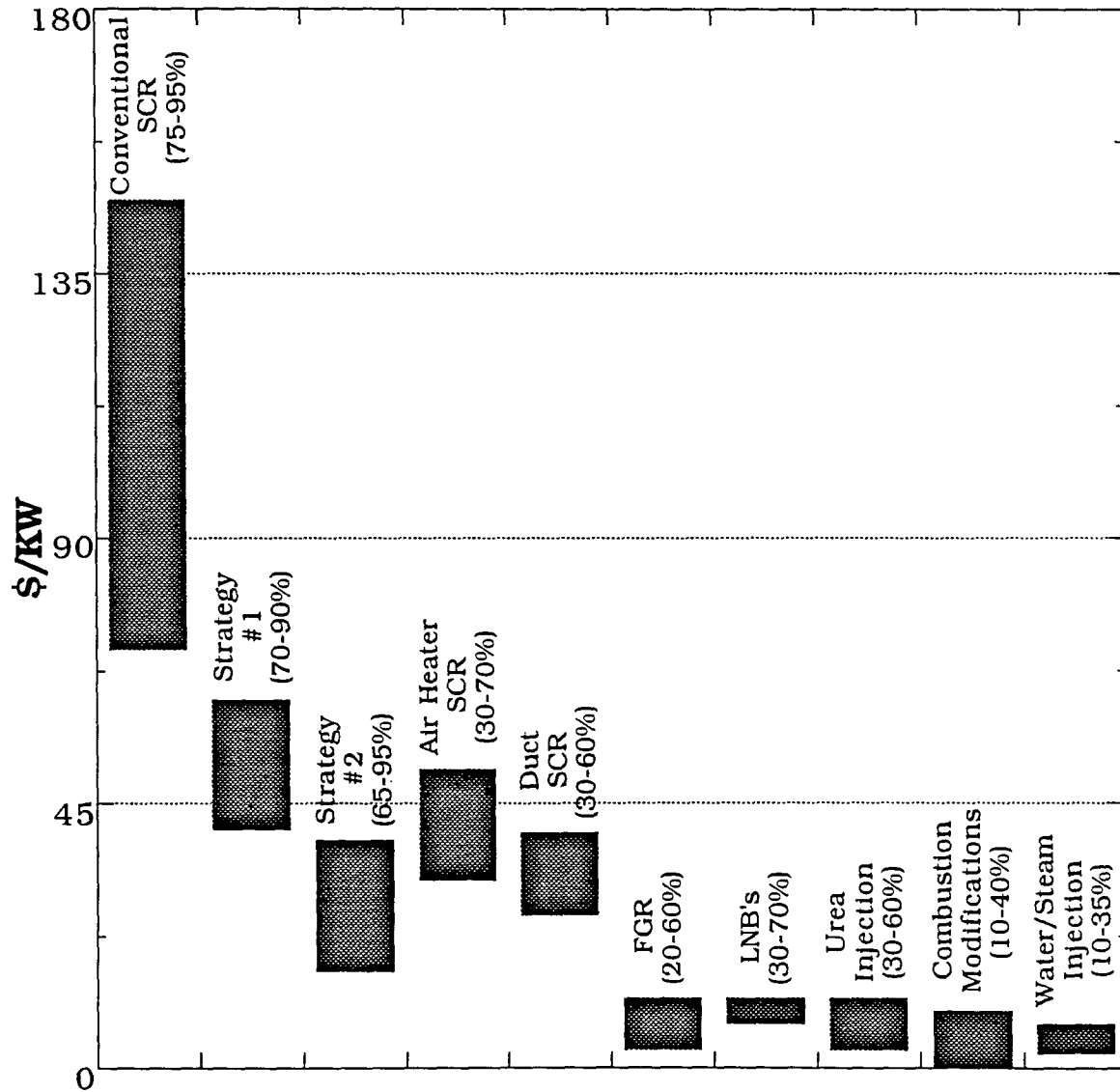


Figure 2.5 Capital cost range for NO_x emission control approaches [11]

- Notes:
1. Strategy #1 consists of urea injection and air heater SCR
 2. Strategy #2 consists of FGR, LNB and urea injection
 3. Option efficiencies are shown in parentheses

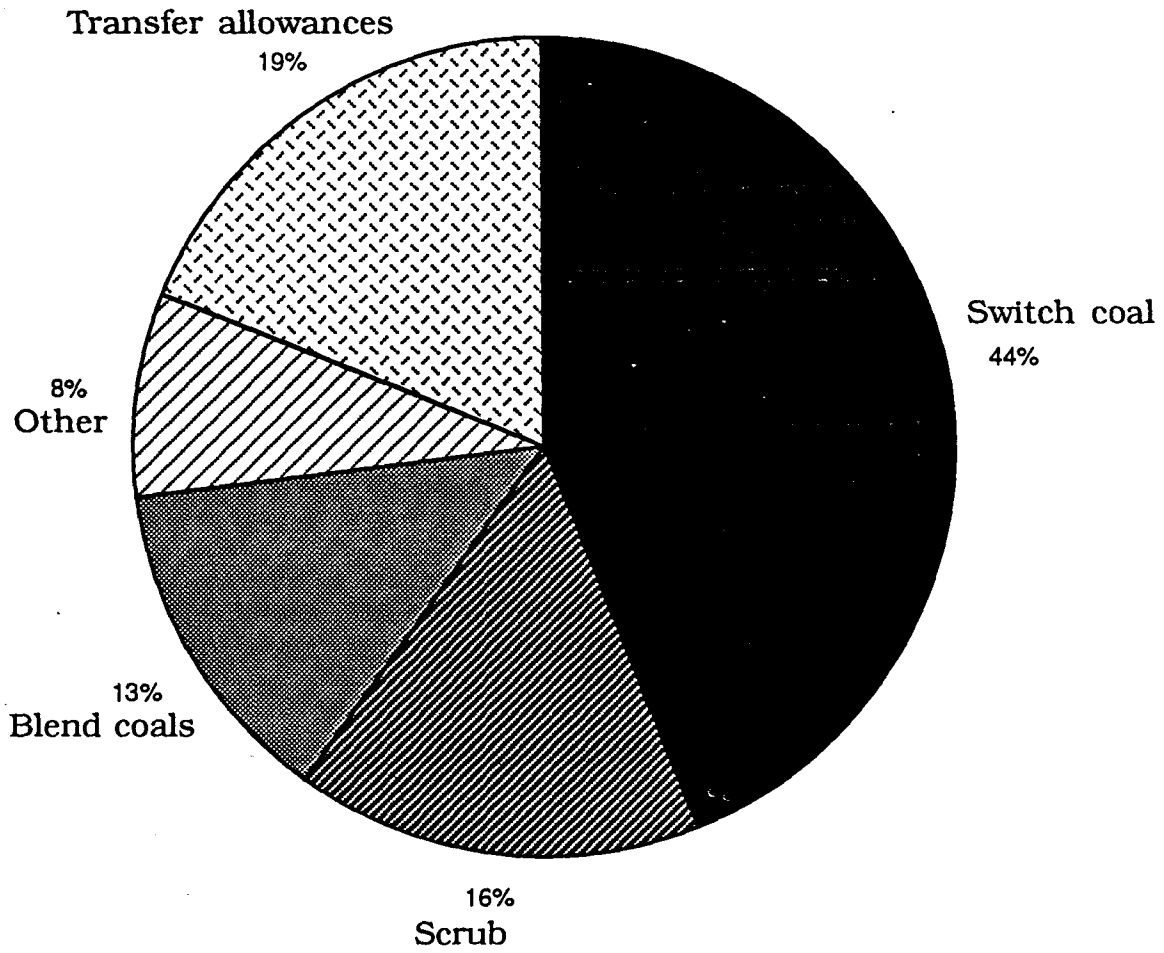


Figure 2.6 Estimate of what utilities will do to meet new clean air regulations [10]

3. EMISSIONS-CONSTRAINED DISPATCHING TECHNIQUES

3.1 Introduction

Dispatching is defined as regulatory action applied to the speed governors of the units operating within a controlled system in order to allocate generation on an optimum basis so that the load is satisfied. Although economics are still the dominant factor and minimization of the operating costs is the ultimate objective, reliability, security, emission limits and other constraints have resulted in the development of several different dispatching methodologies. It has been known for years that the equal incremental cost method, though simple, yields excellent results. Yet, several other approaches, such as mixed integer programming, dynamic programming, Lagrangian relaxation variants, linear programming techniques etc., have been also used [12,13].

The problem of dispatching generation subject to various emission constraints is not a new one. Some states imposed such constraints as early as the mid-1960's. In 1967 the Air Quality Act (AQA) was passed, and in 1970 the Clean Air Act (CAA) was signed into law. The CAA introduced primary and secondary ambient air quality standards (AAQS) nationwide and established a federal regulatory framework that required each state to develop its own state implementation plan (SIP).

To meet the AAQS, the utilities initially adopted plant-level remedies in order to limit their emission output. However, modified dispatching methods were also developed to reduce emissions.

The emission dispatching algorithms previously devised may be grouped into two distinct classes:

1) Methods minimizing emissions: The main characteristic of these techniques is that the “environmental” constraint appears in the objective function. The cost function is no longer the one to be minimized, as is the case in the conventional economic dispatch. A completely new objective function is constructed that is based solely on emission parameters and is optimized without concerns about the operating costs. The emission parameters are based either on i) stack, or ii) ground concentration measurements.

2) Methods minimizing cost subject to emission constraints: These techniques use the cost equation as the objective function and the emission limitations are modeled as additional operating constraint(s). The cost equation may consist of the

- operation cost solely
- operation cost plus an emission tax or cost. In this case, emissions are associated with monetary units and are added to the operating cost, forming a generalized cost equation.
- operation cost plus the value of the emission allowances (in the future).

In a slightly different formulation of the same problem, several objective

functions are simultaneously optimized. One is the cost equation, and the remaining ones are the emissions as functions of power outputs.

3.2 Literature Review

Minimum emission dispatch (MED), developed by Gent and Lamont [14], is probably the most well-documented technique. It minimized emissions using modifications to the traditional economic dispatch. An emission equation derived from stack gas measurements replaced the unit input cost equation. The objective function was the total emission output of the power system. The minimum emission condition occurs when all units are operating at equal incremental emission rates. The method, initially formulated to handle nitrogen oxides (NO_x), did not account for local pollution concentrations. The resulting operating costs were calculated, but not minimized.

In their second paper [15], Gent and Lamont presented the ecological dispatch that was based on then proposed emission taxes involving NO_x , SO_2 and particulates. Emissions were converted to cost values by multiplying by a cost per ton value. The combined cost function was the sum of the emission cost (emission tax) plus the fuel cost. The sum of the combined cost for all units was minimized. In the same paper, a flexible technique for composite emission reduction was presented in which, weighting function were used to provide several different objective functions.

Lamont *et al.* [16], developed an algorithm that minimized operating costs while meeting emission limits. The generating units were divided into pollution groups, depending on the individual geographical characteristics of their location. Therefore, units in urban and rural, or in coastal and inland areas were assigned different environmental limitations and were dispatched accordingly. Each pollution group was dispatched independently from the others, unless one unit belonged to more than one groups.

Sullivan [17] proposed a minimum pollution dispatch (MPD) that minimized local SO₂ ground concentrations. The objective function included SO₂ ground concentration estimates. The method used a SO₂ dispersion model developed for the TVA plants. Following the same concepts, Shepard [18] proposed minimization of fuel cost and human exposure to SO₂ at a specific geographic location. A method for developing population exposure curves was presented in the paper. Mixed integer programming (MIP) was used to find the final solution. Neither paper guarantees overall pollution minimization, and both were mostly oriented towards urban high pollutant concentrations.

Finnigan and Fouad [19] developed the economic dispatch with pollution constraints (EDPC) method. In their approach, they minimized the total cost while the emission constraints were satisfied. Two nonlinear solution procedures were described that were theoretically practical. Yet, as stated in the paper, at least one of the two was not applicable to real-time operations.

Delson [20], in his approach, used conversion factors to transform the environmental constraints into monetary units. Thus emissions were treated as economic entities. A combined cost equation was formulated as the objective function to be minimized. Since an emission market did not exist at that time, the author included a method to choose or “invent” the appropriate emission “prices”.

Economic environmental power dispatch (EPPD), developed by Zahavi and Eisenberg [21,22], was formulated as a multiobjective optimization problem. The cost equation and a combined function of the total emission output were the two objective functions. Because of the existing conflict between them, an overall minimum solution did not exist. To find a compromise, the authors used a trade-off curve to represent all possible dispatch policies and an air pollution diffusion model to evaluate each policy. A golden section based interactive method was used to calculate the final solution.

Another method, also termed minimum emission dispatch (MED), was presented by Vertis and Eisenberg [23]. The main objective of the algorithm was the minimization of the overall pollution. However, local concentrations were simultaneously handled using a dispersion model. Additionally, this technique could also account for rigid economic constraints ensuring that cost would not exceed specified limits.

Gruhl [24] considered the whole scheduling problem. He tried to optimize production and maintenance planning while meeting certain economic and environmental standards. His formulation included the

entire power system and accounted for issues such as reliability, interchange, nuclear plant requirements and hydrothermal coordination. The solution technique used was MIP. He utilized trade-off curves and surfaces to determine the optimum point from the environmental point of view.

Gruhl *et al.* [25], in another paper, discussed the supplementary control systems (SCS), which among other achievements, could control emissions as a function of meteorological conditions to meet the AAQS. The method, initially developed for SO₂, used as input, data from appropriate control devices forecasting meteorological and air quality conditions. The problem was formulated and solved probabilistically.

Cadogan and Eisenberg [26] presented a dynamic emissions management (DEM) system for the control of SO₂ emissions. Using an air pollution diffusion model and simulating several approaches previously discussed, the DEM system gave the power system operator the flexibility to choose the most suitable strategy for air pollution emergencies and contingency evaluation.

Friedmann's paper [27] reviewed the majority of the available methods to achieve pollution control. In his paper a clear distinction is made between emission and environmental control strategies. The first deal with power system emissions but the overall environment effects were not considered. These effects were included in the second class of techniques. More complex functions were used in the environmental control algorithms, but the basic problem structure was similar in every

case. The paper presents a brief yet insightful discussion of each method. The distinction between emission and environmental control strategies was also emphasized in Cadogan and Eisenberg's review paper [28], where they summarized and commented on all available pollution control approaches.

Tsuji [29] developed a method to optimize power dispatch under environmental constraints, and fuel mix. Two solution methods were presented. Another approach, particularly suitable for on-line application, based on the decoupling of the controlled parameters was also discussed.

Hobbs' paper [30] covered the underutilization provision of the 1990 CAA amendments. One more constraining equation, termed the underutilization constraint, was added to the ECD problem. A probabilistic model was formulated and solved using the Lagrangian relaxation method. Two accounting approaches, differing mainly in the time intervals considered, were used to compare the underutilization impact on costs and emissions.

3.3 General Comments

Some general comments regarding emissions-constrained dispatch algorithms should be made:

- Economic dispatch is performed approximately every five minutes in real time operation. For on-line activities, the dispatching

algorithms must i) be computationally fast and ii) have limited input data requirements.

- Dispatching is also an off-line activity consuming approximately 70% of a unit commitment program's computer time. Therefore, the dispatching algorithms must i) simulate the system operation as accurately as possible, and ii) be computationally fast.

- If the objective is to minimize emissions, there is always the risk of reducing them far below the permissible levels, yielding consequently unnecessarily increased costs.

- Assigning realistic monetary values to emissions has been so far a rather unrealistic task.

- The dispatching techniques must be flexible enough to handle all kinds of imposed limits, i.e. local or system limits, hourly or over longer periods of time. Additionally, the techniques must be capable of including various pollutant types and combinations.

- Present technology permits accurate emission measurements associated with existing weather conditions. However, it is not yet possible to allocate portions of the measured emissions to the contributing sources. Furthermore, it is entirely possible that some sources are not yet identified, therefore not yet modeled. Other sources are activities of completely random nature. Thus, conclusions based on such measurements are not to be readily generalized.

- Global minimization of emissions does not necessarily imply local minimization. Quite to the contrary, algorithms minimizing the overall emissions have had adverse effects on local concentrations.

- Emission allowance prices will be a very important factor in the dispatching process. The formation of the EA's market will add to the complexity of the problem. Dispatching algorithms must accommodate the new parameters and include the continuously increasing number of utility-to-utility daily transactions.

3.4 The Proposed Solution Approach

Despite the fact that dispatching methods have limited emission reduction capabilities, a combination of modified dispatch techniques, management-level strategies, and plant-level modifications will be used to comply with the new requirements. Yet, the previous section's comments must be considered in developing robust and flexible dispatch algorithms.

The proposed approach, described in the next chapter, possesses several very attractive characteristics. Within the short-term planning horizon, it can handle local or entire system limitations over short or longer time periods. It requires very limited initial data and does not include complicated air dispersion models or meteorological information. The primary objective is to optimize operating costs while not exceeding the imposed emission limits. Computer implementation

has shown fast solution times, making the proposed approach attractive for on-line application. Although initially developed as a short-term compliance solution, it is applicable to the medium-range timeframe using its convenient matrix formulation and parallel processing.

The proposed method provides a limited-emission minimum-cost dispatch. Emission limits are met by an iterative scheme based on emission shadow prices. As it will be shown in the next chapter, emission shadow prices are formulated as the ratio of incremental emissions over incremental cost. The developed software models economic dispatch, minimum emission dispatch, economic dispatch including EA's values, and limited emission dispatch.

4. SOLUTION OF THE EMISSIONS-CONSTRAINED DISPATCH PROBLEM

4.1 Introduction

Generation scheduling is one of the most important power system operation activities. Depending on the time horizon considered, it may be a long-term (monthly to several years), mid-term (weekly to monthly) or short-term (daily to weekly) activity. The algorithm presented is primarily developed for short-term operational planning. The time period considered extends up to one week (168 hours). The emissions-constrained dispatch (ECD) program schedules generation to meet the forecasted load while satisfying the imposed environmental constraints, and accounting for the system physical limitations and the availability of the individual generating units. Since ECD is primarily an economics optimization problem, it results in the most inexpensive power allocation among the available units.

The ECD algorithm presented consists of two parts:

1) Initialization--Verification: Initial values are chosen or computed for all variables. All the imposed emission constraints are checked to determine they are i) satisfied by the initial power output results (no need exists for modified dispatching), ii) achievable using a modified dispatching scheme, or iii) not achievable because of physical system limits.

II) Solution: An iterative scheme, cycling between two algorithms, is used to solve the problem. Based on the individual operating characteristics of each unit, the *curtailing algorithm* successively alters specific unit operating parameters (e.g. power limits, fuel pseudovalues), until the emission constraints are satisfied. Using the updated parameters, the *economic dispatch algorithm* allocates power among the operating units to meet the load in the most economical way without violating the specified operating constraints.

Figure 4.1 shows the interaction of the proposed algorithms.

4.2 Modeling of the Generating Units

4.2.1 Input-Output characteristic

A fossil generating unit's fuel hourly input is expressed as a function of the power output, $F_i(P_i)$. This expression is referred to as the incremental heat rate or input-output (I/O) characteristic. Several mathematical functions have been used for F_i , such as exponential, polynomial etc. The most commonly used are the reduced cubic representation (eq. 4.1), the quadratic representation (eq. 4.2), and the piecewise segmentation using various polynomial representations for each segment.

$$F_i(P_i) = a_i + b_i P_i + d_i P_i^3 \quad (4.1)$$

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (4.2)$$

where $F_i(P_i)$ = fuel input (MBtu/hr)

P_i = power output (MW)

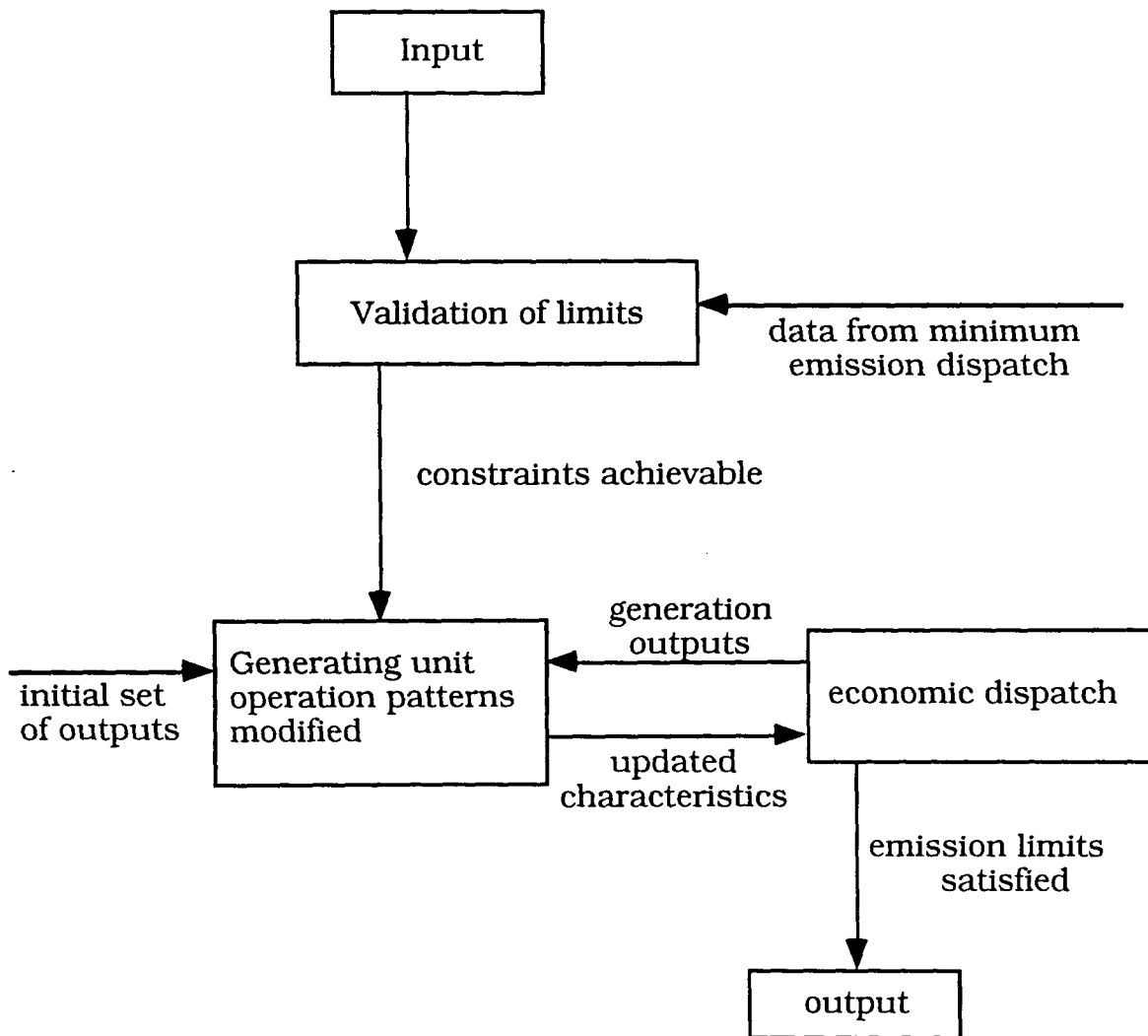


Figure 4.1 Interaction of the algorithms

i = generator index

a_i, b_i, c_i, d_i = input-output characteristic coefficients.

The input-output equation coefficients may be obtained from curve-fitting measured data or typical design data, using the appropriate polynomials.

The incremental fuel rate characteristic, $IF_i(P_i)$, is the first derivative (the slope) of the I/O equation. If $F_i(P_i)$ is described by a reduced cubic equation, then IF is of the form

$$IF_i(P_i) = \frac{dF_i}{dP_i} = b_i + 3d_iP_i^2 \quad (4.3)$$

where $IF_i(P_i)$ = incremental fuel rate (MBtu/hr).

The main advantage of the reduced cubic formulation is that its first derivative is a monotonically increasing, nonlinear function, which shows the actual incremental fuel rate nonlinearities better. The quadratic term is omitted because it is usually negative; thus possible negative slope regions are avoided. In the remainder of this document, I/O unit characteristics are assumed to be described by reduced cubic polynomials.

Multiplying the I/O equation by the fuel price, gives an equation, $C_i(P_i)$, which provides the hourly fuel cost as a function of the power output. Since fuel prices include prorated maintenance and operation costs, $C_i(P_i)$ actually relates the hourly unit operating costs to the generated power. The incremental cost equation, $IC_i(P_i)$ or $\lambda_i C$, is defined as the first derivative of $C_i(P_i)$ with respect to P_i

$$\lambda_i C = IC_i(P_i) = \frac{dC_i}{dP_i} = fp_i \frac{dF_i}{dP_i} = fp_i(b_i + 3d_iP_i^2) \quad (4.4)$$

where $C_i(P_i)$ = operating cost (\$/hr)

$IC_1(P_i) = \lambda_1 C = \text{incremental cost (\$/hr)}$

$fp_1 = \text{fuel price (\$/MBtu)}$.

4.2.2 Emissions modeling

Certain emissions (e.g. SO_2) are directly related to the fuel consumed and, as such, are the product of the unit I/O equation and an emission factor

$$E_i(P_i) = ef_i F_i(P_i) \quad (4.5)$$

where $E_i(P_i) = \text{emission output (ton/hr)}$

$ef_i = \text{emission factor (ton/MBtu)}$.

The emission factor is usually determined from actual measurements. It depends on plant design parameters, fuel quality, and emission types considered. SO_2 is the result of a sulfur (fuel) and oxygen (atmosphere) chemical reaction. The amount of SO_2 that exits the stack is dependent on the percent of sulfur in the input fuel, the ESP and DFG efficiencies, and the molecular weight ratio of the compounds involved in the reaction. Appropriate conversion factors are included in the emission factor. Similar factors can be developed for particulates.

Other emissions (e.g. NO_x) are combustion process dependent and, in general, can not be described accurately using the unit I/O equation. NO_x emissions may be expressed as a function of the power output similar to the I/O curve. A reduced cubic equation is used throughout the remainder of this document to represent the emission output

$$E_i(P_i) = A_i + B_i P_i + D_i P_i^3 \quad (4.6)$$

where $E_i(P_i)$ = emission output (ton/hr)

A_i, B_i, D_i = emissions characteristic coefficients.

The coefficients of $E_i(P_i)$ can be obtained by curve-fitting actual stack gas measurements, or by using typical design and operating parameters.

In either case, incremental emissions, $IE_i(P_i)$ or λ_{iE} , are defined as the first derivative of $E_i(P_i)$ with respect to P_i , and are described by eqs. 4.7 and 4.8 respectively

$$\lambda_{iE} = IE_i(P_i) = \frac{dE_i}{dP_i} = ef_i \frac{dF_i}{dP_i} = ef_i(b_i + 3d_i P_i^2) \quad (4.7)$$

$$\lambda_{iE} = IE_i(P_i) = \frac{dE_i}{dP_i} = B_i + 3D_i P_i^2 \quad (4.8)$$

where $IE_i(P_i) = \lambda_{iE}$ = incremental emissions (tons/hr).

4.3 Mathematical Formulation of the Problem

In economic dispatch, the objective is to minimize the total operating costs subject to a set of constraints. The fundamental constraint of all dispatching methods is that the generated power equals the sum of the system load plus the associated transmission losses. Each generating unit must operate within the manufacturer's specified limits. Furthermore, unit emission may not exceed a maximum permissible value. Emission limits may be applicable to the entire system or a group of units. Individual generating units may be limited by one or more constraints effective over the same or different periods. For a n-unit system, the problem can be expressed mathematically as follows:

$$\text{minimize } C_s = \sum_{i=1}^n C_i(P_i) \quad (4.9)$$

subject to

$$\sum_{i=1}^n P_i = P_{\text{LOAD}} + P_{\text{LOSSES}} \quad (4.10)$$

$$P_{i \text{ min}} \leq P_i \leq P_{i \text{ max}} \quad (4.11)$$

and one or more constraints of the form

$$\sum_t^m \sum_j^k E_j(P_j) \leq L_{\text{max}} \quad (4.12)$$

where C_s = total operating cost (\$/hr)

L_{max} = emission limit (ton)

P_{LOAD} = system load (MW)

P_{LOSSES} = transmission losses (MW)

$P_{i \text{ max}}$ = upper operating limit (MW)

$P_{i \text{ min}}$ = lower operating limit (MW)

i = index of generators

j = index of constrained generators

k = number of constrained generators

m = number of intervals in the constraining time period (hr)

n = number of generators in the system

t = index of time periods (hr).

In inequality (4.12), the first sum covers the applicable time period while the second sum covers the constrained units. Depending on the emission limit L_{max} , the constraint may limit a single unit ($k=1$), a group of units ($1 < k < n$), or the entire system ($k=n$). L_{max} may be expressed either as

a maximum permissible level or as a required percent reduction from the unconstrained case. Since the higher the emissions the lower the cost, eq. (4.12) is actually treated as an equality constraint. In other words, one does not wish to reduce the emissions below the maximum permissible levels because of the associated increase of the system cost.

Assuming that the power outputs lie within the operating limits (inequality constraint (4.11) neglected) and neglecting initially the transmission losses, one forms the Lagrangian function to be minimized

$$\mathcal{L} = \sum_{i=1}^n C_i(P_i) + \lambda(P_{\text{LOAD}} - \sum_{i=1}^n P_i) + \mu(L_{\text{max}} - \sum_t^m \sum_j^k E_j(P_j)) \quad (4.13)$$

where λ, μ = Lagrangian multipliers.

Assuming that the constraining period is 1 hour ($m=1$) and the emission constraint limits the entire system ($k=n$), applying the Kuhn-Tucker conditions yields

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial \mathcal{L}}{\partial \lambda} = \frac{\partial \mathcal{L}}{\partial \mu} = 0 \quad (4.14)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_{\text{LOAD}} - \sum_{i=1}^n P_i \quad (4.15)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = L_{\text{max}} - \sum_{i=1}^n E_i(P_i) \quad (4.16)$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial C_i}{\partial P_i} - \lambda - \mu \frac{\partial E_i}{\partial P_i} \quad (4.17)$$

For a system with n units, eqs. (4.15)--(4.17) are a set of $n+2$ nonlinear equations with $n+2$ unknowns (n power outputs, λ , and μ).

4.4 Definition of the Lagrangian Multipliers

“The Lagrange multipliers associated with a constrained minimization problem have an interpretation as prices, similar to the prices associated with constraints in linear programming. In the nonlinear case the multipliers are associated with the particular solution point and correspond to incremental or marginal prices, that is prices associated with small variations in the constraint requirements” [31]. Other names for the multipliers include shadow prices, imputed values, marginal values and incremental values. The multipliers method is a special case of the larger class of methods, termed penalty function methods, applied to solve constrained minimization problems.

In the emissions-constrained dispatch case, μ is seen as a penalty cost per unit emission and $1/\mu$ is regarded as the rate of emission change per unit cost.

4.5 Economic, Minimum Emission and Ecological Dispatching Cases

The well-known equal incremental cost criterion is derived by applying the Kuhn-Tucker conditions to the optimization problem described by eqs. (4.9)--(4.11). Initially, emission limits and power losses will be neglected. The operating costs are minimized when all generators within the system are operating at an equal incremental cost, λ_C

$$\lambda_C = \frac{dC_i}{dP_i} = fp_i \frac{dF_i}{dP_i} = fp_i(b_i + 3d_i P_i^2) \quad \forall i \quad (4.18)$$

where λ_C = system incremental cost (\$/hr).

If the objective is to minimize emission outputs, neglecting costs, the problem is formulated as

$$\text{minimize } E_s = \sum_{i=1}^n E_i(P_i) \quad (4.19)$$

where E_s = total system emissions (ton/hr),

subject to the constraining equations (4.10)--(4.11). The optimum is again determined by applying the Kuhn-Tucker conditions. The equal incremental emissions criterion states that emissions are minimized when all units are operating at an equal incremental emissions, λ_E . For the two emission representations discussed previously, λ_E is given by

$$\lambda_E = \frac{dE_i}{dP_i} = e f_i \frac{dF_i}{dP_i} = e f_i (b_i + 3d_i P_i^2) \quad \forall i \quad (4.20)$$

$$\lambda_E = \frac{dE_i}{dP_i} = B_i + 3D_i P_i^2 \quad \forall i \quad (4.21)$$

where λ_E = system incremental emissions (ton/hr).

The emissions-constrained dispatch problem may degenerate to either of the two dispatching cases, and be solved accordingly. Generally, it is a compromise between the economic and the minimum emission dispatches.

The ecological dispatch associates emissions with monetary values and formulates a combined cost equation. In this case, the problem may be modeled as

$$\text{minimize } C_{Gs} = \sum_{i=1}^n C_i(P_i) + \sum_{i=1}^n e p_i E_i(P_i) \quad (4.22)$$

where C_{Gs} = total system combined cost (\$/hr)

ep_i = conversion factor (\$/ton),

subject to the constraining equations (4.10)--(4.11). The conversion factors control emissions, acting like penalty factors. If emission limits are violated, very large penalty factors will unload the most polluting units, lower emissions, and reduce the total combined cost. The ecological dispatch originally considered an anticipated emission taxation to convert emissions into economic entities [15,32]. However, the recently approved emission allowances will provide a pseudo monetary value for emissions. The optimum conditions again may be derived by applying the Kuhn-Tucker criterion. The combined cost is minimized when all units are operating at an equal incremental combined cost.

4.6 The Incremental Emissions per Incremental Cost Solution

As previously mentioned, the economic dispatch is the method for solving the problem defined by eqs. (4.9)--(4.11). The ECD problem solution is obtained by enforcing the emission constraint(s) (4.12) during the economic dispatch solution. Certain operating parameters of the unit representations are iteratively modified or replaced by pseudovalues in order to reallocate the demand such that a minimum cost solution is achieved without violating any emission limitations. Assuming that the fuel input equation coefficients and the demand are not variable, two parameters that may be changed are units' maximum power limits and the

fuel prices. Modifying either variable provides a way to reschedule the units to meet the emission limit(s).

A new parameter is defined as the ratio of the incremental emissions to incremental cost. It will be called incremental emissions per incremental cost, λ_{iEC} , and is given by

$$\lambda_{iEC} = \frac{\partial E_i / \partial P_i}{\partial C_i / \partial P_i} = \frac{\partial E_i(P_i)}{\partial C_i(P_i)} = \frac{\lambda_{iE}}{\lambda_{iC}} = K_i(P_i) \quad (4.23)$$

where λ_{iEC} = incremental emissions per incremental cost (ton/\$).

This new parameter identifies the units that would produce the largest emissions reduction per unit cost. In other words, the units whose emission output decrease would cause the least increase in system cost. In optimization theory terms, λ_{iEC} are the shadow prices of the unit emission outputs. Tsuji's approach [29] to optimal power dispatch is partially based on similarly rationalized Lagrangian multipliers.

Based on λ_{iEC} , a two-loop iterative scheme is used to solve the ECD problem. In each iteration, units whose λ_{iEC} values are greater than an updated threshold value $\lambda_{EC}^{(k)}$ (k denotes the iteration number), have their power outputs and resultant emissions reduced. Two specific cases exist:

- i) The first case is when the emissions are directly fuel input dependent, as described by eq. (4.5). In this case, λ_{iEC} reduces to the ratio of the emission factor to fuel price

$$\lambda_{iEC} = \frac{\lambda_{iE}}{\lambda_{iC}} = \frac{ef_i(b_i + 3d_iP_i^2)}{fp_i(b_i + 3d_iP_i^2)} = \frac{ef_i}{fp_i} \quad (4.24)$$

The fuel prices of the units, whose λ_{iEC} values are the largest, are replaced by pseudo fuel prices computed by

$$fp_i = \frac{ef_i}{\lambda_{EC}^{(k)}} \quad (4.25)$$

The fuel prices of the remaining units are not modified. Finally, all units are rescheduled using conventional economic dispatch methods.

- ii) The second case is when emissions are not a direct function of the fuel input rate and can be described by eq. (4.6). In this case, λ_{iEC} is a function of unit power output

$$\lambda_{iEC} = \frac{\lambda_{iE}}{\lambda_{iC}} = \frac{(B_i + 3D_i P_i^2)}{fp_i(b_i + 3d_i P_i^2)} \quad (4.26)$$

The units, whose λ_{iEC} values exceed a threshold value $\lambda_{EC}^{(k)}$, have their outputs reduced to the power levels corresponding to $\lambda_{EC}^{(k)}$. These levels are computed if eq. (4.26) is solved for P_i

$$3(d_i fp_i \lambda_{EC}^{(k)} - D_i) P_i^2 + (b_i fp_i \lambda_{EC}^{(k)} - B_i) = 0 \quad \Rightarrow$$

$$P_i = \sqrt{\frac{B_i - b_i fp_i \lambda_{EC}^{(k)}}{3(d_i fp_i \lambda_{EC}^{(k)} - D_i)}} \quad (4.27)$$

These units may be represented as either fixed output units or units whose maximum power output is restricted to the eq. (4.27) value. The system is then dispatched by conventional economic dispatch methods.

Dispatching with the modified parameters will yield a different power allocation. The procedure iterates until the solved economic dispatch

provides an emissions total equal to the maximum permissible level. In each iteration, the threshold value $\lambda_{EC}^{(k)}$ is updated using an iterative search method. The ECD problem solution is obtained when the unit power outputs satisfy the constraints (4.10)--(4.12).

4.7 Overall Solution Approach

A flowchart of the proposed solution approach is shown in Figure 4.2. The overall solution proceeds as follows:

- Perform a conventional economic dispatch to obtain initial power output values.
- Compute the resultant emissions. If the emissions are less than the permissible level, then the environmental constraints are satisfied, and the problem is solved. Otherwise, a minimum emission dispatch is performed to determine if the emission limits are achievable. If they are not, the problem does not have a feasible solution and the procedure halts.
- Compute each unit's incremental cost, incremental emissions and their ratio. If the emissions are directly dependent on the incremental fuel rate curve, then the pseudo fuel price method is used. If, the emissions are directly independent of the incremental fuel rate curve, the maximum power limit method is used. For each hour, λ_{iEC} values are computed and stored in a matrix. The matrix

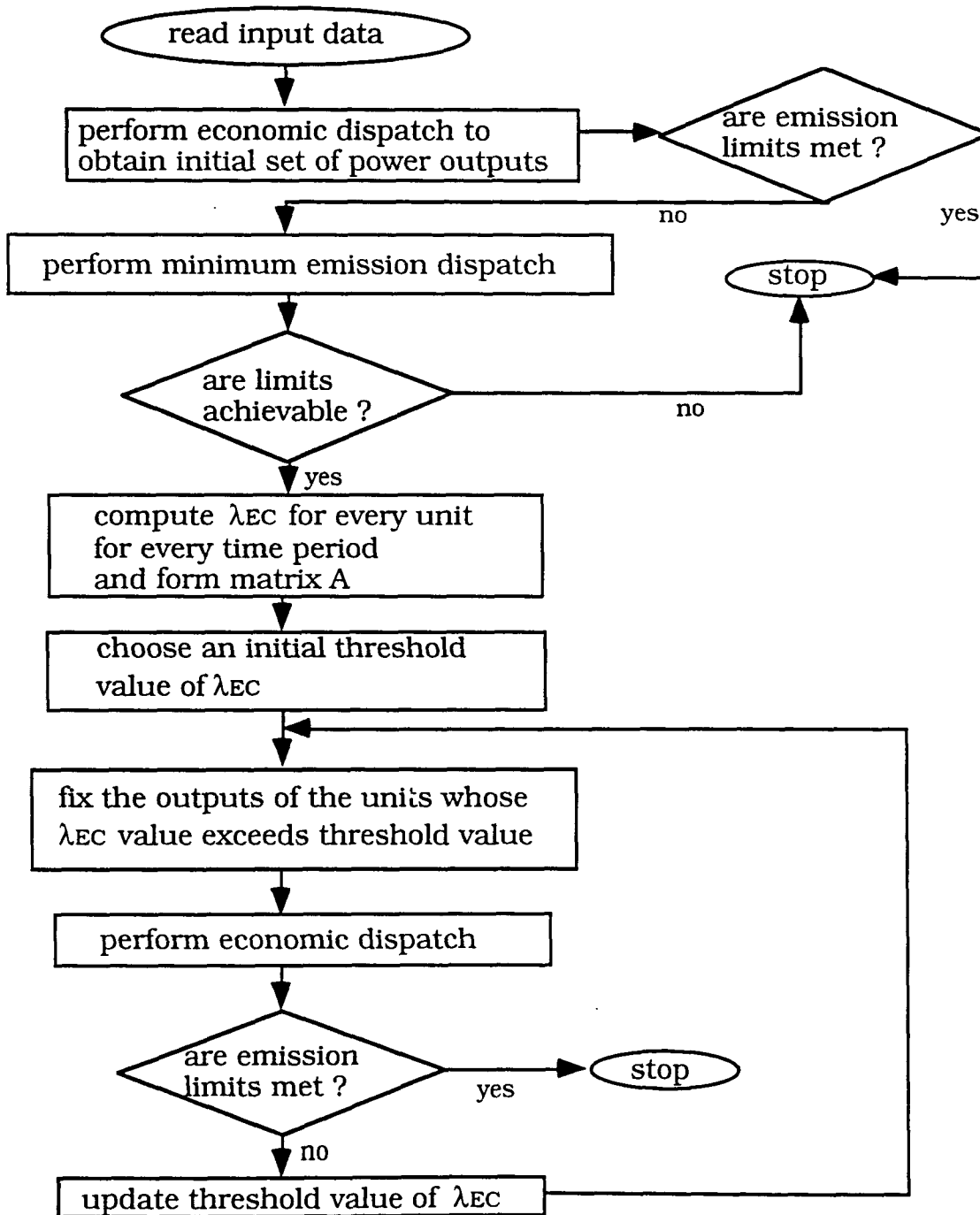


Figure 4.2 Flowchart of the general solution

dimensions are $n \times 1$ (n operating units) in the first case, and $n \times m$ (limits applicable for m hours) in the second case.

- Select an initial threshold value $\lambda_{EC}^{(1)}$ to begin the iterative process.
- Identify matrix elements greater than the threshold value. These elements indicate the units whose emission output should be reduced during certain hour(s). Next, either the maximum power limits or the fuel prices are appropriately modified. The units are then rescheduled using conventional economic dispatch. Only rows corresponding to constrained generating units need to be checked to determine if they exceed the threshold value. If the resultant emissions equal the emission limit, the problem is solved. Otherwise, the threshold value $\lambda_{EC}^{(k)}$ is updated, and the process is repeated.

4.8 Similarities of the Proposed Approach with Other Methods

Several problems in the electric power area, such as regulated margin allocation, hydrothermal coordination, fuel scheduling etc., are described by a set of equations similar to eqs. (4.9)--(4.12).

Several methods have been developed in the optimization theory to solve eqs. (4.15)--(4.17), which result directly from eqs. (4.9)--(4.12) if the inequality constraint is temporarily ignored. One class of methods, called dual methods, is considering the Lagrangian multipliers as the

fundamental unknowns associated with a constrained problem. The proposed algorithm belongs in this class of methods.

An approach similar to the presented technique is used in [33] to solve the regulated margin allocation. However, the similarities of the proposed approach with the γ - λ search method are outlined in the remainder of this section.

The γ - λ search may be used in a fuel scheduling problem for example, where one generating unit (unit T) is constrained under a take-or-pay contract [34]. The problem is stated as follows: determine the minimum production cost for units 1 to n, subject to the constraints that the load must be satisfied and the total fuel consumption of unit T during a period T_{contr} equals a given quantity Q_{contr} . The take-or-pay scheduling problem is described by eqs. (4.9), (4.10)--for $i=1,\dots,n,T$, (4.11)--for $i=1,\dots,n,T$ and an equation of the form

$$\sum_{t=1}^{T_{\text{contr}}} Q_{T_t}(P_T) = Q_{\text{contr}} \quad (4.28)$$

where $Q_{T_t}(P_T)$ = fuel input of unit T for hour t (MBtu)

Q_{contr} = fuel quantity that unit T needs to consume (MBtu)

t = index of time intervals (hr).

Since the total fuel to be consumed by unit T is fixed, unit T is not included in the objective function. Assuming that the power outputs lie within the unit limits and $T_{\text{contr}} = 1$, the Kuhn-Tucker conditions yield

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial C_i}{\partial P_i} - \lambda \quad (4.29)$$

$$\frac{\partial \mathcal{L}}{\partial P_T} = \gamma \frac{\partial Q_{Tt}}{\partial P_T} - \lambda \quad (4.30)$$

where $\gamma, \lambda =$ Lagrangian multipliers,
and P_i 's and P_T are the independent variables of the problem.

The γ - λ search proceeds as follows:

- After selecting initial values for the multipliers λ and γ , an economic dispatch is performed (solve eqs. (4.29)--(4.30)) for every time interval.
- If the total fuel consumption of unit T is close enough to the required quantity Q_{contr} , the problem is solved.
- If the total fuel consumption of unit T is not close enough to the required quantity Q_{contr} , the value of γ is updated.
- With the new value of γ the program cycles and a new set of economic dispatches is performed.

The presented emissions-constrained dispatch method proceeds in a similar way:

- Select initial value for λ_{EC} . It is initially evaluated as the ratio of $\partial E_i / \partial P_i$ over $\partial C_i / \partial P_i$ at a point determined by an economic dispatch.
- Update (or modify) the unit operating limits. This is achieved by solving $\lambda_{\text{EC}} = f(P_i)$ for the P_i 's.
- Perform a set of economic dispatches where the units are dispatched within their updated operating limits.
- If the required emission limit is achieved, the problem is solved.
- If the required emission limit is not achieved, update the value of λ_{EC} .
- Update the unit operating limits and perform a new set of dispatches.

This procedure iterates until the emission limit is satisfied. The results from the most recent set of economic dispatches are taken as final outputs.

4.9 Special Cases

A potential problem that may arise is that several units be limited and sufficient generation seems not to be available to serve the demand. This may occur when constraining the total daily emissions below a certain value. It is probable that the proposed procedure will curtail several units during peak demand periods (e.g. mid-afternoon). The demand, at a first look, may appear as no longer able to be met. This should be interpreted as implying that the emissions during these time periods should be largely reduced. If such a case is recognized, the units are dispatched according to minimum emission dispatch.

Another special case is encountered, when several emission constraints are active at the same time. An example is when one or more units are subject to more than one limits simultaneously. In such a case, the procedure outlined is independently repeated for each constraint. The smallest maximum value of unit power output or largest fuel pseudoprice is retained for the affected units. When all the constraints are satisfied, the retained values are transferred to the economic dispatch module. In following this procedure, all imposed limits are ensured to be satisfied.

4.10 Other Modeled Issues

The software developed contains some features that have not been discussed. Both power and emission losses are represented, jointly-owned units are included, and the startup process may also be included. This section covers these additional features.

4.10.1 Dispatching of the jointly-owned units

To reduce large capital investments for new power plants, multiple utilities installed larger jointly-owned units (JOU's) than those that would be built by individual companies. JOU's are normally operated as base load units rather than load following units. JOU's operation, control and maintenance issues are regulated by the contractual arrangements. Thus, the operation of each JOU depends highly on the contracting parties arrangements. One company, usually the one with the largest ownership share, assumes the managing-owner utility responsibilities. This company actually controls and monitors the JOU's operation.

Each participating company dispatches its part of the unit according to its individual needs. Therefore, each company considers its part of the JOU as a separate unit, whose capacity is equal to that utility's part of the total JOU capacity. Thus, a JOU may be dispatched using the following equations

$$\lambda_{iC} = IC_i(P_i) = \frac{dC_i}{dP_i} = f_{p_i} I F_i \left(\frac{P_i}{\%_i} \right) \%_i \quad (4.31)$$

$$\%_i P_{\min} \leq P_i \leq \%_i P_{\max} \quad (4.32)$$

If the fuel input rate is modeled by a reduced cubic equation, then one obtains

$$\lambda_i C = \frac{dC_i}{dP_i} = fp_i IF_i \left(\frac{P_i}{\%_i} \right) \%_i = fp_i (b_i + 3d_i \left(\frac{P_i}{\%_i} \right)^2) \%_i \quad (4.33)$$

$$P_{\min} \leq \frac{P_i}{\%_i} \leq P_{\max} \quad (4.34)$$

where i = index of co-owning company

$\%_i$ = ownership percentage of company i

P_i = power demanded from the JOU from company i (MW)

The power must be correctly scaled for eqs. (4.33)--(4.34) to yield correct results.

Each participating company notifies the managing utility of its desired schedule. The managing utility sums the schedules from the participating partners to determine the JOU's scheduled output

$$P = \sum_i P_i \quad (4.35)$$

where P = JOU total power output (MW).

The various partners may not use the same percentage of their part of the JOU. This will result in an actual operating cost that differs from the sum of the individual forecasted costs. Thus, the actual costs assessed to each company, will differ from the ones computed from individual companies. In most contracts, actual costs are prorated based on each partner's actual usage

$$C_{i \text{ actual}}(P_i) = C_{\text{JOU actual}}(P) \frac{P_i}{P} \quad (4.36)$$

where $C_{i \text{ actual}}(P_i)$ = actual cost assessed to company i (\$/hr)

$C_{\text{JOU actual}}(P)$ = actual operating cost of the JOU (\$/hr).

4.10.2 Startup procedure

Thermal generating units require gradual temperature changes during start up and shut down. The entire procedure to bring a unit on-line is called startup and requires several hours. Startup is usually modeled as an event occurring at a specific hour. The startup model is essential since it specifies fuel requirements and costs, necessary to begin on-line operation. Because of solution algorithm requirements, shutdown costs are generally included in startup costs [34].

There are two ways to operate a unit while it is out of service. These two ways also dictate two different approaches for returning the unit to service. The first approach is to allow the boiler's temperature to reduce as a function of time. When the unit is returned to service, the boiler's temperature must be increased to its normal operating value. This is called thermal cooling and the associated cost is

$$C_{\text{SUC}} = C_c(1 - e^{-t/a})fp + C_f \quad (4.37)$$

where C_c = cold start cost (MBtu)

C_{SUC} = startup cost when cooling (\$)

fp = fuel cost (\$/MBtu)

C_f = fixed cost--includes crew and maintenance expenses (\$)

a = thermal time constant of the unit (hr)

t = time a unit has been shut down (hr).

The second approach, called banking, maintains the boiler's temperature near to its operating temperature. The cost to start a banking unit is

$$C_{SUB} = C_t t_{fp} + C_i \quad (4.38)$$

where C_t = cost to keep unit at operating temperature (MBtu/hr)

C_{SUB} = startup cost when banking (\$).

For shorter periods, thermal cooling is more expensive than banking. The reverse is true for longer periods. The crossover point depends on the unit design characteristics.

4.10.3 Loss representation

4.10.3.1 Power losses Eq. (4.10) represents the power balance and is the fundamental constraining equation used when solving the economic dispatch problem. In the previous sections, transmission losses were ignored. Including losses, modifies the structure of both the economic and minimum emission dispatching modules. The logic of the curtailing algorithm is not altered.

To solve the complete problem described by eqs. (4.9)--(4.11) (economic dispatch including transmission losses), one formulates the Lagrangian equation to be minimized

$$\text{minimize } \mathcal{L} = C_s + \lambda_c \varphi \quad (4.39)$$

where C_s is given by eq. (4.9) and φ is

$$\varphi = P_{LOAD} + P_{LOSSES} - \sum_{i=1}^n P_i \quad (4.40)$$

To solve this problem, applying well-known calculus principles

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dC_i}{dP_i} - \lambda_c \left(1 - \frac{\partial P_{\text{LOSSES}}}{\partial P_i}\right) = 0 \quad (4.41)$$

Eqs. (4.40)--(4.41) are called the coordination equations. Rearranging eq. (4.41) yields

$$\left(\frac{1}{1 - \frac{\partial P_{\text{LOSSES}}}{\partial P_i}}\right) \frac{dC_i}{dP_i} = pf_i \frac{dC_i}{dP_i} = \lambda_c \quad (4.42)$$

The new parameter pf_i is called the penalty factor of unit i . Three different loss representations are possible depending on the pf_i values:

1) Lossless case where

$$P_{\text{LOSSES}} = 0 \Rightarrow pf_i = 0 \quad \text{and} \quad \frac{dC_i(P_i)}{dP_i} = \lambda_c \quad (4.43)$$

and is the case described in section 4.5. The losses are assumed to be included in the load demand values.

2) Constant penalty factors representation where pf_i have constant values independent of the unit power outputs. These values are derived somewhat heuristically using actual data and past experience. Losses are either considered to be included in the load demand values or a function of the load demand as calculated using a polynomial.

3) B matrix loss representation. In this case P_{LOSSES} is given by

$$P_{\text{LOSSES}} = P^T[B]P + P^T B_0 + B_{00} \quad (4.44)$$

or

$$P_{\text{LOSSES}} = \sum_i \sum_j P_i B_{ij} P_j + \sum_i B_{i0} P_i + B_{00} \quad (4.45)$$

where P = vector of all power outputs

$[B]$ = square matrix of same leading dimension as P

B_0 = vector of same length as P

B_{00} = constant.

In this case, pf_i is given by

$$pf_i = \frac{1}{1 - 2 \sum_j B_{ij} P_j - B_{i0}} \quad (4.46)$$

This is the most complex, yet complete loss representation, short of detailed modeling of the actual transmission network. The B matrix coefficients are derived using the equivalent total load center approach. The B matrix loss representation was developed mainly by L. Kirchmayer [35,36] and G. Kron [37]. The penalty factors penalize those units that are farther from the load center. Penalty factors may also be derived using the reference bus approach. It can be shown [34] that the reference bus penalty factors are a constant times the loss matrix penalty factors.

The inclusion of losses couples the coordinating equations. A two loop approach is often employed to solve the problem, as is schematically shown in Figure 4.3. The inner loop is a conventional economic dispatch module that solves eq. (4.42) with given pf_i values, whereas the outer loop updates the penalty factors using eq. (4.46). The whole procedure iterates until convergence is achieved. An alternative

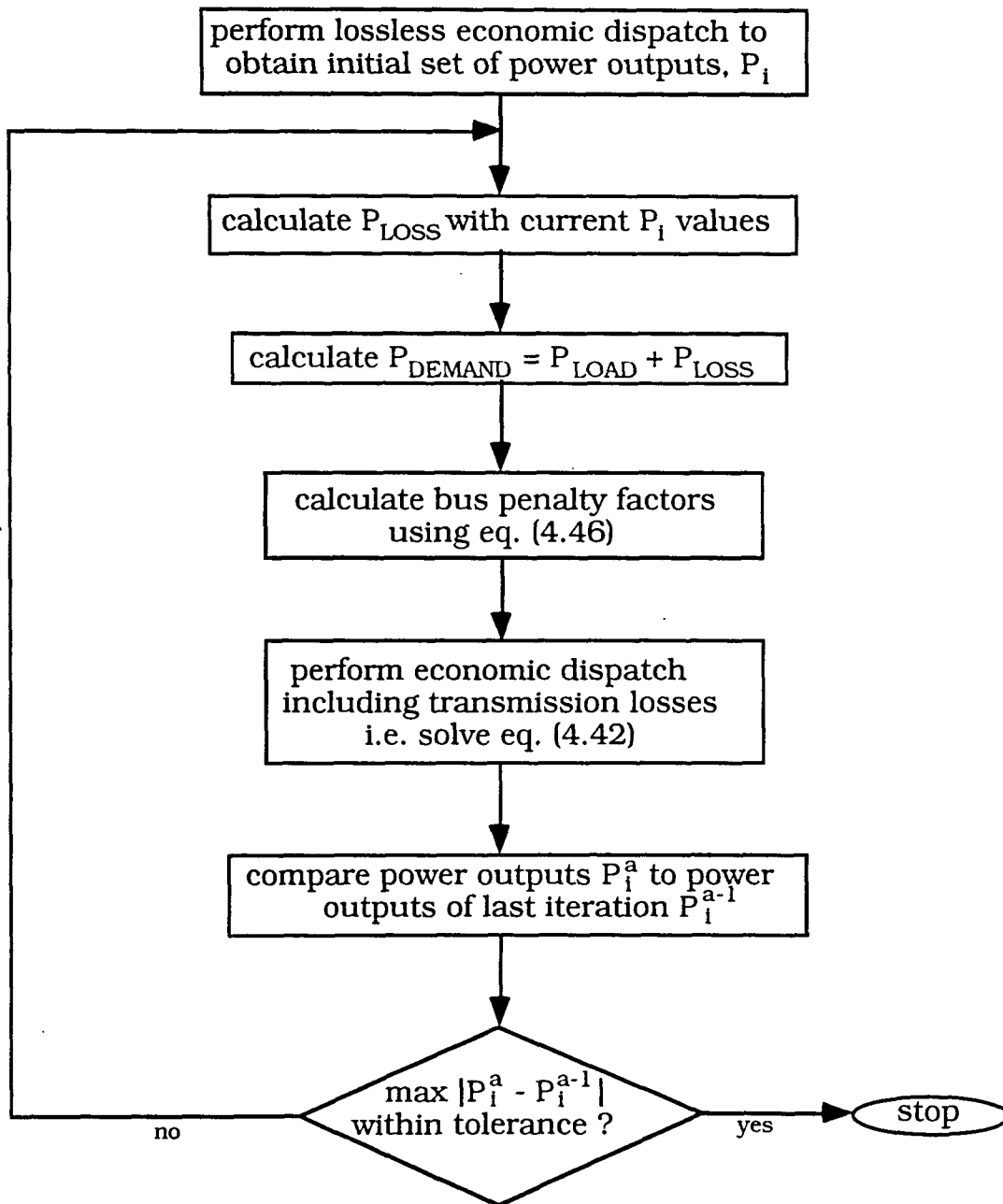


Figure 4.3 Economic dispatch with updated penalty factors

approach is to vary the penalty factors in the inner loop while the outer loop updates the incremental cost.

4.10.3.2 Emission losses In the minimum emission dispatch one may wish to penalize the units in the load center proximity. These units significantly contribute to the load center's emission levels. In practice, this may be accomplished by introducing emission penalty factors that are the reciprocal of the power penalty factors

$$epf_i = \frac{1}{pf_i} = 1 - \frac{\partial P_{LOSSES}}{\partial P_i} \quad (4.47)$$

To solve the problem including losses, the following coordinating equation is solved

$$epf_i \frac{dE_i}{dP_i} = \lambda_E \quad (4.48)$$

As previously discussed, there are three emission losses representations. The lossless case is when $epf_i = 1$. The constant penalty factors case is when epf_i have constant values independent from the unit power outputs. The B matrix case is when power losses are given by eq. (4.44). In such a case, epf_i are given by

$$epf_i = 1 - 2 \sum_j B_{ij} P_j - B_{i0} \quad (4.49)$$

Similarly to the power losses case, the minimum emission dispatch including losses problem, may be solved using a two loop iterative process, as shown in Figure 4.3. In this case, the outer loop updates the emission penalty factors using eq. (4.49) and the inner loop solves eq. (4.48). The

alternative approach of updating the penalty factors in the outer loop and the incremental emissions in the inner loop also works.

4.11 Implementation of the Algorithm

Figure 4.4 is a block diagram of the proposed algorithm. The usual three-part structure is employed: i) input, ii) solution and iii) output.

The input data required includes:

1) Unit representations:

- Minimum and maximum operating limits
- Fuel price
- Fuel input characteristic coefficients
- Emission factors
- Emissions characteristic coefficients
- Startup parameters

2) Unit availability for the period under consideration. This information is readily available from a unit commitment program. Units committed at fixed output are represented as having equal minimum and maximum power output limits.

3) Forecasted load requirements

4) Emission limits

5) System information:

- Number of companies (if more than one)
- Number of jointly-owned units (if any)

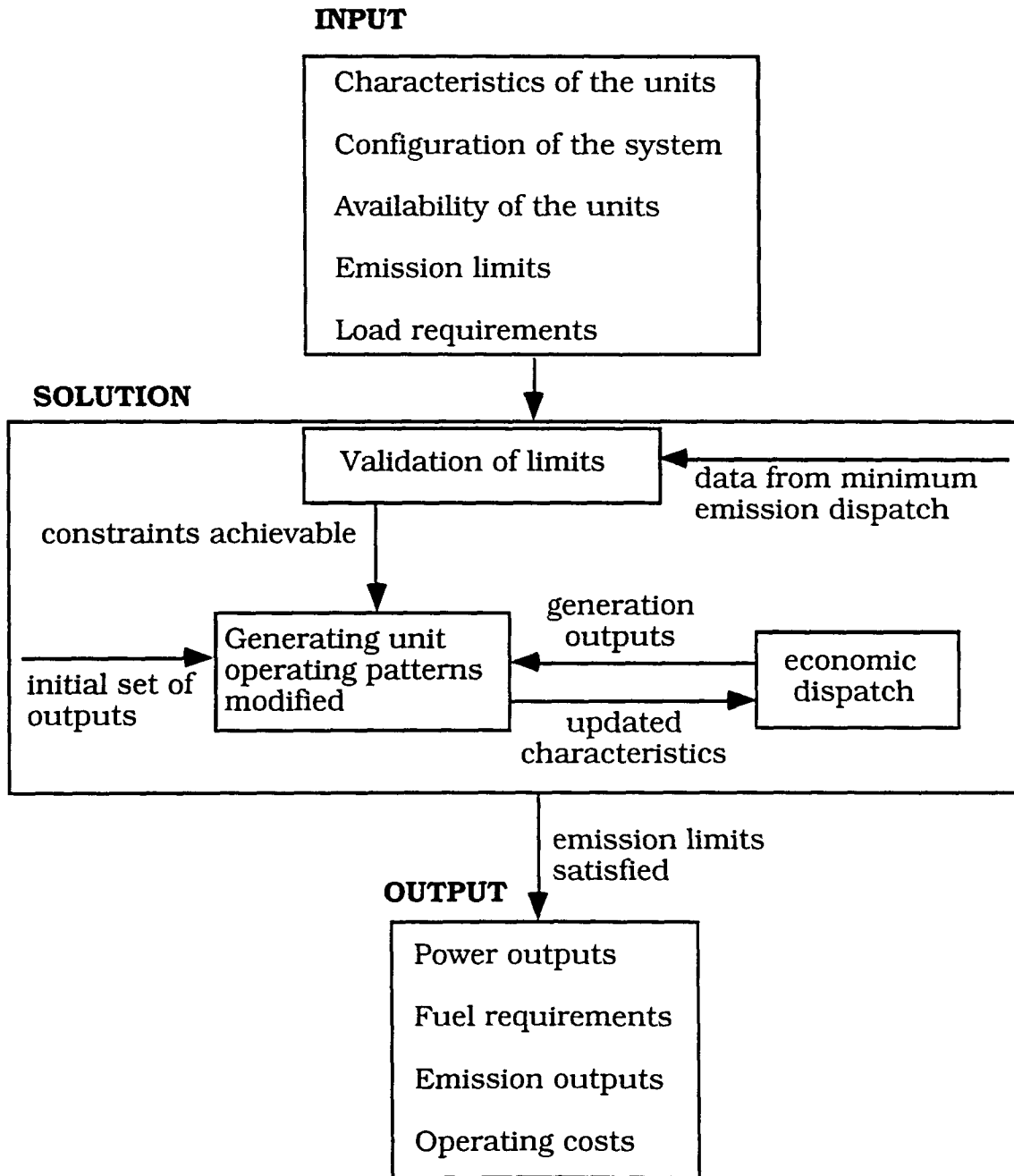


Figure 4.4 Block diagram of the solution implementation

- Transmission losses representation. If losses are to be modeled, adequate information, e.g. B matrix coefficients or constant penalty factors, must be furnished.

6) Number of hours to be simulated

7) If the constraints are not affecting the entire system, constrained units must be identified.

The solution procedure is as described previously.

The last section provides total or individual unit results. Output may also be grouped by plants or companies. The following data may be output:

- 1) Unit, plant, company and system power outputs
- 2) Energy
- 3) Unit, plant, company and system emission outputs
- 4) Operating and startup fuel requirements and costs
- 5) Jointly-owned unit values.

4.12 Advantages of the Proposed Algorithm

- The general objective is to minimize the operating costs while meeting the imposed environmental constraints. Obviously, overcomplying with regulations (overreducing emissions) results in unnecessarily increased costs. The proposed procedure will not overcomply.
- Overall emission reduction does not imply reductions in all geographical areas. On the contrary, several approaches have successfully constrained total emissions, but created new or increased existing local

problems. The proposed algorithm is capable of handling both system and local limitations simultaneously.

- All kinds of pollutants may be handled in combinations or separately. Even emissions for which power industry is not currently regulated probably could be handled.
- Limited initial data is required and few assumptions are made. Although this analysis uses reduced cubic unit representations, other representations may be used. Neither weather forecasting data nor complicated air dispersion models are required. All necessary pieces of information are available from tests, measurements or analyses.
- Although they increase problem complexity, transmission losses can be included. Actually, only the economic and the minimum emission dispatches are affected, whereas the logic and structure of the curtailing module are not altered.
- Algorithm implementation has shown reasonable solution times. Therefore the proposed solution is suitable for both real-time scheduling and unit commitment.
- The proposed solution approach was originally developed to handle problems within the short-term horizon. However, by use of parallel computers, it might be extended to medium-range planning and stochastic power models.

5. NUMERICAL RESULTS

5.1 Introduction

A twenty-two generating units system was used to test the emissions-constrained dispatch algorithm explained in the previous chapter. Twenty units are solely-owned and the remaining two are jointly-owned. Several cases were considered in order to cover a wide variety of dispatching problems. Fictitious names were used for the purpose of original data security. The system units are divided into two companies:

- Company 1, named Alright Power and Light (AP&L), owns six units equally divided into two plants: SPA and VER. The same company owns 40% and 50% of units FET1 and FET2 respectively.
- Company 2, named Neighboring Electric Utility (NEU), owns fourteen units grouped into four plants: LAS, MAC, RAV and TOR. NEU also owns the remaining 60% and 50% of units FET1 and FET2 respectively.

Data for unit economic modeling is given in Table 5.1, where

i = unit index

fp_i = fuel price (\$/MBtu).

a_i, b_i, c_i, d_i = input-output characteristic coefficients

$P_{i \max}$ = upper operating limit (MW)

$P_{i \min}$ = lower operating limit (MW).

Table 5.1 Economic modeling data

	$P_i \max$	$P_i \min$	a_i	b_i	c_i	d_i	fp_i
AP&L							
SPA1	50	240	7.5921e+01	8.6935e+00	0.00	4.7585e-06	1.40
SPA2	45	240	8.6079e+01	8.6831e+00	0.00	4.5553e-06	1.40
SPA3	275	450	2.2494e+02	8.7515e+00	0.00	1.3154e-06	1.40
VER1	150	350	2.3114e+02	7.3452e+00	0.00	5.1558e-06	1.75
VER2	150	350	2.3088e+02	7.2330e+00	0.00	6.1154e-06	1.75
VER3	350	750	6.5000e+02	8.5700e+00	0.00	1.3000e-06	1.75
NEU							
LAS1	35	175	5.3250e+01	9.5269e+00	0.00	4.4109e-06	1.80
LAS2	35	175	5.8249e+01	9.2874e+00	0.00	7.6203e-06	1.80
LAS3	45	240	1.7312e+02	7.6848e+00	0.00	1.3949e-05	1.80
MAC1	40	180	1.3851e+02	7.6546e+00	0.00	3.0531e-05	1.60
MAC2	40	180	1.0675e+02	7.9722e+00	0.00	2.9051e-05	1.60
MAC3	45	240	1.8339e+02	7.7118e+00	0.00	1.3787e-05	1.60
RAV1	105	200	5.5843e+02	4.8614e+00	0.00	6.1135e-05	1.25
RAV2	80	250	1.0807e+02	8.9273e+00	0.00	4.7677e-06	1.25
RAV3	75	245	1.0536e+02	8.9593e+00	0.00	4.2816e-06	1.25
RAV4	75	255	9.3883e+01	8.8851e+00	0.00	2.7227e-06	1.25
TOR1	60	190	1.1371e+02	8.7018e+00	0.00	1.4934e-05	1.20
TOR2	100	350	1.6191e+02	9.3131e+00	0.00	3.5665e-06	1.20
TOR3	90	360	3.1062e+02	9.0475e+00	0.00	9.5326e-07	1.20
TOR4	325	575	3.4035e+02	8.3992e+00	0.00	1.5331e-06	1.20
Jointly-Owned Units							
FET1	275	450	6.5933e+02	6.5740e+00	0.00	8.2349e-06	1.50
FET2	320	750	9.2590e+02	7.7113e+00	0.00	1.8205e-06	1.50

Data for unit emissions modeling is given in Table 5.2, where

A_i, B_i, C_i, D_i = emissions characteristic coefficients

H_{val} = fuel heating value (Btu/lb)

%S = percent of sulfur contained in fuel.

Startup data is given in Table 5.3, where

C_c = cold start cost (MBtu)

a = thermal time constant of the unit (hr)

C_f = fixed cost-- includes crew and maintenance expenses (\$)

C_t = cost to keep unit at operating temperature (MBtu/hr).

Although startup costs were computed, startup emissions were not calculated.

Figures 5.1 (A) and (B) show typical weekly load curves for AP&L and NEU respectively. In some cases, both companies are considered together as a power pool serving the sum of the individual company loads.

Two time periods are considered: i) 24 hours and ii) 168 hours, with emphasis given on the daily operation.

First, several unconstrained cases were run to determine the system limits with respect to SO_2 and NO_x emissions. Furthermore, the unconstrained dispatches demonstrate the effect of the different loss representations and of the different unconstrained dispatching approaches on daily operation. Results for several emissions-constrained dispatching examples are presented. Results are presented either in tables or in graphs. In all tables, power is given in MW, fuel in MBtu, cost in dollars, SO_2 in tons and NO_x in tons.

Table 5.2 Emissions modeling data

	A_i	B_i	C_i	D_i	H_{val}	%S
AP&L						
SPA1	5.0732e-02	4.3111e-04	0.00	1.4790e-08	10,000	0.6
SPA2	5.0732e-02	4.3111e-04	0.00	1.4790e-08	10,000	0.6
SPA3	3.2163e-01	2.9966e-05	0.00	6.1471e-09	10,000	0.6
VER1	-9.3024e-02	7.7763e-04	0.00	8.6433e-09	10,000	0.6
VER2	-9.3024e-02	7.7763e-04	0.00	8.6433e-09	10,000	0.6
VER3	3.0395e-02	3.3649e-04	0.00	1.9262e-09	10,000	0.6
NEU						
LAS1	1.0571e-01	1.3030e-03	0.00	1.4881e-09	10,000	0.5
LAS2	1.0571e-01	1.3030e-03	0.00	1.4881e-09	10,000	0.5
LAS3	2.0417e-02	4.0667e-04	0.00	1.4458e-08	10,000	0.6
MAC1	1.2786e-02	6.2659e-04	0.00	2.4004e-08	10,000	0.5
MAC2	1.2786e-02	6.2659e-04	0.00	2.4004e-08	10,000	0.5
MAC3	2.0417e-02	4.0667e-04	0.00	1.4458e-08	10,000	0.6
RAV1	1.6496e-02	1.3853e-03	0.00	5.9102e-08	10,000	0.5
RAV2	5.8713e-02	7.8394e-04	0.00	6.6790e-09	10,000	0.5
RAV3	1.3428e-01	8.9624e-05	0.00	2.5954e-08	10,000	0.5
RAV4	1.4293e-01	9.7775e-05	0.00	2.5073e-08	10,000	0.5
TOR1	9.7052e-02	5.1905e-05	0.00	4.0032e-08	10,000	0.5
TOR2	7.4479e-02	7.1032e-04	0.00	3.0876e-09	10,000	0.5
TOR3	1.3712e-01	3.6216e-04	0.00	7.4993e-09	10,000	0.5
TOR4	4.9346e-01	8.6930e-05	0.00	2.3419e-09	10,000	0.6
Jointly-Owned Units						
FET1	4.1377e-01	2.8745e-05	0.00	1.8885e-09	10,000	0.5
FET2	2.8955e-02	3.4152e-04	0.00	1.8885e-09	10,000	0.5

Table 5.3 Startup data

	C _c	C _t	a	C _f
AP&L				
SPA1	2246	234	8	7500
SPA2	2,294	239	8	7,500
SPA3	19,138	1,329	12	10,000
VER1	9,720	675	12	10,000
VER2	9,619	668	12	10,000
VER3	53,366	1,853	24	12,500
NEU				
LAS1	1,853	193	8	7,500
LAS2	1,843	192	8	7,500
LAS3	2,496	260	8	7,500
MAC1	2,141	223	8	7,500
MAC2	2,054	214	8	7,500
MAC3	2,554	266	8	7,500
RAV1	5,472	570	8	7,500
RAV2	3,955	412	8	7,500
RAV3	3,744	390	8	7,500
RAV4	5,486	381	12	10,000
TOR1	3,072	320	8	7,500
TOR2	7,891	548	12	10,000
TOR3	8,107	563	12	10,000
TOR4	22,478	1,561	12	12,500
Jointly-Owned Units				
FET1	28,490	1,319	24	12,500
FET2	50,314	1,747	24	12,500

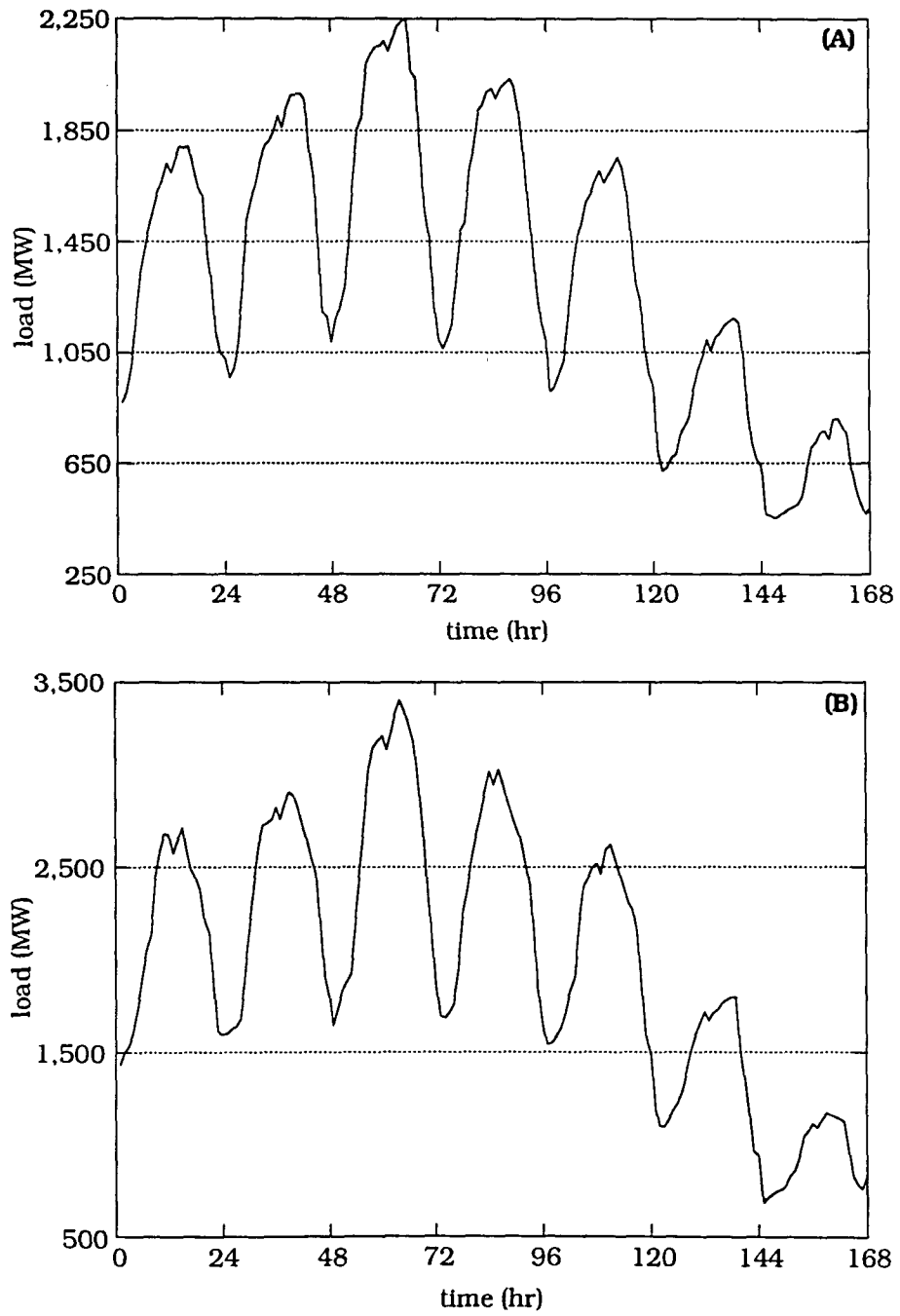


Figure 5.1 Typical weekly load curves: (A) AP&L (B) NEU

5.2 Unconstrained Dispatches

Table 5.4 shows results for a 168-hour example. The percentages shown are with respect to economic dispatch values. The units are dispatched according to different dispatching techniques. From this table, one derives that up to 14.1% NO_x emissions reduction or up to 2.1% SO₂ emissions reduction from economic dispatch may be achieved through the use of modified dispatching techniques. The low percentage in the SO₂ case is because of the low sulfur content of the input fuel.

Table 5.4 Results for 168-hour examples: total minimization dispatches, power pool

	Cost	Fuel	NO _x	SO ₂
Economic	7,689,247 (100%)	5,476,087 (100%)	814.16 (100%)	2,986.57 (100%)
Min fuel	7,879,200 (102.5%)	5,396,100 (98.54%)	782.24 (96.08%)	2,954.80 (98.94%)
Min NO _x	7,903,200 (102.8%)	5,462,200 (99.75%)	699.17 (85.88%)	2,974.50 (99.60%)
Min SO ₂	7,876,500 (102.5%)	5,439,000 (99.32%)	811.77 (99.71%)	2,923.90 (97.90%)

Figures 5.2 and 5.3 show the effect of different unconstrained dispatches on daily SO₂ and NO_x emissions respectively. It is clear that minimum emission dispatches achieve a more (NO_x case) or less (SO₂ case) significant decrease in daily emissions.

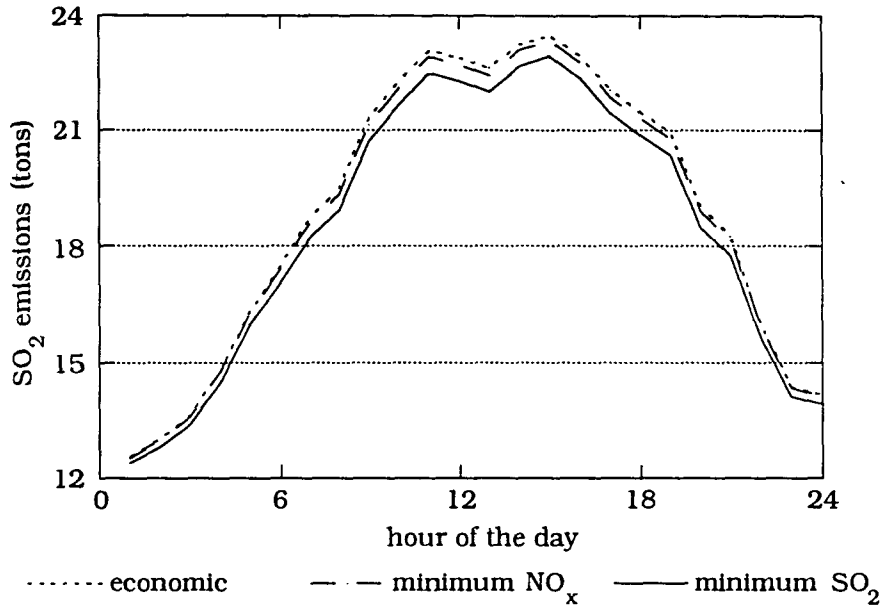


Figure 5.2 Effect of various dispatching techniques on daily power pool SO₂ emissions

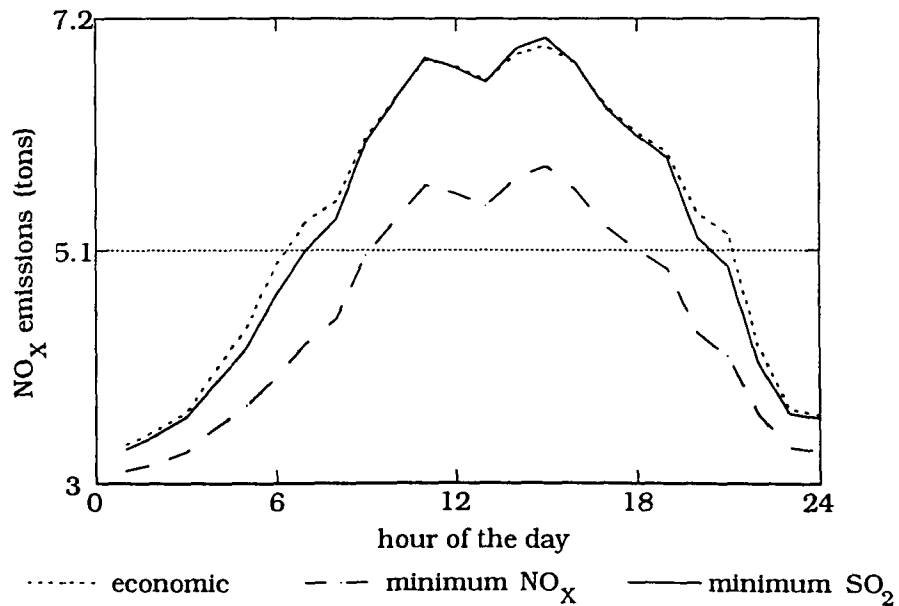


Figure 5.3 Effect of various dispatching techniques on daily power pool NO_x emissions

Figure 5.4 shows to what extent different loss representations affect daily costs. Daily costs are increased by 3.13% if losses are represented. Both, B matrix and constant penalty factors representations show similar results. Indeed, carefully chosen constant penalty factors may result in an accurate loss representation, thus avoiding the computational difficulties involved in the calculation of the B matrix.

Table 5.5 shows typical weekly startup costs and fuel requirements. As already mentioned, startup emissions were not calculated. Since turning a unit on and off is determined from a unit commitment program, several units may stay on or off for the entire week. It is common practice to try to keep the larger units (base load units) on continuously.

5.3 Emissions-Constrained Dispatches

Table 5.6.a shows results for a 24-hour example. A constraint, shown in the table in bold characters, is imposed on unit SPA1 to limit its NO_x emissions for the 24-hour period to 5.55 tons from 6.49 tons that results if the units are scheduled under economic dispatch. If minimum NO_x dispatch is used, unit SPA1 emits 4.7 tons during the 24-hour period. Total results are also compared percentagewise with the values resulting from unconstrained economic dispatch.

Table 5.6.b shows results for the same 24-hour period where an additional constraint is imposed. The total NO_x emissions are limited to 41.5 tons. For comparative reasons it is worthy to mention that NO_x

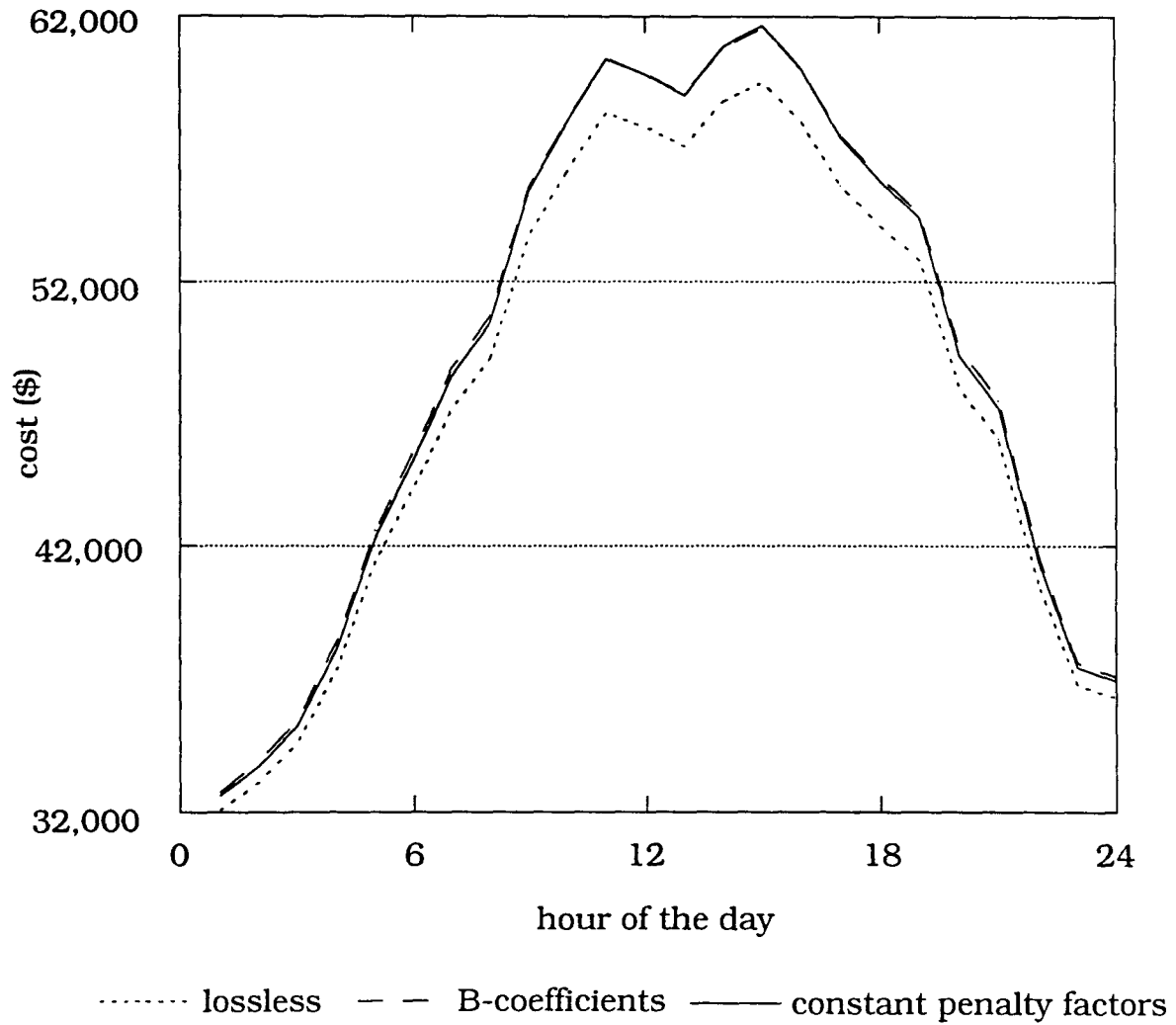


Figure 5.4 Effect of loss representations on daily power pool costs

Table 5.5 Typical weekly startup costs and startup fuel requirements

	Startup cost	Startup fuel
SPA1	0.0	0.0
SPA2	9,631.6	1,450.1
SPA3	55,566.7	24,195.1
VER1	21,213.2	6,144.2
VER2	0.0	0.0
VER3	74,064.1	33,733.8
FET1	0.0	0.0
FET2	0.0	0.0
TOTALS AP&L	160,480.0	65,523.1
LAS1	0.0	0.0
LAS2	9,666.9	1,165.0
LAS3	0.0	0.0
MAC1	0.0	0.0
MAC2	0.0	0.0
MAC3	0.0	0.0
RAV1	0.0	0.0
RAV2	0.0	0.0
RAV3	0.0	0.0
RAV4	0.0	0.0
TOR1	0.0	0.0
TOR2	0.0	0.0
TOR3	33,426.5	10,249.2
TOR4	0.0	0.0
FET1	0.0	0.0
FET2	0.0	0.0
TOTALS NEU	43,093.4	11,414.2

Table 5.6 Results for AP&L 24-hour example

a) 1 NO_x constraint

	Power	Fuel	Cost	SO ₂	NO _x
SPA1	4,344.8	40,383.8	56,537.3	24.23	5.55
SPA2	4,775.9	44,566.8	62,393.6	26.74	6.63
SPA3	9,173.3	87,638.0	122,693.1	52.58	17.15
VER1	3,580.7	31,572.5	55,251.9	18.94	2.01
VER2	4,173.5	36,568.5	63,994.9	21.94	2.20
VER3	0.0	0.0	0.0	0.00	0.00
FET1	2,942.4	28,001.7	42,002.6	14.00	4.59
FET2	5,432.0	55,275.9	82,913.9	27.64	4.56
TOTALS	34,422.7	324,007.3	485,787.3	186.08	42.69
		99.9%	100.05%	99.84%	99.35%

b) 2 NO_x constraints

	Power	Fuel	Cost	SO ₂	NO _x
SPA1	4,344.8	40,383.8	56,537.3	24.23	5.55
SPA2	4,678.9	43,651.0	61,111.4	26.19	6.34
SPA3	8,384.9	80,195.4	112,273.6	48.12	14.59
VER1	3,903.2	34,169.5	59,796.6	20.50	2.65
VER2	4,455.8	38,839.2	67,968.5	23.30	2.74
VER3	0.0	0.0	0.0	0.00	0.00
FET1	3,005.3	28,591.5	42,887.3	14.30	4.63
FET2	5,649.7	57,315.9	85,973.9	28.66	5.01
TOTALS	34,422.6	323,146.2	486,548.6	185.30	41.51
		99.63%	100.21%	99.42%	96.51%

emissions total 43 tons and 39.7 tons if the units are scheduled using economic and minimum NO_x dispatches respectively.

Tables 5.7.a and 5.7.b show similar results for SO₂ emissions. First, emissions from unit SPA3 are limited to 47 tons from 52.45 tons emitted under economic dispatch. Afterwards, an additional constraint limits the total SO₂ emissions to 185 tons from 186.4 tons emitted under economic dispatch.

In another example, total SO₂ emissions of NEU are limited to 266 tons and total NO_x emissions are limited to 78 tons for a 24-hour period. Results are shown in Table 5.8.

Figure 5.5 shows graphically how NEU's daily emissions were limited with increasing emission constraints. Cases for 2%, 4% and 6% NO_x emissions reduction, from the emission levels resulting under economic dispatch, were run. The corresponding costs show an increase of less than 1% from the unconstrained economic dispatch values.

Figure 5.6 shows total daily cost versus maximum allowable daily SO₂ emissions. Emissions vary from a maximum, determined by economic dispatch results (point 1), to a minimum, identified by the results of a minimum SO₂ dispatch (point 2). By inspecting Figure 5.6 some important conclusions may be derived. Namely, emissions may be reduced up to a certain point (point 3) with a corresponding cost increase rate. If, however, one desires to further decrease emissions, the system cost increases at considerably higher rates. This implies that there are several units in the system, whose emissions may be reduced with a reasonable effect on the

Table 5.7 Results for AP&L 24-hour example

a) 1 SO₂ constraint

	Power	Fuel	Cost	SO ₂	NO _x
SPA1	4,844.2	45,074.1	63,103.7	27.04	6.85
SPA2	4,908.9	45,805.7	64,127.9	27.48	6.95
SPA3	8,170.0	78,294.9	109,612.8	46.98	14.49
VER1	3,671.8	32,277.6	56,485.8	19.37	2.14
VER2	4,264.6	37,271.9	65,225.9	22.36	2.33
VER3	0.0	0.0	0.0	0.00	0.00
FET1	2,981.9	28,350.6	42,525.9	14.18	4.61
FET2	5,581.5	56,595.3	84,892.9	28.30	4.79
TOTALS	34,422.9	323,670.0	485,974.9	185.71	42.17
		99.79%	100.09%	99.64%	98.14%

b) 2 SO₂ constraints

	Power	Fuel	Cost	SO ₂	NO _x
SPA1	4,462.6	41,580.0	58,212.0	24.95	6.13
SPA2	4,568.9	42,705.1	59,787.1	25.62	6.33
SPA3	8,173.6	78,308.9	109,632.4	46.99	14.41
VER1	3,768.4	33,036.7	57,814.2	19.82	2.30
VER2	4,385.7	38,215.5	66,877.1	22.93	2.52
VER3	0.0	0.0	0.0	0.00	0.00
FET1	3,061.6	29,050.1	43,575.2	14.53	4.66
FET2	6,002.3	60,239.9	90,359.9	30.12	5.35
TOTALS	34,423.0	323,136.3	486,258.1	184.95	41.70
		99.63%	100.15%	99.20%	97.04%

Table 5.8 Results for NEU 24-hour example: 2 constraints

	Power	Fuel	Cost	SO ₂	NO _x
LAS1	0.0	0.0	0.0	0.00	0.00
LAS2	0.0	0.0	0.0	0.00	0.00
LAS3	1,191.8	13,360.1	24,048.1	8.02	1.02
MAC1	1,791.5	17,505.4	28,008.6	8.75	1.80
MAC2	1,522.9	14,979.0	23,966.4	7.49	1.49
MAC3	2,456.0	23,963.1	38,340.9	14.38	2.14
RAV1	2,520.0	27,351.6	34,189.4	13.68	5.53
RAV2	5,093.5	49,357.8	61,697.3	24.68	7.21
RAV3	4,680.0	45,333.5	56,666.9	22.67	8.95
RAV4	5,069.9	47,934.2	59,917.7	23.97	9.77
TOR1	3,720.9	36,655.8	43,987.0	18.33	6.67
TOR2	5,733.6	59,110.0	70,932.0	29.56	7.44
TOR3	0.0	0.0	0.0	0.00	0.00
TOR4	8,198.4	78,525.4	94,230.5	47.12	14.84
FET1	4,352.9	41,469.8	62,204.7	20.73	6.85
FET2	5,188.1	53,176.0	79,764.0	26.59	4.25
TOTALS	51,519.6	508,721.5	677,953.4	265.95	77.97
		99.66%	101.5%	98.52%	91.78%

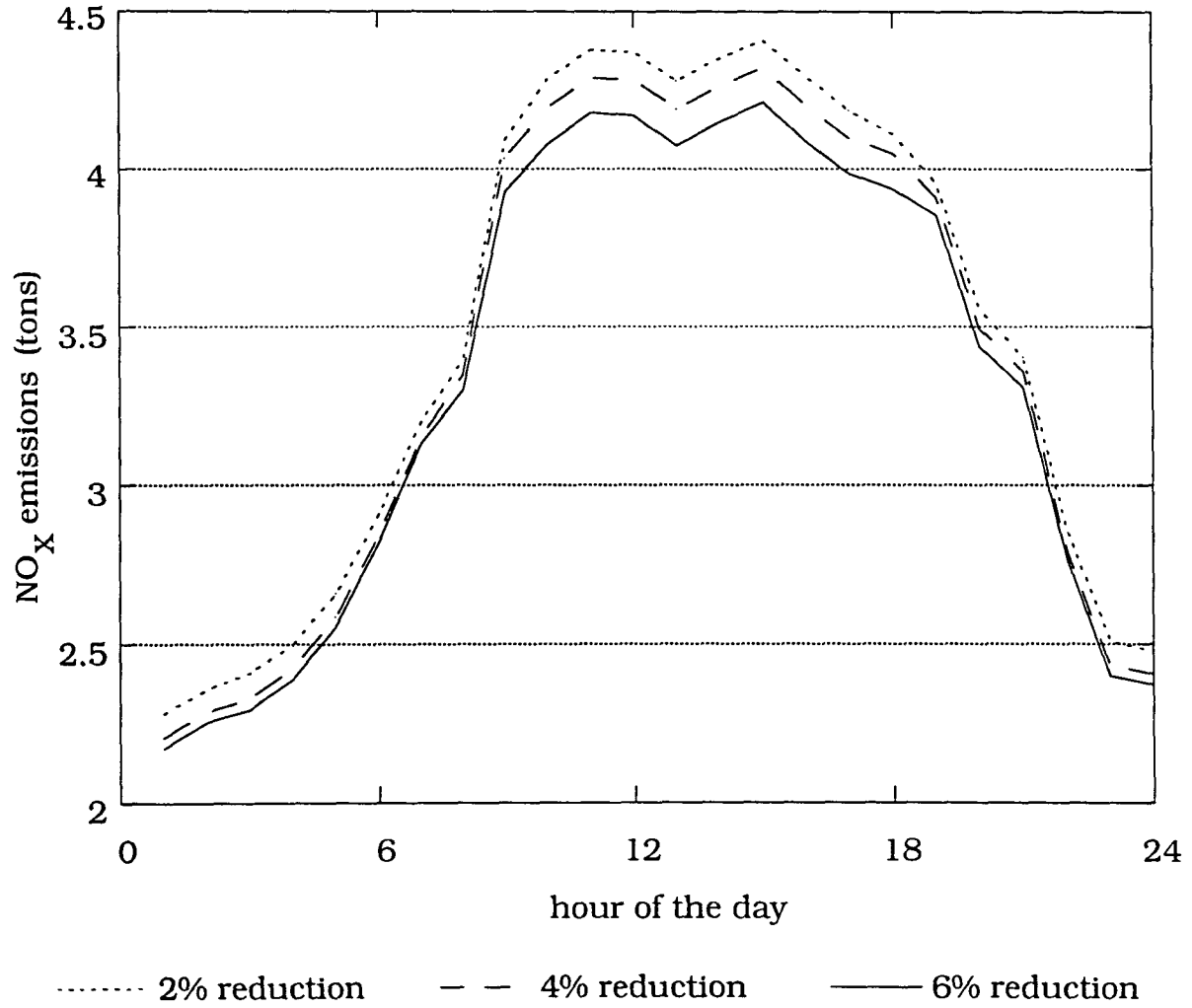


Figure 5.5 Effect of various NO_x limits on daily NEU NO_x emissions

system total cost. After these units are curtailed, a further emission decrease requires curtailment of more “environmentally” costly units, thus resulting in a substantial increase of the total cost.

Figure 5.7 shows daily costs versus maximum allowable NO_x emissions. Emissions vary from a maximum, again identified by the results of an economic dispatch (point 1), to a minimum, corresponding to results from a minimum NO_x dispatch (point 2). As in the case of SO₂, if emissions are limited up to a certain point (point 3), system total cost increases at a moderate rate. Further emission decrease however, results in a significant increase of the system cost.

As a final example, two constraints are imposed on the entire pool for a 168-hour period. Total NO_x emissions are limited to 800 tons and total SO₂ emissions are limited to 2965 tons. For comparison reasons, under economic dispatch, the system emits 2975 tons of SO₂ and 819 tons of NO_x. Results are shown in Table 5.9.

5.4 Validation of Results

To investigate the validity of the results yielded by the presented emissions-constrained economic dispatch, the system of nonlinear equations (4.15)--(4.17) was solved by the Newton-Raphson method for a number of cases. In most cases, the results from the Newton-Raphson process were identical with the ones yielded by the proposed method. In some cases though, a difference of not more than .04% in operating cost

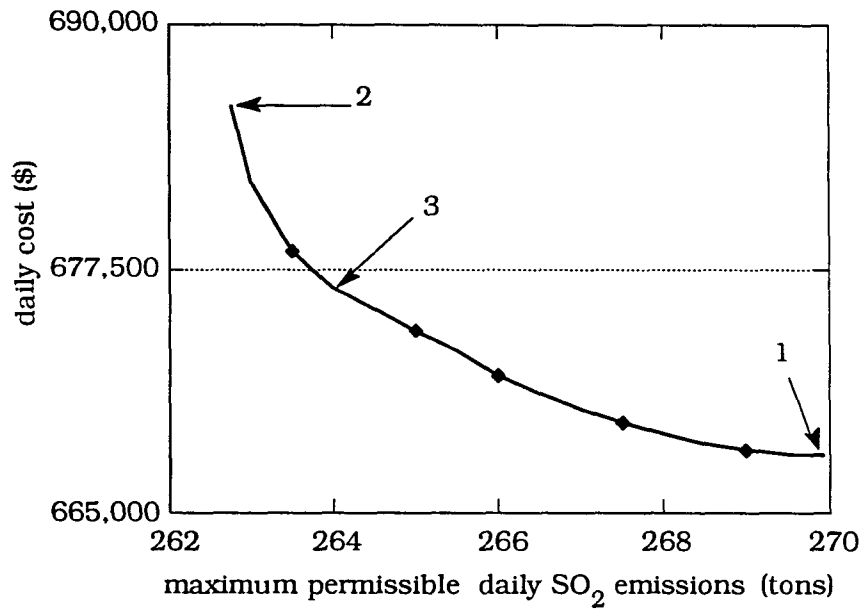


Figure 5.6 NEU daily operating cost subject to various SO₂ emission constraints

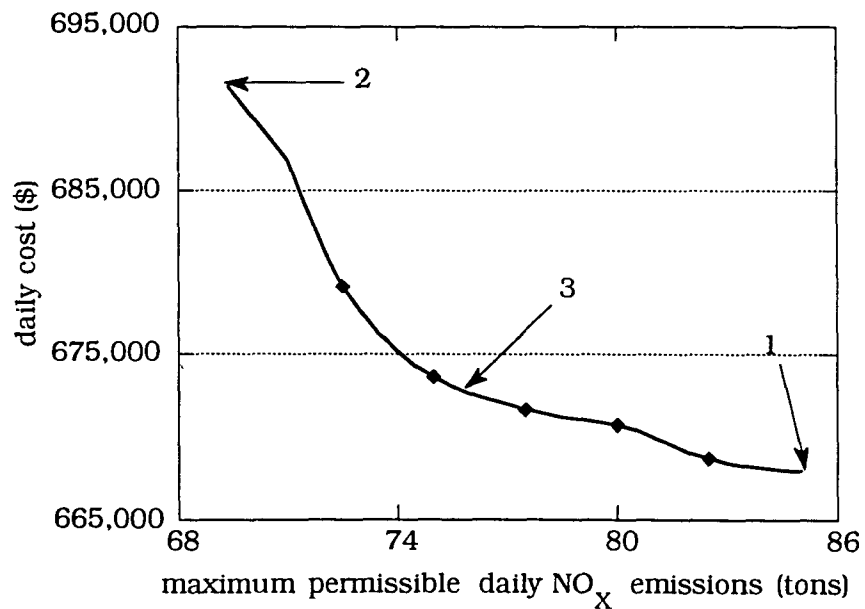


Figure 5.7 NEU daily operating cost subject to various NO_x emission constraints

Table 5.9 Results for power pool 168-hour example: 2 constraints

	POWER	FUEL	COST	NO _x	SO ₂
SPA1	24,861.4	233,778.1	327,289.1	140.27	34.44
SPA2	21,513.7	203,720.1	285,207.9	122.23	29.65
SPA3	17,898.7	171,250.0	239,749.9	102.75	33.06
VER1	23,955.2	212,265.9	371,466.0	127.36	13.19
VER2	17,171.1	150,165.8	262,789.9	90.10	10.22
VER3	30,100.0	318,650.5	557,638.1	191.19	19.84
LAS1	0.0	0.0	0.0	0.00	0.00
LAS2	2,205.0	24,169.1	43,504.4	12.08	9.54
LAS3	6,674.4	72,042.7	129,676.9	43.23	5.62
MAC1	9,056.5	88,206.0	141,129.6	44.10	9.27
MAC2	7,756.1	76,012.4	121,620.0	38.01	7.75
MAC3	12,480.0	121,304.0	194,086.3	72.78	11.24
RAV1	15,315.3	155,833.0	194,791.6	77.92	38.66
RAV2	31,703.2	306,802.4	383,503.4	153.40	45.00
RAV3	31,223.3	301,920.9	377,401.2	150.96	65.62
RAV4	34,462.3	325,261.0	406,575.9	162.63	76.42
TOR1	25,214.0	247,873.0	297,447.8	123.94	48.11
TOR2	41,878.6	428,440.6	514,128.8	214.22	53.68
TOR3	12,512.0	125,861.0	151,033.2	62.93	21.08
TOR4	76,891.0	731,243.3	877,492.8	438.75	132.63
FET1	49,907.9	476,329.4	714,495.0	238.16	79.54
FET2	69,314.2	716,107.6	1,074,160.0	358.05	55.56
TOTALS	562,093.9	5,487,237.0	7,665,189.0	2,965.06	800.12
		100.08%	100.25%	99.66%	97.70%

(the minimizing quantity) was noticed. In that sense, the presented emissions-constrained dispatch algorithm is not guaranteed to yield optimal results; however, it yields results satisfactorily close to optimal.

6. SUGGESTIONS FOR FUTURE WORK, CONCLUSIONS AND SUMMARY

6.1 Suggestions for Future Work

This research focused basically on the development of an emissions-constrained dispatch algorithm. Although the basic development is complete, many enhancements could be added and many features may be further investigated.

- Since the environmental restrictions are an ongoing issue and several crucial decisions are still pending, modifications may be necessary to implement future legislative provisions.
- The emissions-constrained dispatch method described previously yields reasonable results for a wide variety of cases. However, the optimality of the method needs further exploration. Furthermore, other optimization techniques may be applied to the emissions-constrained dispatch problem.
- Modeling of startup and shutdown emissions should be investigated. Such emissions should be included in the system emission output.
- Ways that would improve unit and emissions modeling should be investigated.
- Other iterative methods, instead of the bisection method, may be used to reduce the number of iterations.

- After an allowances market is working, the described method may be incorporated in an emission brokerage program.
- The emissions-constrained dispatch may be interfaced with other software such as a unit commitment program or a fuel scheduling program.
- Modifying the method so that it can be used on parallel computers, provided that such computers are readily available, would improve its efficiency and would make it suitable for medium range planning.

6.2 Conclusions and Summary

Recent legislation imposed additional environmental restrictions on the electric utilities. In order to comply with the new constraints, the electric power industry needs to come up with strategies that will reduce their emission output with a minimum increase in operating costs. Modified dispatching techniques seem to be an attractive tool for this task.

An emissions-constrained dispatch algorithm was presented that succeeds in meeting preset environmental constraints. The algorithm has some very attractive characteristics:

- the overall objective is cost minimization, thus emission overreduction is avoided
- local and system constraints may be handled
- limited input data requirements
- reasonable solution times

- different kinds of pollutants may be handled.

A new variable, named incremental emissions per incremental cost was introduced. It was defined as the derivative of emission output with respect to cost. This variable identified the units that would produce the largest emission change per unit cost. An iterative scheme successively altered the unit operating patterns, based on their incremental emissions per incremental cost, until the imposed emission constraints were satisfied.

Since it has shown reasonable solution times, the presented method is suitable for on-line application and could be of assistance to power system operators in order to comply with the new legislation.

BIBLIOGRAPHY

1. "News Analysis: The Impact of the New Acid-Rain Laws." *Electrical World*, December 1990: 9-10.
2. U.S. Congress. House. *Summary of H.R. 3030 "Clean Air Act Amendments of 1990."* April 20, 1990.
3. U.S. Congress. Senate. *Summary of S. 1630 "Clean Air Act Amendments of 1990."* April 20, 1990.
4. Lamarre, L. "Clean Air Act Amendments." *EPRI Journal*, April-May 1991: 21-29.
5. U.S. Environmental Protection Agency, Office of Air Quality Planning and Standards, *National Air Pollution Estimates*, Research Triangle Park, North Carolina, 1982.
6. Boutacoff, D. "Clean Air Act Amendments." *EPRI Journal*, March 1991: 5-13.
7. Niemeyer, V. "Emission Trading: Effects on Utility Planning and Operations." EPRI, Palo Alto, California, June 1991.
8. Desai, M. S., D. J. King, and A. M. Elgawhary. "Clean Air Act Amendments and their Impacts." *Proceedings of the American Power Conference*, vol. 53 I, Chicago, Illinois, 1991: 339-344.
9. Makansi, J. "Special Report. Clean Air Act Amendments. The Engineering Response." *Power*, June 1991: 11-66.

10. Bretz, E. A. "Equipment Options for Meeting the New Clean-Air Laws." *Electrical World*, October 1991: 51-59.
11. Mansour, M. N., D. W. Nass, J. Brown, and T. M. Jantzen. "Integrated NO_x Reduction Plan to Meet Staged SCAQMD Requirements for Steam Electric Power Plants." *Proceedings of the American Power Conference*, vol. 53 II, Chicago, Illinois, 1991: 964-970.
12. Chowdhury, B. H., and S. Rahman. "A Review of Recent Advances in Economic Dispatch." *IEEE Transactions*, vol. PWRS-5, 1990: 1248-1257.
13. Happ, H. H. "Optimal Power Dispatch - A Comprehensive Survey." *IEEE Transactions*, vol. PAS-96, 1977: 841-854.
14. Gent, M. R., and J. W. Lamont. "Minimum-Emission Dispatch." *IEEE Transactions*, vol. PAS-90, 1971: 2650-2660.
15. Lamont, J. W., and M. R. Gent. "Environmentally-Oriented Dispatching Techniques." *Proceedings of the 8th PICA Conference*, Minneapolis, Minnesota, 1973.
16. Lamont, J. W., K. F. Sim, and E. P. Hamilton III. "A Multi-Area Environmental Dispatching Algorithm." *Proceedings of the 9th PICA Conference*, New Orleans, Louisiana, 1975.
17. Sullivan, R. L. "Minimum Pollution Dispatch." C 72 468-7. IEEE PES Summer Meeting, San Francisco, California, July 1972.
18. Shepard, D. S. "A Load Shifting Model for Air Pollution Control in the Electric Power Industry." *Journal of the Air Pollution Control Association*, vol. JAPCA-20, 1970: 757-761.
19. Finnigan, O. E., and A. A. Fouad. "Economic Dispatch with Pollution Constraints." C 74 155-8. IEEE PES Winter Meeting, New York, New York, January 1974.

20. Delson, J. K. "Controlled Emission Dispatch." *IEEE Transactions*, vol. PAS-93, 1974: 1359-1366.
21. Zahavi, J., and L. Eisenberg. "Economic-Environmental Power Dispatch." *IEEE Transactions*, vol. SMC-5, 1975: 485-489.
22. Zahavi, J., and L. Eisenberg. "An Application of the Economic-Environmental Power Dispatch." *IEEE Transactions*, vol. SMC-7, 1977: 523-530.
23. Vertis, A. S., and L. Eisenberg. "An Approach to Emissions Minimization in Power Dispatch." *Journal of the Franklin Institute*, vol. 296, 1973: 443-449.
24. Gruhl, J. "Generator Maintenance and Production Scheduling to Optimize Economic-Environmental Performance." MIT Energy Laboratory, Cambridge, Massachusetts.
25. Ruane, M. F., J. Gruhl, F. C. Schweppe, B. A. Egan, D. H. Fyock, and A. A. Slowik. "Supplementary Control Systems - A Demonstration." F 75 474-7. IEEE PES Summer Meeting, San Francisco, California, July 1975.
26. Cadogan, J. B., and L. Eisenberg. "Sulfur Oxide Emissions Management for Electric Power Systems." *IEEE Transactions*, vol. PAS-96, 1977: 393-400.
27. Friedmann, P. G. "Power Dispatch Strategies for Emission and Environmental Control." ISA Power Instrumentation Symposium, Chicago, Illinois, 1973.
28. Cadogan, J. B., and L. Eisenberg. "Environmental Control of Electric Power Systems." 74-502. ISA International Instrumentation-Automation Conference and Exhibit, New York, New York, 1974.

29. Tsuji, A. "Optimal Fuel Mix and Load Dispatching under Environmental Constraints." 81 WM 103-1. IEEE PES Winter Meeting, Atlanta, Georgia, February 1981.
30. Hobbs, B. F. "Emissions Dispatch under the Underutilization Provision of the 1990 U.S. Clean Air Act Amendments: Models and Analysis." Case Western Reserve University, Cleveland, Ohio.
31. Luenberger, D. G. *Linear and Nonlinear Programming*. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 2nd edition, 1984.
32. Chapman, D. "A Sulfur Emission Tax and the Electric Utility Industry." *Energy Systems and Policy*, vol. 1, No. 1, 1974: 1-30.
33. Stadlin, W. O. "Economic Allocation of Regulating Margin." *IEEE Transactions*, vol. PAS-90, 1971: 1776-1781.
34. Wood, A. J., and B. F. Wollenberg. *Power Generation, Operation and Control*. New York: John Wiley and Sons, Inc., 1984.
35. Kirchmayer, L. K., and G. W. Stagg. "Analysis of Total and Incremental Losses in Transmission Systems." *AIEE Transactions*, vol. 70, 1951: 1197-1205.
36. Kirchmayer, L. K., H. H. Happ, G. W. Stagg, and J. F. Hohenstein. "Direct Calculation of Transmission Loss Formula." *AIEE Transactions*, vol. 79, 1960: 962-969.
37. Kron, G. "Tensorial Analysis of Integrated Transmission Systems - Part I: The Six Basic Reference Frames." *AIEE Transactions*, vol. 70, 1951: 1239-1248.
38. Brodsky, S. F., and R. W. Hahn. "Assessing the Influence of Power Pools on Emission Constrained Economic Dispatch." *IEEE Transactions*, vol. PWRS-1, 1986: 57-62.

39. Heslin, J. S., and B. F. Hobbs. "A Multiobjective Production Costing Model for Analyzing Emissions Dispatching and Fuel Switching." 89 WM 151-2 PWRS. IEEE PES Winter Meeting, New York, New York, January 1989.
40. Buttorff, L. A, and A. W. Latti. " Meeting Environmental Standards in an Integrated Resource Planning Process." *Proceedings of the American Power Conference*, vol. 53 I, Chicago, Illinois, 1991: 317-320.
41. Slump, D. J. "Economic Operation of a Power System under the Clean Air Act Amendments of 1990." Final report, American Public Power Association, 1991 Demonstration of Energy-Efficient Developments Scholarship program, Electric Power Research Center, I.S.U., Ames, Iowa, February 1992.
42. Schweizer, P. F. "Determining Optimal Fuel Mix for Environmental Dispatch." *IEEE Transactions on Automatic Control*, short papers, 1974: 534-537.
43. Huang, W., and B. F. Hobbs. "Estimation of Marginal System Costs and Emissions of Changes in Generating Unit Characteristics." *IEEE Transactions*, vol. PWRS-7, 1992: 1251-1258.
44. Zmuda, J. T. "Acid Rain Compliance Planning: Compliance Issues and Options." *Proceedings of the American Power Conference*, vol. 54 I, Chicago, Illinois, 1992: 371-376.
45. Obessis, E. V., and J. W. Lamont. "Emission-Constrained Economic Dispatch." *29th Annual Power Affiliate Report*, Electric Power Research Center, Iowa State University, Ames, Iowa, 1992.
46. Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Wetterling. *Numerical Recipes. The Art of Scientific Computing (FORTRAN Version)*. Cambridge, U.K.: Cambridge University Press, 1989.

47. Perkins, H. C. *Air Pollution*. New York: McGraw-Hill, Inc., 1974.
48. Linder, K., and E. Comer. "Economic Regulation of Allowance Trading." *Electric Perspectives*, Sep.-Oct. 1991: 30-42.
49. Garrity, T. F. "Clean Air Act Economics. GE Review." An Address to the Power Generation Committee of the Association of Edison Illuminating Companies, Atlanta, Georgia, April 3, 1991.
50. Himmelblau, D. M. *Applied Nonlinear Programming*. New York: McGraw-Hill, Inc., 1972.
51. Simmons, D. M. *Nonlinear Programming for Operations Research*. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1975.