EFFECT ON NEUTRON FLUX OF SOURCE GEOMETRY IN A URANIUM GRAPHITE SUBCRITICAL ASSEMBLY

by

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I. INTRODUCTION

The subcritical assembly of uranium and moderator has widely known application in the fields of nuclear research and education. The theory of its operation is standard material for reactor physics textbooks, and the results of many subcritical experiments have appeared in scientific and engineering journals. Recently the history, advantages and limitations of subcritical assemblies have been reviewed by Baughman (1).

In January 1957, a uranium graphite subcritical assembly was erected at Iowa State College. The research for this thesis was undertaken as a part of the initial program of work with this subcritical assembly. The part contained herein is concerned with the variation of source conditions and environment, and was first of interest because of the necessity of surrounding the original neutron source with a gamma ray shield which might have affected the results measurably. The review of the literature made it evident that source arrangements applicable to many specific conditions have been neglected by the theorists and virtually unmentioned by the experimenters. Then the problem of determining the optimum source conditions for the specific assembly at Iowa State College became important. With this in mind, a variety of conditions were applied to the source. The results and comparisons are contained herein.

II. REVIEW OF THE LITERATURE

The theory of the application of data from subcritical assemblies to critical reactors was developed by Anderson *et al*. (2) whose report was not published until 1952. In that same year Glasstone and Edlund (3) published a far more detailed analysis which appears to be the basic reference for most authors writing on the subject. Their development, though very complete, deals with none of the problems with which a researcher might be confronted. These problems are discussed at some length by Clayton (4) who covers the theory of the various calculations possible on a subcritical assembly, the experimental procedures used at Hanford, and the calculations and measurements of a large series of experiments. His treatment of source effects, however, is sketchy. Richey (5) introduces a reflection coefficient and deals with harmonics as correction factors. Duvall (6) also is concerned with harmonics and develops in detail the theory of the subcritical assembly for split sources. Hughes (7) gives a fairly good general discussion of the theory, boundary conditions and application of the assembly.

A brief treatment of the theory and of the laboratory techniques used with the subcritical assembly by the students at the Oak Ridge School of Reactor Technology is given by Klema (8). McCorkle's recent article (9) is an excellent but general treatment of the history, theory and uses of subcritical assemblies. Dopchie *et al*. (10) describe a typical uranium graphite subcritical assembly experiment conducted in Belgium, giving detailed accounts of the experimental procedure. Baughman (1) and Hoganson (11) both conducted a series of tests on the same subcritical assembly described herein, and their methods and results were of considerable assistance. None of the authors reviewed discussed in detail the physical location of the neutron source in relation to the assembly or to the source environment.

III. THEORY OF THE SUBCRITICAL ASSEMBLY

The theory of the subcritical reactor is based on the steady state diffusion equation for a critical reactor

$$\nabla^2 \phi + B_m^2 \phi = 0 \qquad (1)$$

which is a simplified form of the asymptotic reactor equation valid only at distances from the boundaries which are large in comparison with the slowing down length of the neutrons (3, p. 361). Here ϕ is the neutron flux in $cm^{-2} \cdot sec^{-1}$ and B_m^2 is the material buckling in cm^{-2}. Because in regions far from the source nearly all neutrons have resulted from fissions, Eq. 1 will represent closely the neutron balance in a large subcritical assembly maintained in a steady state by an outside source of neutrons. As will be shown, B_m^2 of a subcritical assembly may be obtained by experimental procedure. Since B_m^2 is only dependent upon the characteristics of the material of the assembly, it may be set equal to B_g^2 of a critical reactor (3, p. 199), and from this may be determined the critical size of a reactor of the same lattice and materials.

For this development an assembly will be assumed in the shape of a parallepiped such as is shown in Fig. 1 where a, b and c represent the extrapolated boundaries of the assembly. The steady state is maintained by a source of neutrons at one end. This source will be assumed to be a point source of fast neutrons centered below the z = 0 plane. If the material of the assembly is a reasonably good moderator most of the fast neutrons will be slowed down within two or three slowing down lengths of the source (3, p. 117). They will then behave as if they had originated from a distributed source of thermal neutrons. This hypothetical plane source will be assumed to be at $z = 0$.

The boundary conditions are as follows:

1. The flux everywhere in the assembly is finite and non-negative.
2. The flux is zero at the extrapolated boundaries except at $z = 0$.
3. For each value of m and n (to be introduced below) the number of neutrons flowing out of the plane $z = 0$ into the assembly must equal the number produced for that value of m and n in the source.

Eq. 1 in rectangular coordinates is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + B_m^2 \phi = 0. \qquad (2)$$

If one assumes the variables x, y, z are separable, Eq. 2 becomes

$$\phi = X(x)Y(y)Z(z) \qquad (3)$$

where $X(x)$ is a function of x alone, $Y(y)$ is a function of y alone and $Z(z)$ is a function of z alone.

With this substitution Eq. 2 becomes

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + B_m^2 = 0 \qquad (4)$$

after division by ϕ.

Since B_m^2 is a constant dependent only on the physical characteristics of the assembly, and the other terms may be assumed to be independent of one another, they must all be constant for Eq. 4 to be valid. If the terms in X, Y and Z are set equal to $-\alpha^2$, $-\beta^2$ and γ^2, respectively, where these terms are real and positive, Eq. 4 becomes

$$\alpha^2 + \beta^2 - \gamma^2 = B_m^2 . \qquad (5)$$

The equation of definition,

$$-\alpha^2 = \frac{1}{X}\frac{d^2X}{dx^2} , \qquad (6)$$

is a standard form of differential equation the solution of which under the boundary conditions imposed is

$$X = E \cos \alpha x , \qquad (7)$$

where E is a constant of integration, $\alpha = \frac{m\pi}{a}$, and m is any odd positive integer. Similarly

$$Y = F \cos \beta y \qquad (8)$$

where F is a constant of integration, $\beta = \frac{n\pi}{b}$, and n is any odd positive integer.

The solution for Z, given in detail by Glasstone and Edlund (3, p. 120) leads to a solution in hyberbolic functions which may be reduced to

$$Z = Ge^{-\gamma z}\left[1 - e^{-2\gamma(c - z)}\right] \qquad (9)$$

with the constants combined in G.

Since α and β take on a series of values and by Eq. 5, γ is dependent on α and β, Eq. 5 now becomes with substitution

$$\gamma_{mn}^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 - B_m^2 , \qquad (10)$$

and Eq. 9 becomes

$$Z_{mn} = C_{mn}\, e^{-\gamma_{mn} z}\left[1 - e^{-2\gamma_{mn}(c - z)}\right] . \qquad (11)$$

A substitution of X, Y and Z into Eq. 3 gives the general solution

$$\phi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \cdot e^{-\gamma_{mn} z}\left[1 - e^{-2\gamma_{mn}(c - z)}\right], \qquad (12)$$

where all arbitrary constants have been combined into A_{mn}.

The evaluation of A_{mn} depends on the source condition. There are two standard methods of treatment. One employs the Dirac delta function (3, p. 121); the other makes use of the method of images (6, p. 3). The results obtained by both methods differ from one another in magnitude but show the source term to be proportional to $\frac{S}{abD\gamma_{mn}}$ where S is defined as the number of neutrons emitted per second by a thermal point source at the origin and D is the diffusion coefficient in cm. Since S is hypothetical the difference in magnitude is of no consequence, and it is interesting to note that in the original development by Anderson *et al*. (2), no evaluation of the effect of source strength was made.

If all the source term constants are combined for simplicity into a single term Q so that $Q = \frac{KS}{abD}$ where K is a constant of proportionality, Eq. 12 becomes

$$\phi = Q \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\gamma_{mn}} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cdot$$

$$e^{-\gamma_{mn} z}\left[1 - e^{-2\gamma_{mn}(c - z)}\right]. \qquad (13)$$

Inspection of Eq. 10 shows that γ_{mn} has its lowest value when m = 1 and n = 1 because B_m^2 is constant. The term in brackets rapidly approaches unity with increasing γ_{mn} and with the values of γ_{11} encountered only differs significantly from unity when z approaches c. This term in brackets is called the end correction factor. For further discussion it will be regarded temporarily as equal to unity.

At a central vertical axis with x = 0 and y = 0, the cosine terms will equal unity. Then Eq. 13 becomes

$$\phi = Q \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\gamma_{mn}} e^{-\gamma_{mn} z} . \qquad (14)$$

Since γ_{mn} increases with m and n and $e^{-\gamma_{mn} z}$ approaches zero with increasing γ_{mn} and z, at some value of z flux due to harmonics above m = 1 and n = 1 will be negligible. This value occurs at a z of approximately two diffusion lengths (3, p. 124). Beyond this z, but less than a z where end corrections would be significant, Eq. 14 becomes

$$\phi = Q' e^{-\gamma z} , \qquad (15)$$

where $\gamma = \gamma_{11}$ and $Q' = \frac{Q}{\gamma}$. Thus within the limitations mentioned, the flux in a subcritical assembly will decrease exponentially with increasing distance from the source of neutrons. Since the cosine terms eliminated previously would be constant for constant x and y, the preceding statement holds for any value of x and y not close to the boundaries.

If the flux on the z axis is measured with equal efficiency at all points, the results will agree with the equation

$$A = Ce^{-\gamma z} \qquad (16)$$

where A is measured activity and C relates actual flux to measured activity. The results may be plotted as ln A versus z, and the slope of the line formed will be γ. This value may then be substituted into Eq. 10 with m = 1 and n = 1 to determine B_m^2. This value, dependent only on the material characteristics of the assembly, is equal to B_g^2, the geometric buckling, for a critical reactor. In this manner a subcritical assembly may be used to determine the critical size of a reactor of the same lattice configuration.

If the assembly is relatively small, both harmonics and end effects may have to be taken into account. The end correction factor

$$1 - e^{-2\gamma_{mn}(c - z)}$$

is only applicable when z approaches c in value. In this range, except in an assembly which is small in relation to its diffusion length, the flux due to the harmonics higher than the fundamental is negligible. Therefore the end correction may be taken as

$$1 - e^{-2\gamma_{11}(c - z)} .$$

If measurements may be taken above a value of z where the harmonics beyond the 33 harmonics are negligible, Eq. 13 for the z axis may be expanded to

$$\phi = Q\left[\frac{e^{-\gamma_{11}z}}{\gamma_{11}} + \frac{e^{-\gamma_{13}z}}{\gamma_{13}} + \frac{e^{-\gamma_{31}z}}{\gamma_{31}} + \frac{e^{-\gamma_{33}z}}{\gamma_{33}}\right] \cdot$$

$$\left[1 - e^{-2\gamma_{11}(c - z)}\right] . \qquad (17)$$

Here $\frac{1}{\gamma_{11}} e^{-\gamma_{11}z}$ is the predominant term. If one considers a square based assembly where $\gamma_{31} = \gamma_{13}$, Eq. 17 may be rewritten

$$\phi = \frac{Q}{\gamma_{11}} e^{-\gamma_{11}z} \cdot$$

$$\left[1 + \gamma_{11}e^{\gamma_{11}z}\left(\frac{2}{\gamma_{13}} e^{-\gamma_{13}z} + \frac{1}{\gamma_{33}} e^{-\gamma_{33}z}\right)\right] \cdot$$

$$\left[1 - e^{-2\gamma_{11}(c - z)}\right] . \qquad (18)$$

In this expression the first term in brackets is called the harmonic correction factor and the second term in brackets is the end correction factor referred to previously. If both sides of Eq. 18 are divided by the two correction factors, an expression similar to Eq. 16 results. Thus flux measurements taken on the z axis of a square based assembly may be divided by the appropriate correction factors, and if the natural logarithm of the corrected activities is plotted against z, the resultant curve should be a straight line of slope $-\gamma$.

If the source is divided into four equal parts placed at $(\frac{a}{4},0,0)(0,\frac{b}{4},0)(-\frac{a}{4},0,0)$ and $(0,-\frac{b}{4},0)$, respectively, with the total source strength equal to that of the single centralized source previously postulated, it may be shown (6, p. 13) that the equation for flux corresponding to Eq. 13 is

$$\phi = \frac{Q}{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\cos \frac{m\pi}{4} + \cos \frac{n\pi}{4}\right) \frac{1}{\gamma_{mn}} \cos \frac{m\pi x}{a} \cdot$$

$$\cos \frac{n\pi y}{b} e^{-\gamma_{mn} z} \left[1 - e^{-2\gamma_{mn}(c - z)}\right] . \qquad (19)$$

If one assumes the same conditions which allowed the development of Eq. 18 from Eq. 13, the equation corresponding to Eq. 18 is

$$\phi = \frac{\sqrt{2}}{2} \frac{Q}{\gamma_{11}} e^{-\gamma_{11} z} \left[1 - \gamma_{11} e^{\gamma_{11} z} \left(\frac{1}{\gamma_{33}} e^{-\gamma_{33} z}\right)\right] \cdot$$

$$\left[1 - e^{-2\gamma_{11}(c - z)}\right] . \qquad (20)$$

Note that the term in parenthesis in Eq. 19 has caused the cancellation of the 13 and 31 harmonics which were several times larger than the 33 harmonics. A comparison of Eq. 20 with Eq. 18 shows that dividing the source has considerably reduced the effect of harmonics at the expense of reducing the flux by a factor of 0.707.

Developments with two sources placed at $(\frac{a}{4},0,0)$ and $(-\frac{a}{4},0,0)$, respectively, and with four sources placed at $(\frac{a}{4},\frac{b}{4},0)$, $(-\frac{a}{4},\frac{b}{4},0)$, $(\frac{a}{4},-\frac{b}{4},0)$ and $(-\frac{a}{4},-\frac{b}{4},0)$, respectively, have shown that the first of these configurations has the disadvantage of reducing the total flux but eliminates the 13 and 31 harmonics only at the center (6, p. 8), and that the second of these configurations eliminates no harmonics while reducing the flux by 0.500 (6, p. 13).

IV. STATEMENT OF THE PROBLEM

The application of the theory of the subcritical assembly to actual physical conditions presents several problems. These include those associated with the boundary conditions, size of source, strength of source, size of the assembly, end corrections, harmonics, and the presence of fast neutrons, which are taken up in order below.

The boundary conditions require a zero flux at the extrapolated boundaries. This is closely approximated at a boundary of graphite and air because air is a poor moderator and reflects few neutrons. Such a boundary can be brought even closer to the theoretical requirement by covering the assembly with a strong absorber of thermal neutrons, such as cadmium (7, p. 81). This cover of cadmium is not primarily to prevent thermal neutrons from leaking out of the assembly but to prevent those which do leak out from being reflected back into the assembly, thereby preventing any secondary extraneous neutron source from changing the flux pattern originated by the primary source. Such a cadmium cover is normally put on the sides and top of the subcritical assembly. The cover may be put outside of the source on the bottom, but this is not necessary as is explained below. Any standard neutron source used with a subcritical assembly will emit neutrons in all directions. Many originally directed away from the assembly may be reflected back into it. Since these neutrons have suffered at least one collision, they will have less energy than those source neutrons which entered the assembly unhindered. Such neutrons will soon be thermalized and will add to the thermal flux. This should cause no concern for the following reason. The flux due to the unreflected neutrons will in general follow the form

$$\phi_2 = \phi_1 e^{-\gamma(z_2 - z_1)} \qquad (21)$$

where z_1 and z_2 are any distances measured vertically in the assembly far from the boundaries. For the same z_1 and z_2 the flux due to the reflected neutrons will in general follow the form

$$\phi_2' = \phi_1' e^{-\gamma(z_2 - z_1)} . \qquad (22)$$

The γ's are the same because γ is solely dependent on the physical constants of the assembly. Adding Eq. 21 and Eq. 22,

$$\phi_2 + \phi_2' = (\phi_1 + \phi_1') e^{-\gamma(z_2 - z_1)} .$$

This shows that the flux due to reflected source neutrons will not change the rate of attenuation of the primary source neutrons. In fact, the reflected neutrons will increase the total flux and thus aid in reducing errors due to low counting rates. Consequently a cadmium shield on the base of a subcritical assembly, assuming the source is located at the base, is neither necessary nor desirable.

Reflected source neutrons were not considered in the theory as developed earlier. It appears that being reflected they would have a different harmonic distribution than unreflected neutrons and would therefore have a different harmonic correction. Most authors ignore this. Richey (5, p. 5) gives for this effect a correction factor of ($\frac{1}{2}$ coth $\cdot Z\gamma_{mn}$ + $\frac{1}{2}$) which is to be divided into measured activity. Here Z is the distance from the source to the bottom of the moderating pedestal. Although this correction factor was not directly applicable to all the conditions concerned here, it does indicate that this correction factor for all but the fundamental mode is essentially equal to one. A correction of this type multiplies each measured activity by a constant and does not alter the buckling or the expected cosine distribution of the flux. Therefore there is little error introduced in considering all the activity as due to primary source neutrons.

A cadmium shield on the base of the assembly would increase the leakage of fission neutrons from the base and would definitely decrease the neutron multiplication in the lower levels ($0 < z < 24$ in.) of the assembly. This would give lower activity at these levels. Since the flux higher in the assembly is directly dependent on the activity at the lower levels where the source neutrons become thermal, the presence of cadmium at the base would decrease overall activity, which is not a desired effect.

During the experiment presented here, cadmium sheeting was placed on the top and sides of the assembly. There was none at the base; instead, tanks of water were placed directly underneath the assembly to act as a reflector. The flux in the assembly was greater when these tanks were filled than when they were empty which indicated increased leakage of fission neutrons and decreased reflection of source neutrons as the air space between water and assembly increased.

The usefulness of a subcritical assembly is greatly dependent on the source strength, that is, the number of neutrons emitted per second by the source. It is necessary to have a source strong enough to produce in the foils sufficient beta radiation to be measured without excessive deviation by the counters available. The source used produced an average of 100 counts per minute at a distance of 54 in. from the base of the assembly. Rates under this were avoided whenever possible when determining buckling values. To obtain higher rates by means of increasing the source strength generally results in an increase in shielding difficulties with a consequent loss of experimental flexibility. The source used required a minimum of shielding and produced strengths considered adequate for the purpose. This source consisted of five small cylinders arranged as shown in Fig. 2 when most closely approximating a point source. This is not the point source called for in the theory but in comparison with the assembly size of 165 cu. ft. could be assumed to be a point source without serious error.

The physical size of the assembly was 60 in. by 60 in. by

79 in. Since the neutron diffusion length in graphite is approximately 20 in., these dimensions seem rather small. Most of the authors are vague on the minimum required size of a uranium graphite subcritical assembly (7, p. 219; 3, p. 361), but they agree that the larger the assembly the greater the accuracy. The effect of size makes itself felt in two ways. For m = 1, n = 1, Eq. 10 becomes

$$B_m^2 = (\frac{\pi}{a})^2 + (\frac{\pi}{b})^2 - \gamma^2 , \qquad (23)$$

For this assembly B_m^2 was considerably smaller than either $\left[(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2 \right]$ or γ^2. This meant that a slight error in either of these two factors resulted in a large error in B_m^2. For a particular fuel and moderator configuration there should be only one value of B_m^2. Therefore, if an assembly of a given lattice configuration is increased in size, there should be no change in B_m^2 as a and b increase. If a and b increase, then $\left[(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2 \right]$ must decrease. Then to keep the right hand side of Eq. 23 constant, γ^2 must also decrease. Since both $\left[(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2 \right]$ and γ^2 decrease as the assembly increases in size and since B_m^2 is equal to the difference of these two quantities, a percentage error in determining the value of either of these quantities will produce a smaller error in the value of B_m^2 for a large assembly than for a small assembly.

Length of the assembly in the z direction is important because of harmonics. If the assembly is long enough, measurements may be taken far enough from the source to obviate the need for harmonic corrections. Whether or not this is possible depends on the strength of the source and on the lateral dimensions. To decrease the length of side in order to increase height is not necessarily advantageous, because smaller sides allow increased leakage.

Application of end corrections to measurements made on a subcritical assembly is relatively simple. The assembly here discussed was large enough so that in the area where the end corrections were to be applied, the harmonics above the fundamental were damped out. Therefore the end correction could be applied directly to the measured activity.

There is no great mathematical difficulty in the application of harmonic corrections to the measured activity except that an iteration method is necessary. A serious problem occurs in choosing the location of the base of the assembly (z = 0 plane) for the purpose of applying the harmonic corrections. Those applied herein begin between 1.5 and 1.6 at z = 6 in., decrease rapidly with increasing z, and are not significantly different from 1.00 at z = 54 in. These factors are divided into the measured activity at their respective z levels. If the z = 0 plane is shifted upwards, each measured activity is divided by a larger factor. This causes a larger relative decrease of the measured activities at the lower levels which will tend to decrease the slope of the curve of ln A vs. z. The result is a definite increase in buckling. The opposite result occurs when the z = 0 plane is shifted downward.

The theory makes two apparently contradictory assumptions concerning the source. The first assumption is that a point source of fast neutrons placed in a moderator will produce neutrons which a few slowing down lengths from the source will behave as if they originated from a plane source of thermal neutrons (3, p. 117). This assumption is based on the similarity of the flux distribution due to point and plane sources of thermal neutrons (3, p. 106) and on the similarity beyond a few slowing down lengths of the flux distribution due to plane sources of fast and thermal neutrons (3, p. 187). The postulated thermal plane should be the location of the z = 0 plane.

The second assumption, used for evaluating the source strength term of the resultant equation of the flux distribution, is that the source is actually a point source of thermal neutrons located at the assumed plane of thermal neutrons (3, p. 121). Both assumptions lead to a resultant equation in terms of a Fourier cosine series. Neither assumption definitely locates the thermal plane source in relation to the physical location of the fast point source or in relation to the assembly base. None of the authors of the references reviewed for this thesis discussed in much detail the problem of locating the z = 0 plane. Most of them assumed it to be the physical base of the assembly proper, whether the source was located there (2, p. 12) or three slowing down lengths from it (3, p. 284).

For this thesis z = 0 was taken at the physical base of the assembly regardless of the distance to physical location of the fast source below the base. This was done for several reasons.

1. Other researchers have used this assumption (4, p. 20; 5, p. 2).
2. This assumption is not unreasonable as long as measurements near the source are not used to determine the flux attenuation.
3. This assumption eliminated a possible variable and allowed easier comparison of related results.
4. Since the actual location of the z = 0 plane is coincident with the assumed thermal source plane which is hypothetical, any definite location of the z = 0 plane is open to serious question.

As mentioned above, theory requires a source of thermal neutrons. No such usable point source exists. The fast neutron source used in its place puts fast neutrons into the assembly. These must be and are for the greater part thermalized with two or three slowing down lengths of the moderator. Normally this is accomplished by placing the source in a pedestal of moderator material similar to that of the assembly. When the neutrons enter the assembly proper, they are generally thermalized. The foils then count few if any epithermal source neutrons. The resonance neutron cross section of indium for 1.3 ev neutrons is 28000 barns, whereas its cross section for thermal neutrons is only 300 barns. It is expected that the indium will count epithermal fission neutrons, but as these will be produced at the same relative rate throughout the assembly, this will not alter the vertical flux attenuation. The presence of resonance source neutrons in the lower levels of the pile will, because of the ninety times higher cross section of indium for resonance neutrons, produce higher measured activity in the lower level. This would not be reflected relatively in the activity at higher levels because the assembly multiplication is not directly proportional to the number of epithermal neutrons. This erroneous high count tends to steepen the attenuation curve and indicate a value of buckling lower than the true value.

A measure of the effect of these resonance neutrons is the cadmium ratio. This is defined as

$$C.R. = \frac{A\ \text{resonance} + A\ \text{thermal}}{A\ \text{resonance}}$$

where A is measured activity, and the right hand side is equal to the activity of a bare foil divided by the activity of an equal foil wrapped in cadmium at the same location in the assembly. At a distance from the source where there is a negligible amount of epithermal source neutrons, the resonance activity will be due to fission neutrons. Then the cadmium ratio should be constant since the relative amounts of thermal flux and epithermal flux due to fission is constant. In a region of constant cadmium ratio, therefore, no corrections need be made for the presence of epithermal source neutrons.

The change in lattice at z = 54 in. is of no consequence. The distance between fuel channels decreased only from 8.5 in. to 7.8 in. No measurements used in computing bucklings were taken inside the region of smaller lattice.

The general undertaking of this experiment was to determine the effect on the flux due to variation of several source parameters. This was done by determining the assembly buckling as the source was raised or lowered, centralized or spread, and immersed in water or dry. Regardless of the source variation, if the requirements of the theory were fulfilled the buckling should have remained constant. Due to many of the factors discussed above, the buckling did not remain constant. Those factors which did affect the buckling variations are reviewed briefly below before discussing them in terms of specific results.

It may be assumed that a change in measured buckling is due only to a change in the measured flux at the lower levels of the assembly relative to the measured flux at the upper levels. For simplicity it will be further assumed at this point that the upper level flux will remain constant and that any change is due to a fluctuation in measured flux at the levels near the source. In this region then higher activity indicates lower buckling and lower activity indicates higher buckling, providing this change in activity is characteristic only of the low levels.

Leakage from the base reduces multiplication at low levels. Since multiplication will improve further within the assembly, leakage here decreases activity at lower levels out of proportion to its overall effect, and consequently increases buckling.

Reflection may be considered the normal condition from which leakage is a variant. Reflection then serves to increase activity throughout the assembly and causes no change in buckling.

Unmoderated neutrons passing through the base of the assembly serve to increase the measured activity at lower levels which indicates a decrease in buckling. This effect is reduced when the source neutrons are moderated or reflected and a greater number of thermal neutrons flow into the assembly.

V. EXPERIMENTAL EQUIPMENT

The subcritical assembly used for this thesis is pictured in Fig. 3. Each side of the square base of the assembly was 60 in. and the height was 79 in. Graphite blocks with a specific weight of 97.3 lb./cu.ft. were the principal component. These 60 in. long blocks were of two sizes, 6 in. by 6 in. or 6 in. by 5 in., and were stacked horizontally in rows of ten each. The bigger blocks formed the lower nine rows, while the upper five rows were comprised of the smaller blocks. The lengthwise corners of the blocks were rounded so that at the junction of every four there was formed traversing the length of the assembly a hole shaped like a rhombus with concave sides (Fig. 4). These holes had a minimum dimension of 1.38 in. and were designed to hold the fuel rods.

The fuel used was natural uranium in the form of cylindrical rods 1.00 in. in diameter and 8.00 in. long. These rods were encased in aluminum 0.035 in. thick on the sides and 0.200 in. thick at the ends. Seven fuel rods were placed in a fuel rod hole when the hole was filled. For the tests described herein every second hole was filled so that a graphite uranium lattice of 8.5 in. was formed.

The assembly was supported by a wooden framework which allowed three open topped aluminum tanks 11.75 in. deep to be slipped directly underneath a sheet of plywood upon which the bottom layer of graphite blocks rested. Normally water was placed in these tanks for moderating and shielding purposes. An 8 in. by 18 in. by 11.75 in. compartment was placed in the center of the middle tank. This could be kept dry if desired while the rest of the tank was filled. The neutron source was normally placed in this compartment (Fig. 5).

The four sides and top of the assembly were covered with a sheet of 0.010 in. thick cadmium sandwiched between plywood and masonite. The cadmium provided a black boundary to thermal neutrons and prevented reflection of scattered neutrons back into the assembly from the surroundings.

The assembly was designed for flux measurement by foil activation. For this purpose single 1.25 in. by 0.125 in. slots were cut lengthwise in the upper sides of thirty-one of the graphite blocks (Fig. 4). In these slots were inserted thin notched strips of aluminum holding indium foils glued on small rectangular aluminum planchets. The indium foils were 0.003 in. by 1.0 in. by 1.5 in. and weighed an average of 0.5953 gm. The same foil was used in the same position throughout all runs.

Because there was an even number of graphite blocks per row and the foil slots were cut in the center of the top of each block, there were no foils directly over the center of the assembly base. The vertical flux distribution closest to the z axis was taken at x = -3 in., y = 0 (Fig. 6). The horizontal distribution in the y direction was taken at x =

-3 in., z = 30 in. The x direction distribution was taken at y = 0, z = 30 in. (Fig. 6).

After irradiation the foils were counted in a nearby room where the average measured background activity was 45 counts per minute. A glass wall Geiger tube in a wooden counting box (Fig. 8) was used. Counting geometry was reproduced by means of a simple aluminum holder which fixed the location of every foil counted. The scaler used was a Nuclear Chicago Model 181A. The reliability of the tube and counter was checked before each counting session with a strontium-yttrium source with an average measured activity of 2850 counts per minute.

In several runs cadmium covered foils were used. The cadmium used for each foil was a 0.010 in. thick rectangular planchet 1.50 in. by 2.25 in. folded once to make an envelope into which the indium foil on its aluminum backing was slipped. The average weight of the cadmium envelope was 5.24 gm.

The neutron source used for the subcritical assembly consisted of five separate one curie plutonium-beryllium capsules encased in tantalum and stainless steel cylinders 1 in. in diameter and 1.125 in. high. The rated strength of the individual sources averaged 1.63×10^6 neutrons per second. All five sources were used during each run. For Runs 1 through 11 the individual sources were held in a wooden frame in a cruciform shape adjacent to one another as shown in Fig. 2. For Runs 12 and 13, each source was held individually in a wooden holder, and the sources were placed as shown in Fig. 7. Throughout this paper these two configurations will be referred to as centralized or spread, respectively. Both configurations were centrally symmetrical, to facilitate mathematical treatment of the data. The centralized source was always placed in the center compartment of the middle tank previously discussed. This will be referred to as the source compartment.

VI. EXPERIMENTAL PROCEDURE

The experimentation was conducted in thirteen separate foil loadings or runs. For all runs the assembly was so loaded that on a vertical line through $x = -3$ in., $y = 0$, there was a foil on top of each block every 6 in. from $z =$ 6 in. to $z = 54$ in. and every 5 in. from $z = 54$ in. to $z =$ 74 in. (Fig. 6). There was a corresponding number of cadmium covered foils at $x = -3$ in., $y = -3$ in. during Runs 1 through 8 only. In the $z = 30$ in. plane for all runs there were foils at 6 in. intervals across the width of the assembly at $x =$ -3 in. and at $y = 0$ (Fig. 6).

For the first four runs, the source was centralized. The source compartment was dry while the rest of the tanks were filled with water. For Run 1 the source was placed against the plywood base, putting the source center 1.25 in. from the base of the graphite. This distance from source center to the $z = 0$ plane will be designated z'. For Runs 2, 3 and 4 all conditions stayed the same except that z' increased to 3.75 in., 6.25 in. and 10.25 in., respectively.

At the conclusion of Run 4, the source compartment was filled with water. The source was raised progressively for the next four runs so that the source geometries for Runs 5, 6, 7 and 8 were identical to those for Runs 4, 3, 2 and 1, respectively. Conditions for Run 9 were identical to those of Run 8 except that the cadmium covered foils had been removed from the assembly. Run 10 was a replicate of Run 9. For Run 11 water was drained from all of the tanks. Otherwise conditions were the same as for Runs 8 through 10, that is, source centralized and $z' = 1.25$ in.

The source configuration was spread for Runs 12 and 13 (Fig. 7) while z' remained at 1.25 in. All tanks were dry for Run 12 and all were filled for Run 13.

A summary of the variable source parameters is included with the tabulated results in Table 2.

The immediate object of the measurements in each set of data was to evaluate γ, the inverse relaxation length of the neutron flux in the z direction. If the source conditions for each set of data fall within the limits of the theory and the various correction factors are properly applied, γ should be the same for each set. The variation which can be tolerated is extremely small because of the great dependence of the buckling, and consequently the critical size, upon the value of γ.

From Eq. (10)

$$B_m^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 - \gamma^2 \qquad (24)$$

where a and b are the lengths of the sides of the assembly including extrapolation lengths. Since both are equal to 62 in. in this case,

$$B_m^2 = 51.40 \times 10^{-4} - \gamma^2 \qquad (25)$$

with dimensions of in^{-2}. For a critical cubical reactor with length of side L,

$$L^3 = \frac{161}{B^3} \text{ cm}^3 \qquad (3, \text{ p. } 208) \qquad (26)$$

so that for this assembly

$$L = 0.484 \, (51.40 \times 10^{-4} - \gamma^2)^{-\frac{1}{2}} \text{ ft.} \qquad (27)$$

This equation is plotted as Fig. 9. Since for a critical reactor the material buckling is equal to the geometric buckling, B^2 has been substituted for both $B_m^{\ 2}$ and $B_g^{\ 2}$. Because the average experimental value of γ^2 equals 46.60×10^{-4} in^{-2} B^2 is equal to the difference of two nearly equal numbers ten times larger than itself. Therefore the experimental values of B^2 and L will vary greatly with small changes in γ, as is shown in Fig. 9.

The procedure followed for determining γ, with data from Run 1 is given below:

1. Indium foils, after being irradiated were removed from the assembly four at a time, and after the decay of an indium isomer with a 13 second half life had been allowed 3 minutes to decay to less than 0.01 per cent, the activity of each one was counted for 3 minutes. After the raw count was converted to counts per minute, the background rate was subtracted from it. The count then was corrected back to the time of removal from the neutron flux and was adjusted for difference between the weight of the counted foil and the average weight of the foils.

2. The natural logarithms of these measured activities were plotted against z. The slope of the best straight line

drawn by inspection through these points was measured. This was designated as trial γ_{11}.

3. The trial value of γ_{11} was used to determine the value of the end correction factor

$$1 - e^{-2\gamma_{11}(c - z)} \tag{28}$$

and the harmonic correction factor developed from Eq. 18

$$1 + \frac{\gamma_{11}\, e^{z\gamma_{11}}}{\cos \frac{3\pi}{a}} \cdot$$

$$\left(\frac{e^{-z\gamma_{13}}}{\gamma_{13}} \cos \frac{3\pi}{a} + \frac{e^{-z\gamma_{31}}}{\gamma_{31}} \cos \frac{9\pi}{a} + \frac{e^{-z\gamma_{33}}}{\gamma_{33}} \cos \frac{9\pi}{a}\right) \tag{29}$$

which is applicable only to the vertical row of foils through $x = -3$ in., $y = 0$. Since $a = 62$ in. and $\gamma_{31} = \gamma_{13}$, correction factor (29) was simplified to

$$1 + \frac{\gamma_{11}\, e^{z\gamma_{11}}}{0.988} \left(\frac{1.884}{\gamma_{13}} e^{-z\gamma_{13}} + \frac{0.895}{\gamma_{33}} e^{-z\gamma_{33}}\right) \tag{30}$$

where the γ_{13} and γ_{33} terms are determined from Eq. 10 after B^2 has been evaluated from Eq. 25. For the split source the corresponding harmonic correction factor was

$$1 + \frac{\gamma_{11}\, e^{z\gamma_{11}}}{0.988} \left(\frac{0.493}{\gamma_{13}} e^{-z\gamma_{13}} - \frac{0.427}{\gamma_{33}} e^{-z\gamma_{33}}\right) . \tag{31}$$

End corrections for harmonics above the fundamental were found to be negligible. Harmonic corrections above the 33 harmonic were negligible in the range of z used for determining γ and for simplicity were not applied to any activities.

4. The end correction factor and the applicable harmonic correction factor were multiplied together and divided into the measured activity. The resultant corrected activities in the range of $z = 18$ in. to $z = 54$ in. were used to determine a value of γ. Activities below $z = 18$ in. were not used because of the suspected presence of epithermal neutrons and excess leakage. Activities above $z = 54$ in. were avoided because of the change of lattice at $z = 54$ in. and because the low counts in that range allowed too much possible statistical deviation. γ was determined by means of a method of least squares (12, p. 166) applicable to an equation of the type $A = Ce^{-\gamma z}$, with steps as follows:

a. Tabulate the values of A, ln A, z, z^2 and (z ln A).

b. Add the columns to determine $\sum \ln A$, $\sum z$, $\sum z^2$ and $\sum (z \ln A)$.

c. Insert these summations in following equations where n is the number of measurements:

$$\sum \ln A = -\gamma(\sum z) + (n \ln C) \qquad (32)$$

$$\sum (z \ln A) = -\gamma(\sum z^2) + (\ln C)\sum z \,. \qquad (33)$$

d. Solve these equations simultaneously for γ.

For the data of Run 1, the above steps were as shown in Table 1.

Table 1. Data for least squares approximation

z	z^2	A (corrected)	ln A	z ln A
18	324	1954.7	7.5780	136.404
24	576	1399.5	7.2439	173.854
30	900	917.0	6.8211	204.633
36	1296	605.8	6.4065	230.634
42	1764	384.8	5.9527	250.013
48	2304	258.3	5.5531	266.549
54	2916	157.1	5.0569	273.073
Σ 252	10080		44.6122	1535.160

Substituting:

$$44.612 = -\gamma(252) + 7 \ln C \qquad (32')$$

$$1535.160 = -\gamma(10080) + 252 \ln C \ . \qquad (33')$$

Solving these equations simultaneously,

$$\gamma = .07032 \ .$$

It should be noted that like B^2, γ is dependent upon the small difference of two large numbers and that ordinary slide rule calculations were not accurate enough for this computation.

5. If the computed γ from step 4 equalled the trial γ_{11}, the solution was completed to find C, and the equation $A = Ce^{-\gamma z}$ was plotted in the range of the corrected activ-

ities.

6. If the computed γ did not equal the trial γ_{11}, a second trial γ_{11} equal to the computed γ was chosen, and the correction factors were recalculated. The measured activities were adjusted with the new correction factors, and steps 4 and 5 were repeated.

7. This procedure continued until the computed γ equalled the last trial γ_{11}. Usually three repetitions were sufficient. Using this best value of γ, step 6 was followed. The corrected values of the activities were plotted on the same graph as the equation $A = Ce^{-\gamma z}$ to indicate the fit of the line to the points and to give an idea of the fit of the activities at the top and bottom of the assembly.

Once the true value of γ for each run was computed there remained two other steps. The first was to convert the individual values of γ into terms of the more familiar buckling. This was done by means of Fig. 9 and the results are shown on Fig. 10. The second step was the application of the computed γ in determining correction factors for the horizontal flux distribution. These were readily determined from Eq. 13 with γ_{mn} and z constant.

With the activity of every point now corrected so that the activity of each run could be fitted to the general equation $A = Ce^{-\gamma z}$, a series of graphs were drawn comparing the flux distribution for related source conditions. This concluded the experimental procedure.

VII. DISCUSSION OF RESULTS

Table 2 contains the experimental results of each run together with the variable source parameters. The buckling values are compared graphically in Fig. 10. Graphical comparisons of vertical flux distributions for related runs are shown beginning with Fig. 11. Graphical comparisons of horizontal flux distributions for several runs are shown from Fig. 17 to Fig. 19. Cadmium ratio curves are shown in Fig. 20.

It was postulated previously that since the physical characteristics of the assembly remained constant there should be only one value of buckling for all runs regardless of source configuration even though the magnitude of the flux will vary with the absorbing and reflecting characteristics of the source environment. In discussing the results of the runs, the object will be to explain the differences in computed buckling values and flux magnitude of related runs in terms of the variable source parameters.

For Runs 1, 2, and 4, in which the source was progressively lowered while the source compartment was kept dry, the buckling values were very close and averaged $24 \times 10^{-6} cm^{-2}$. This unexpected low value may be attributed to a large percentage of resonance neutrons increasing the measured activity in the lower levels. The energy distribution, as could be expected in a poorly moderating medium like air, apparently

Table 2. Summary of results

Run	z' (in.)	Source location	Source compartment water level	Outer tanks water level	Cadmium present?	Corrected activity (c/m) at z=40 in.	γ (in^{-1})	Buckling ($\times 10^{-5} cm^{-2}$)
1	1.25	Central	Empty	Full	Yes	433	0.0703	31
2	3.75	Central	Empty	Full	Yes	340	0.0709	18
3	6.25	Central	Empty	Full	Yes	321	0.0657	128
4	10.25	Central	Empty	Full	Yes	240	0.0708	20
5	10.25	Central	Full	Full	Yes	42	0.0633	167
6	6.25	Central	Full	Full	Yes	93	0.0668	106
7	3.75	Central	Full	Full	Yes	218	0.0680	81
8	1.25	Central	Full	Full	Yes	488	0.0688	64
9	1.25	Central	Full	Full	No	563	0.0680	81
10	1.25	Central	Full	Full	No	548	0.0683	74
11	1.25	Central	Empty	Empty	No	455	0.0665	112
12	1.25	Spread	Empty	Empty	No	335	0.0644	154
13	1.25	Spread	Full	Full	No	376	0.0667	108

remained constant. The buckling value for Run 3 was six times that of the others and is attributed to unknown experimental error. The measured activity of all four runs decreased with increasing z' due primarily to geometrical attenuation (Fig. 11).

Runs 5, 6, 7, 8 produced buckling values which started at $167 \times 10^{-6} cm^{-2}$ for Run 5 (z' = 10.25 in.) and decreased with z' to $64 \times 10^{-6} cm^{-2}$ for Run 8 (z' = 1.25 in.). This decrease may be directly attributed to the thermalizing effect on the source neutrons of the water in the filled source compartment. This thermalizing effect progressively reduced the percentage of epithermal neutrons entering the assembly. The activity of the four runs decreased with increasing z' (Fig. 12) as could be expected, and for the three runs with largest z', the activity was less than that of the corresponding runs with the neutron source in the dry compartment (Fig. 13, 14 and 15). This may be attributed to the strong absorption of the water between source and assembly. The activity of Run 8 was noticeably above that of Run 1 (Fig. 16), and this is readily attributed to the reflective effect of the water, which at a z' of 1.25 in. far outweighed the absorption of the water.

For these first eight runs a cadmium covered foil was placed at each level at x = -3 in., y = -3 in. The presence of the cadmium produced a noticeable deviation in the curves of the horizontal flux distributions for these runs as shown in Fig. 17 and 18. When the cadmium was removed for Runs 9 and 10 which were otherwise identical to Run 8 this deviation was not present and the now higher activities when plotted produced the expected cosine curve (Fig. 17 and 18).

The average cadmium ratio for the lower six foils of Run 1 through 4 are shown in Fig. 20. For foils located higher than z = 36 in. in the assembly the results were too sporadic to be meaningful. The curve shows that the percentage of resonance neutrons increases from z = 6 in. to z = 18 in. where the percentage reaches a maximum and then decreases at about the same rate. If the first two points are disregarded, the results support the postulated effect of the resonance neutrons upon buckling measurements. The first two points indicate that in the near vicinity of the source the fast neutrons have not yet been slowed to resonance energy and that most neutrons reflected back into that vicinity are at thermal energy. A second possibility is that the leakage of epithermal neutrons approaching resonance near the base occurs at a much greater rate than that of the slower moving thermal neutrons, and that neutrons which do leak out the base are for the greater part thermalized by the time they may reenter the assembly. This argument is supported by the results of Runs 5 through 8. The cadmium ratio for Run 8 is similar to that of Runs 1 through 4, whereas for Runs 5, 6 and 7 the curve indicates a much lower percentage of resonance neutrons at z = 6 in., a rapid increase to a maximum at z = 18 in. and a slight steady decrease to z = 36 in. where the activity of the covered foils became so low as to be meaningless.

The cadmium apparently affected the buckling which for Run 8 was 17.4% lower than the average buckling of Runs 9 and 10 (Fig. 21). This increase in buckling as well as the increased flux for Runs 9 and 10 is attributed to an increase in multiplication which was not expected to be noticeable. The total amount of the cadmium (69.1 gm.) then is considered to have been sufficient to cause a change in the physical characteristics of the pile, and the results obtained in the runs with cadmium present cannot be compared to the results of runs without cadmium unless this is taken into consideration. Baughman (1, p. 19) showed that the effect of the indium on multiplication is negligible. What little effect there might be need not be considered in a comparison because the amount and position of the indium foils was reproduced for each run regardless of whether or not a particular foil was to be counted for a particular run.

For Run 11 all tanks were drained while the source configuration and height remained as for Run 10. The measured activity was considerably reduced (Fig. 22), evidently due to increased leakage. This leakage apparently caused a sharp increase in the buckling value as was postulated previously.

The next two runs introduced a new source parameter with the spreading of the individual source cylinders. Run 12 (Fig. 23), with z' = 1.25 in. and all tanks dry produced a high buckling, whereas Run 13, with identical conditions except that the tanks were full, produced a buckling value nearly 50% lower, about equal to that of the centralized source with empty tanks used in Run 11. The explanation of the comparative buckling values for Runs 12 and 13 is identical to that of Runs 11 and 10, but the greater increase in buckling values in the case of the spread source in relation to that of the centralized source deserves explanation. Since the leakage and the source neutron local moderation effects are apparently identical, the difference in measured buckling is apparently due to the distance the source neutrons must travel to reach the centrally located foils. From the first foil used in buckling determination, located at z = 18 in., the distance to the centrally located source was 19.25 in. The same distance to one of the outer sources in the spread configuration was 24.8 in. Therefore fewer resonance source neutrons could reach the foils used in computing buckling.

According to the theory (6, p. 12) the activity produced by a source whose four equal components are located at ($\pm \frac{a}{2}$,0) and (0,$\pm \frac{b}{2}$) should be 0.707 that of a source of equal total strength located at (0,0) providing z' is the same in both cases. Using the corrected activity values at z = 40 in. and subtracting that activity due to the fifth (center) source, the ratio of the activities due to the spread and centralized sources were found to be 0.680 when the tanks were empty (Fig. 24) and 0.635 when the tanks were full (Fig. 25). The difference of these values from 0.707 may be attributed to excess lateral leakage from the non-centrally located sources in the comparatively small assembly. The larger difference in the case of the full tanks may be explained as follows. When there is no water in the tanks there will be virtually no reflection from the base in either case and the difference in activity due to spread and centralized sources is due only to lateral leakage from the assembly proper. When the tanks are filled, besides this leakage from the assembly proper for the spread configuration, there will be a larger percentage of neutrons leaking from the sides of the water tanks than there was when the source was located in the center.

The completely corrected activity of Run 1 is compared in Fig. 26 with the activity of Run 1 corrected only for background activity, foil weight differential and time of removal from flux. This gives an indication of the effect of harmonic and end corrections. The same is done for Run 12 in Fig. 27. A comparison of the two figures shows that using the spread source configuration enables one to determine γ and consequently B^2 to a fair degree of accuracy without considering harmonic corrections. This is not true for the centralized source.

A comparison of corrected activities of Run 4 using the base of the assembly as the z = 0 plane as compared to the corrected activities of the same run using a plane 12 in. higher as the z = 0 plane is shown in Fig. 28. Similar results for Run 13 are shown on the same figure. A comparison shows that another advantage of the spread source is that the location of the z = 0 plane is not nearly as critical as it is for the centralized source.

VIII. CONCLUSIONS

The results of the research and experimentation reported herein appear to verify the postulated theory that the presence of epithermal source neutrons in a subcritical assembly tends to lower the measured buckling whereas leakage at the base of an assembly raises buckling. Specifically, the results may be summarized as follows:

1. Placing the source in a nonabsorbing nonmoderating medium and changing source to assembly distance does not alter buckling but decreases the flux with increasing distance.

2. Placing the source in a good moderating medium and changing source to assembly distance increases buckling and decreases flux with increasing distance.

3. A small amount of a strong absorber in a subcritical assembly can measurably change the flux pattern and decrease buckling and flux.

4. Spreading the source so that it is located at points of harmonic suppression will increase buckling and decrease flux in comparison to a centralized source.

5. Arbitrary raising or lowering of the assumed thermal neutron source plane will respectively increase or decrease buckling considerably for a centralized source. The effect on a spread source is negligible for small movements of the plane.

6. Harmonic corrections are necessary for measurements

taken within two diffusion lengths of a centralized fast neutron source. This distance can be halved without serious error for a spread source.

7. Based on conclusions 5 and 6, a source spread for harmonic suppression gives far more reliable results than a centralized source and is preferable to use providing the induced activity is high enough to count without a large statistical deviation.

IX. SUGGESTIONS FOR FURTHER STUDY

In the variation of source geometry of a subcritical reactor, an obvious difficulty arises in the evaluation of results when the actual buckling of the particular lattice configuration is unknown. Further study and experimentation must be undertaken to determine actual buckling for the assembly described herein. This is best done experimentally by so designing the source conditions that the necessity of estimating a $z = 0$ plane and applying harmonic corrections may be eliminated. Replacement of the water pedestal, a relatively strong neutron absorber, with graphite is suggested as a first step. The sources could be placed in the spread configuration near the bottom of this pedestal. In that way harmonic corrections and the variation of buckling due to movement of the $z = 0$ plane would be minimized. The true buckling could then be evaluated within a small range.

The range of fast source neutrons in the assembly should be studied more closely. A possible method of investigating this range is to place the source in the volumetric center of the assembly and take cadmium ratio measurements radially from it. The assembly should also be changed so that a multitude of foil slots are available at and near the base of the assembly to give more information of the behavior of neutrons in this critical area.

Attempts might be made to place between the source and the assembly a combination of moderators so that there will be more absorption and moderation of neutrons in the center, where the source is located, than near the edges. This would tend to approximate the assumption of source more closely.

Since making use of the results of the measurements on the subcritical assembly requires a process of iteration, it is suggested that the process be programmed for an electronic computer in order to save many hours of mathematical manipulation.

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APPENDIX

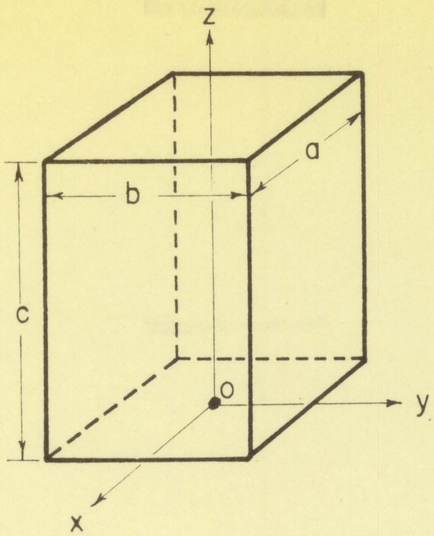


Fig. 1. Diagram of subcritical assembly

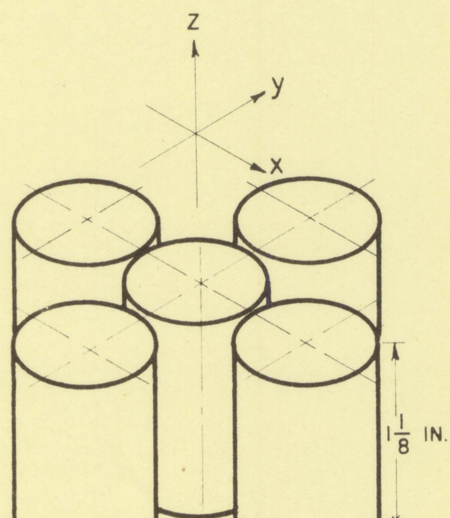
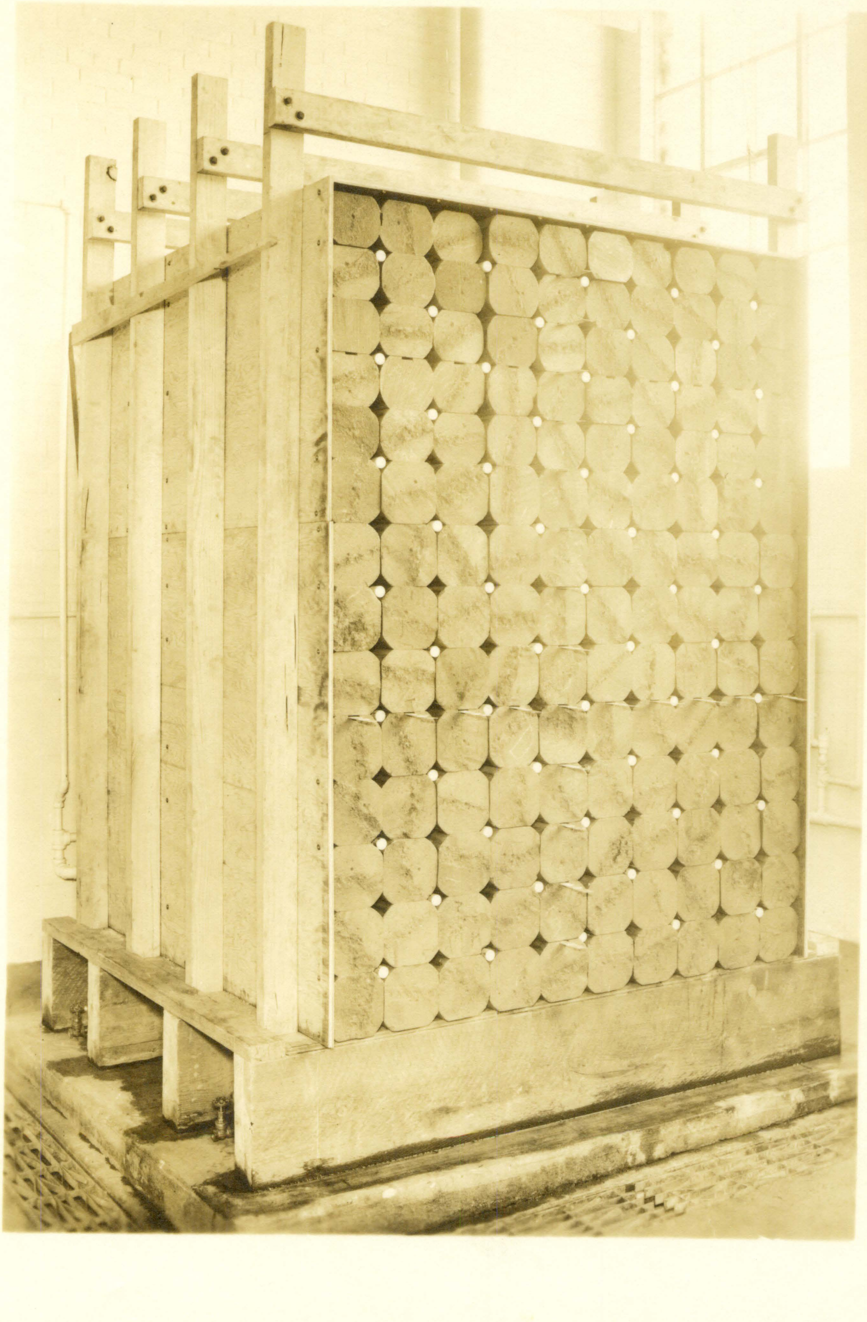


Fig. 2. Source configuration for point source approximation

Fig. 3. Subcritical assembly



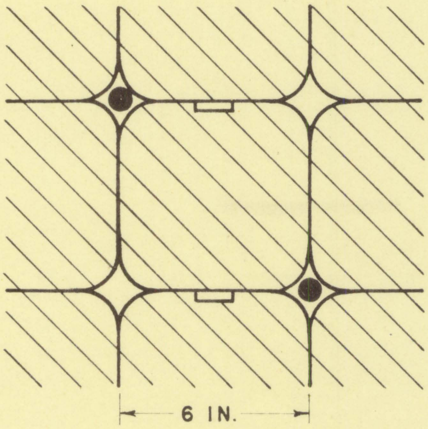


Fig. 4. Section of subcritical assembly showing fuel rods and foil slots

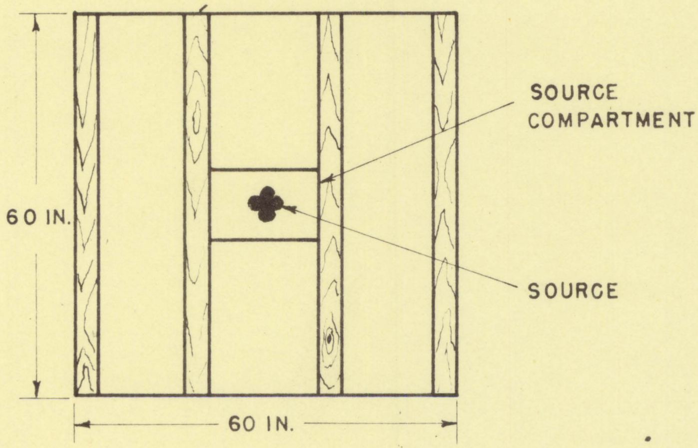


Fig. 5. Pedestal of subcritical assembly showing water tanks, source compartment and assembly supports

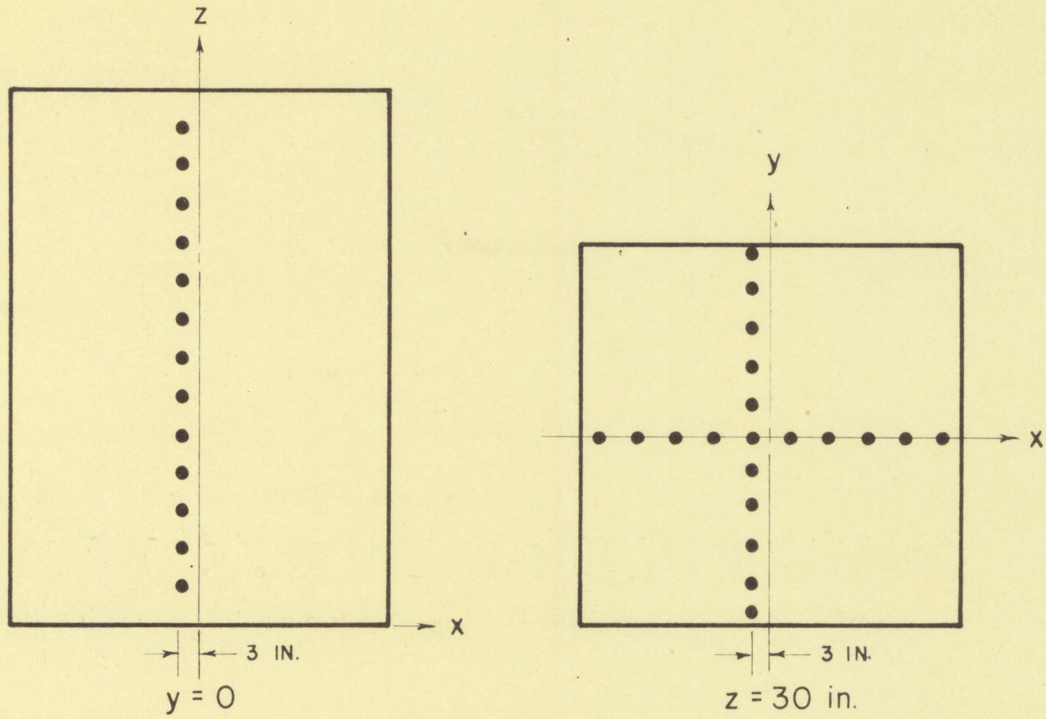


Fig. 6. Vertical and horizontal foil distribution

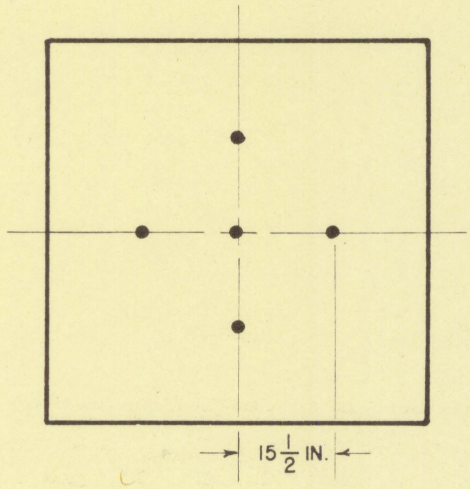
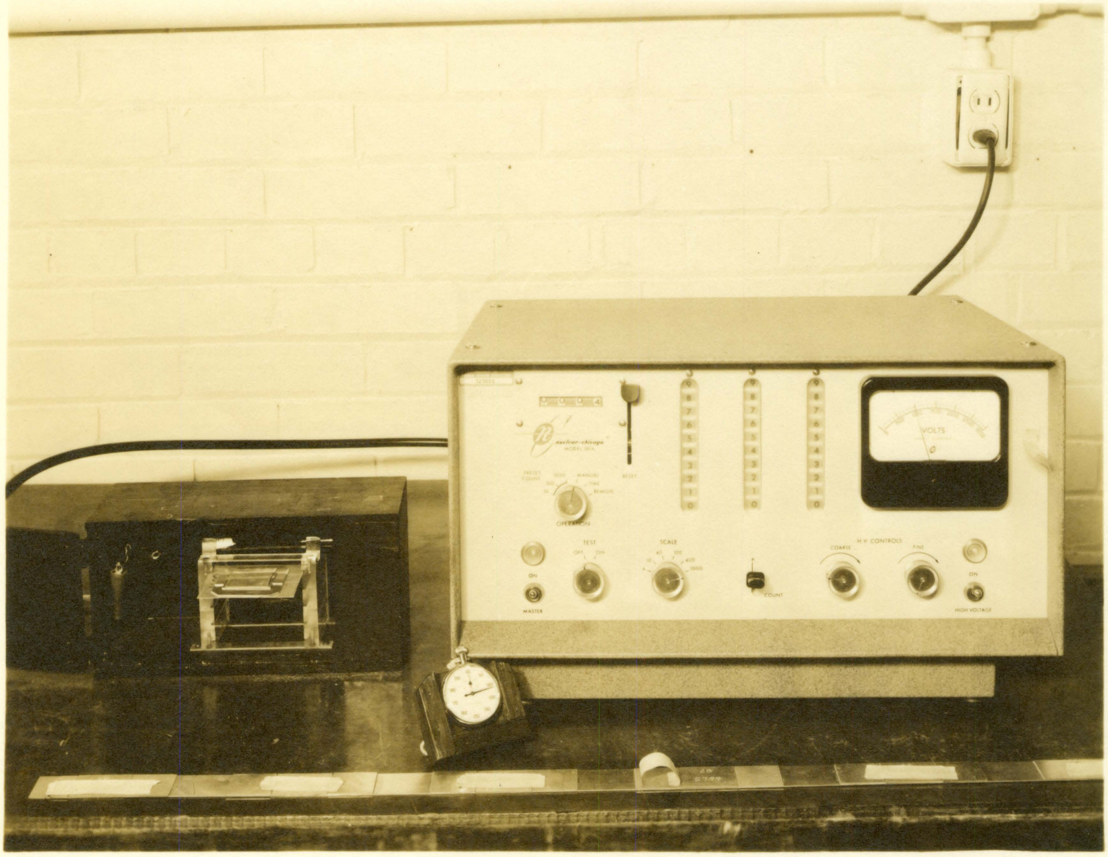


Fig. 7. Source distribution for spread configuration

Fig. 8. Counting equipment including foil holder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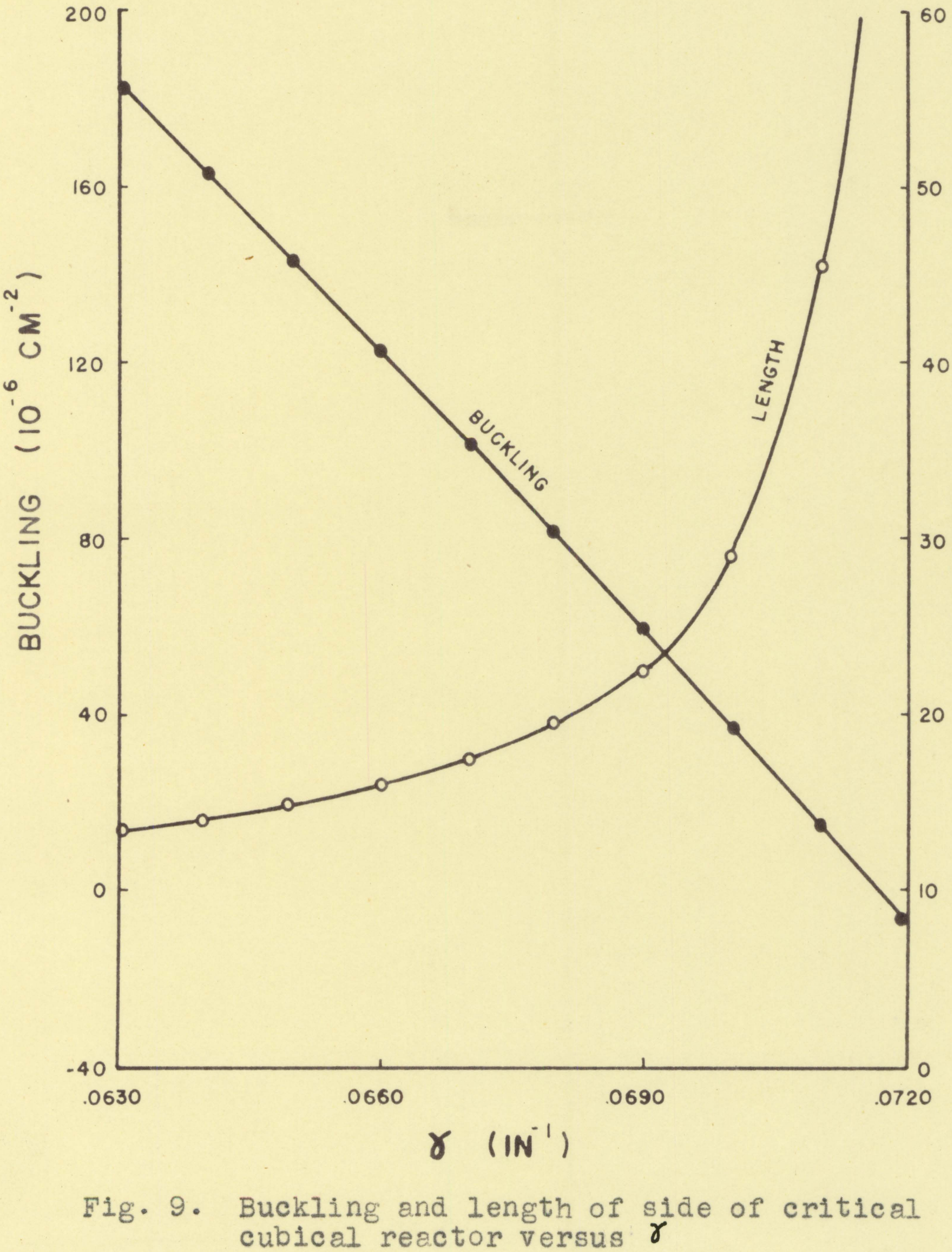


Fig. 9. Buckling and length of side of critical cubical reactor versus γ

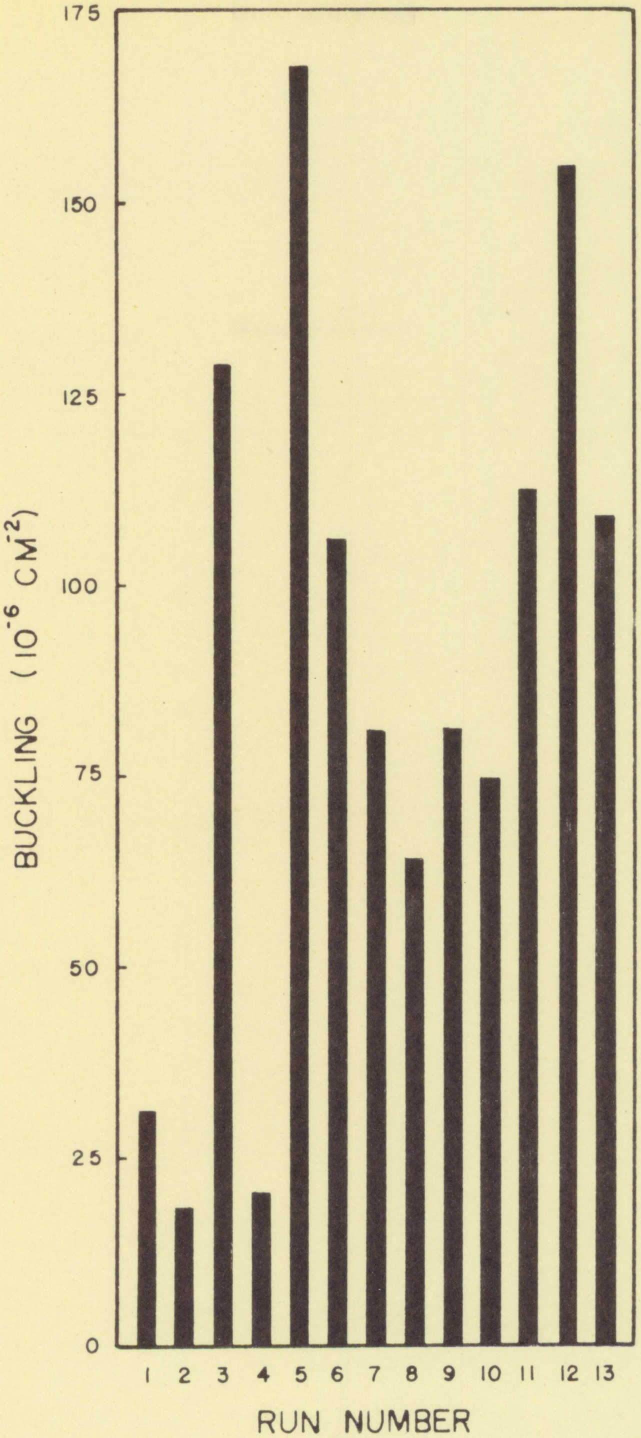


Fig. 10. Experimentally determined values of buckling for each run

Fig. 11. Vertical flux distribution for Runs 1, 2 and 4

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
1	z' = 1.25 in.	31
2	z' = 3.75 in.	18
3	z' = 6.25 in.	128
4	z' = 10.25 in.	20

(The results of Run 3 appear to be in error. For clarity this run has been omitted from this figure but is shown graphically in Fig. 14.)

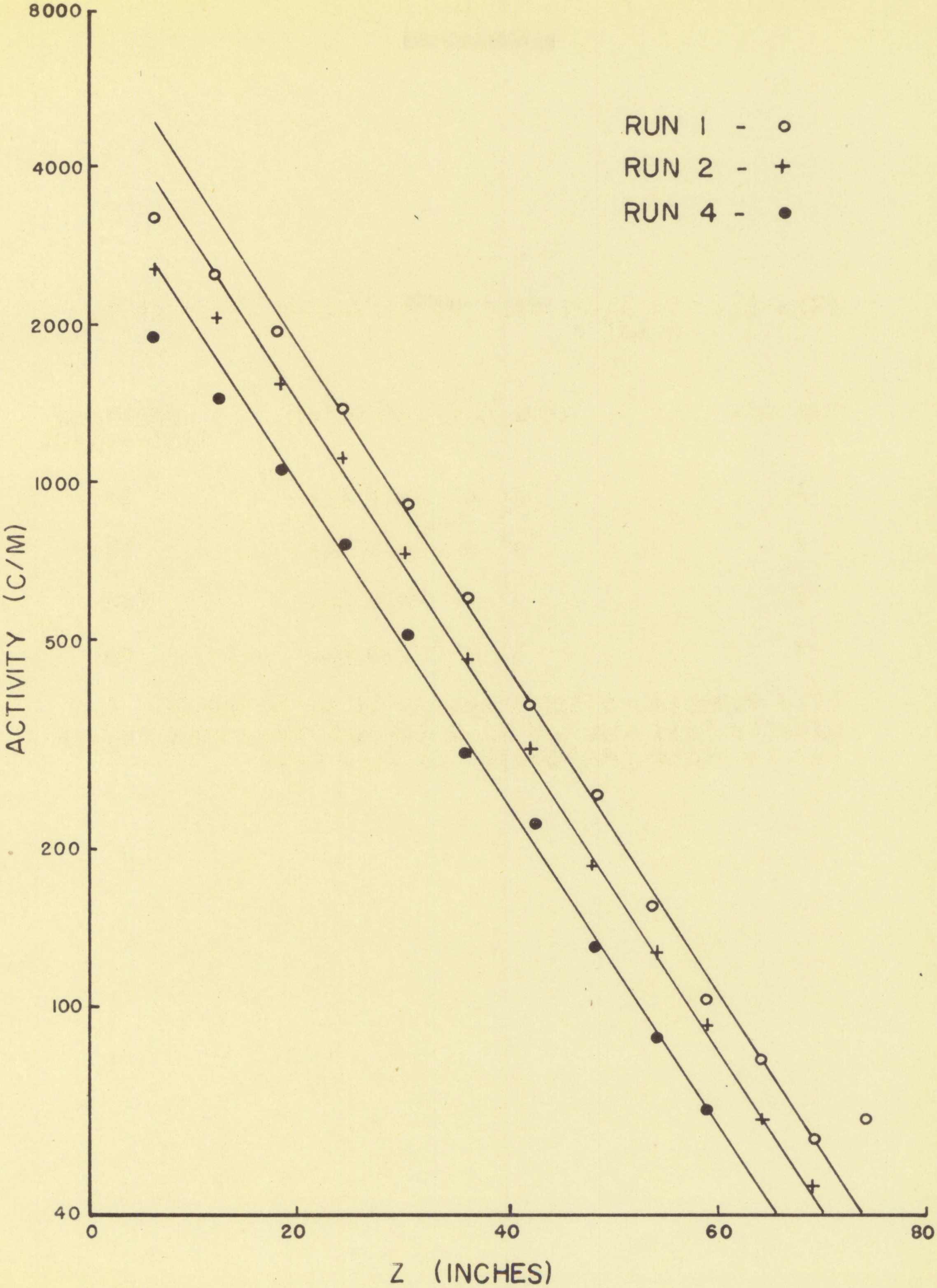

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)
RUN 1 - o
RUN 2 - +
RUN 4 - ●

Fig. 12. Vertical flux distribution for Runs 5, 6, 7 and 8

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
5	z' = 10.25 in.	167
6	z' = 6.25 in.	106
7	z' = 3.75 in.	81
8	z' = 1.75 in.	64

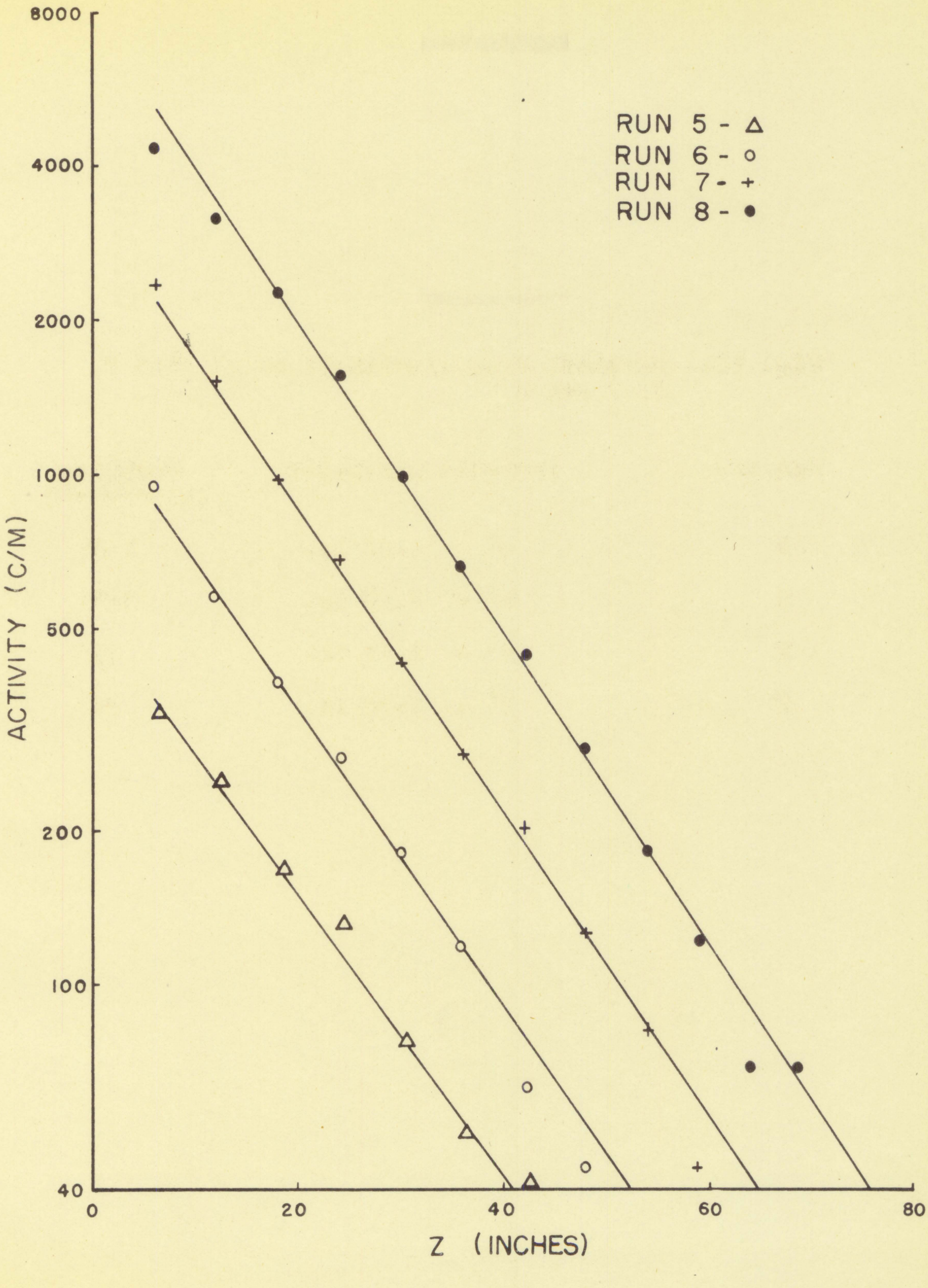

RUN 5 - △
RUN 6 - ○
RUN 7 - +
RUN 8 - ●
8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)

Fig. 13. Vertical flux distribution for Runs 2 and 7

Run no.	Variable parameter	Buckling ($x10^{-6}cm^{-2}$)
2	Source compartment empty	18
7	Source compartment full	81

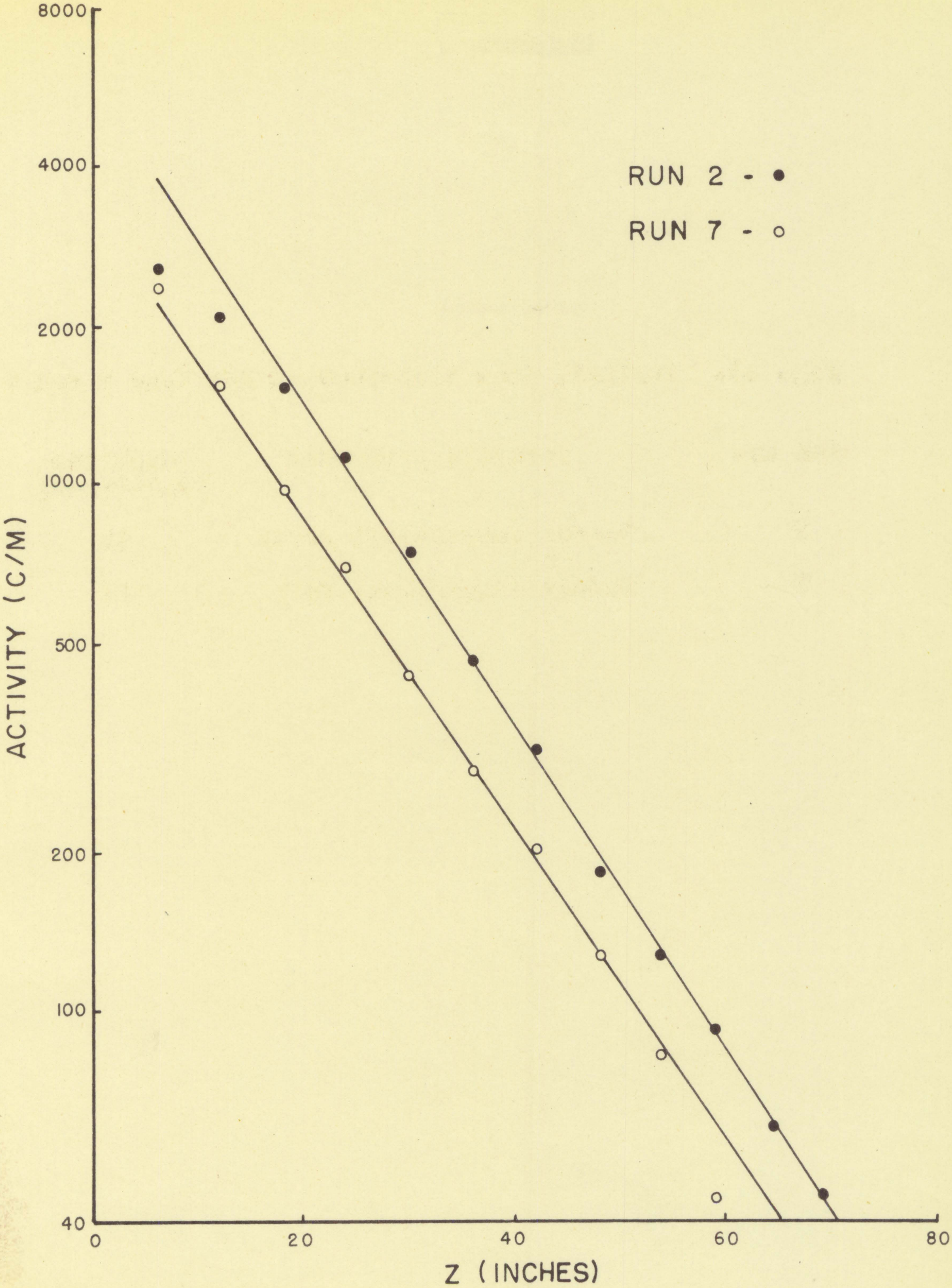

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
RUN 2 - ●
RUN 7 - ○
0
20
40
60
80
Z (INCHES)

Fig. 14. Vertical flux distribution for Runs 3 and 6

Run no.	Variable parameter	Buckling ($\times 10^{-6}cm^{-2}$)
3	Source compartment empty	128
6	Source compartment full	106

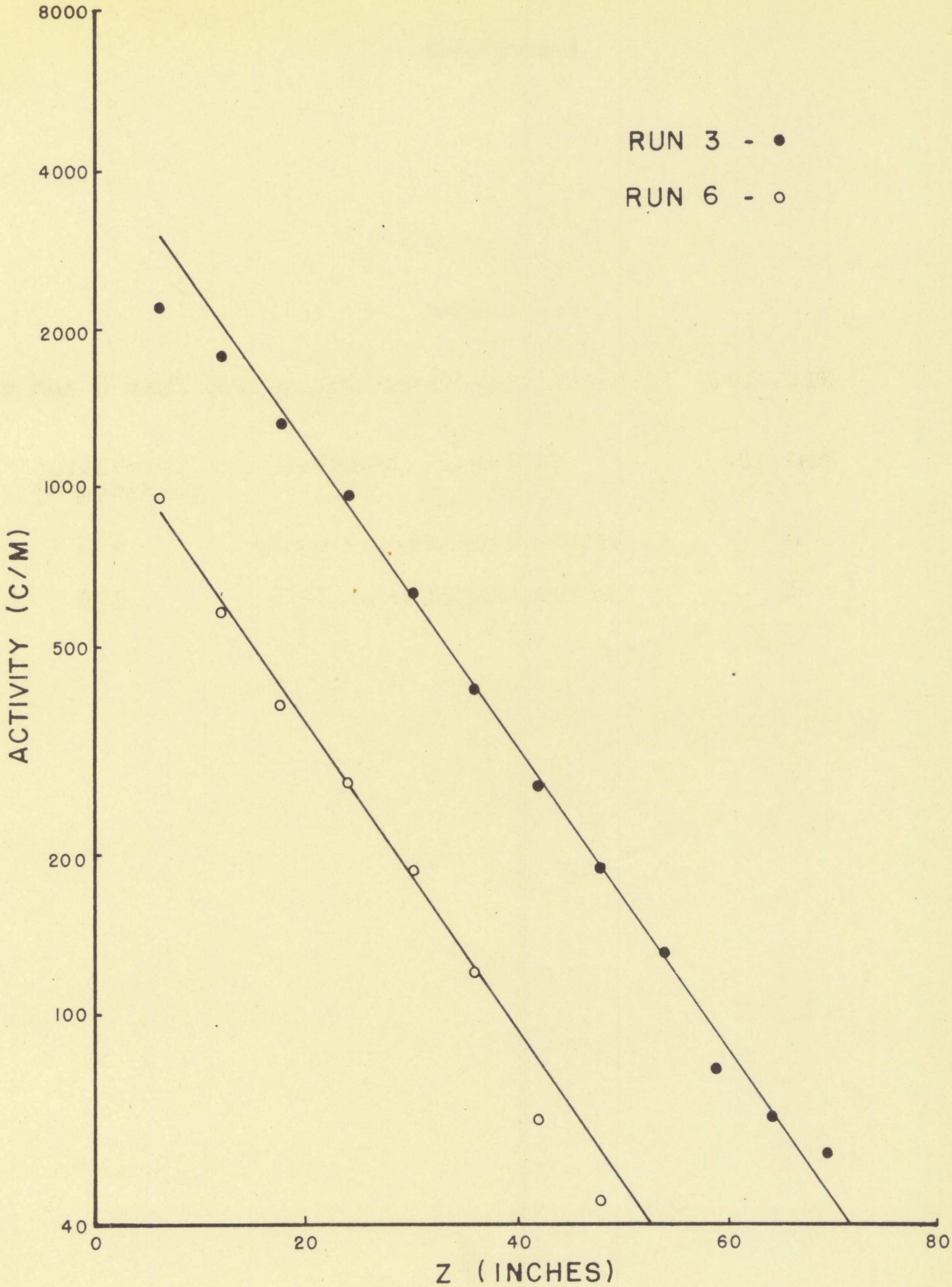

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
RUN 3 - ●
RUN 6 - o
0
20
40
60
80
Z (INCHES)

Fig. 15. Vertical flux distribution for Runs 4 and 5

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
4	Source compartment empty	20
5	Source compartment full	167

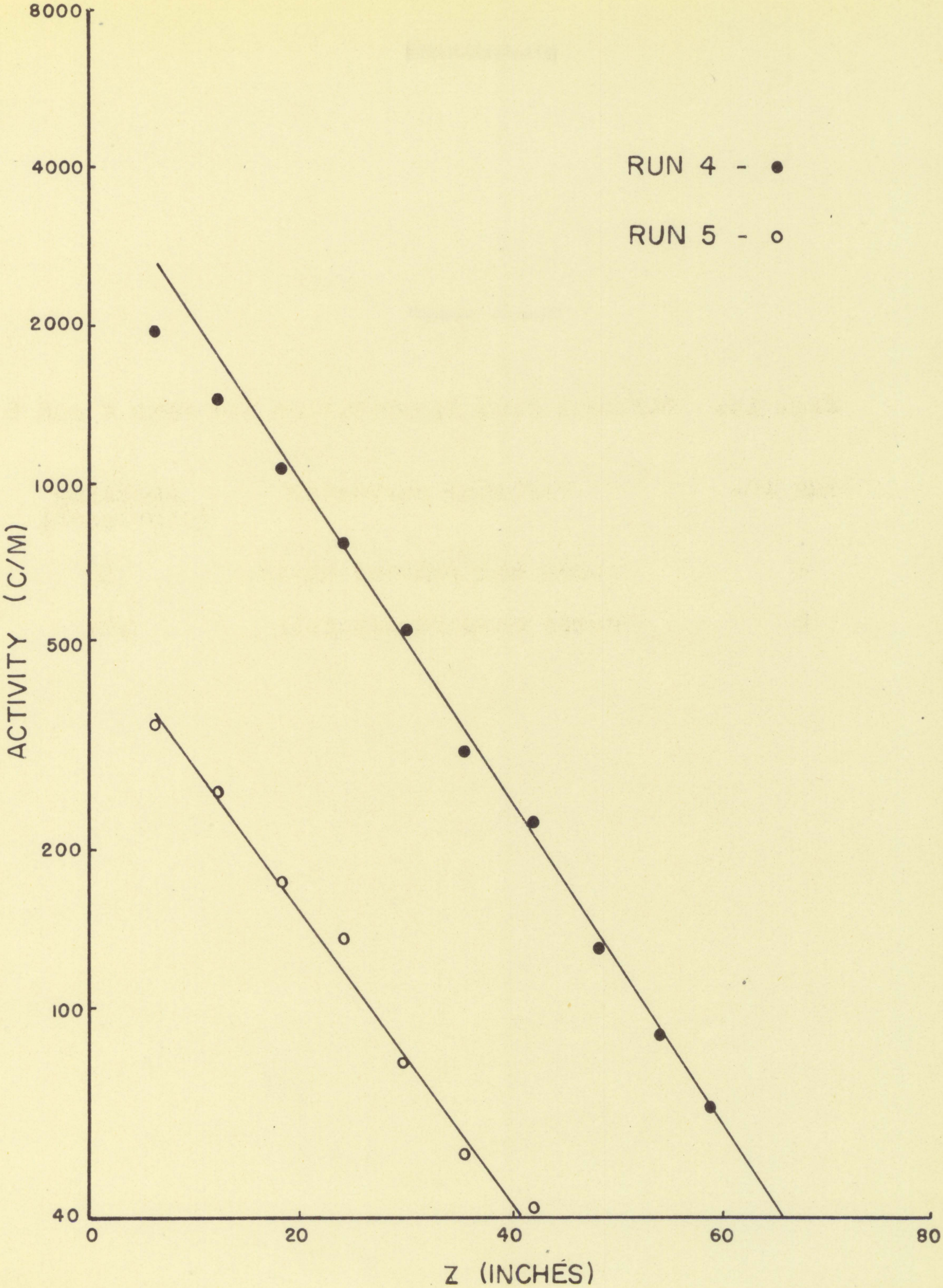
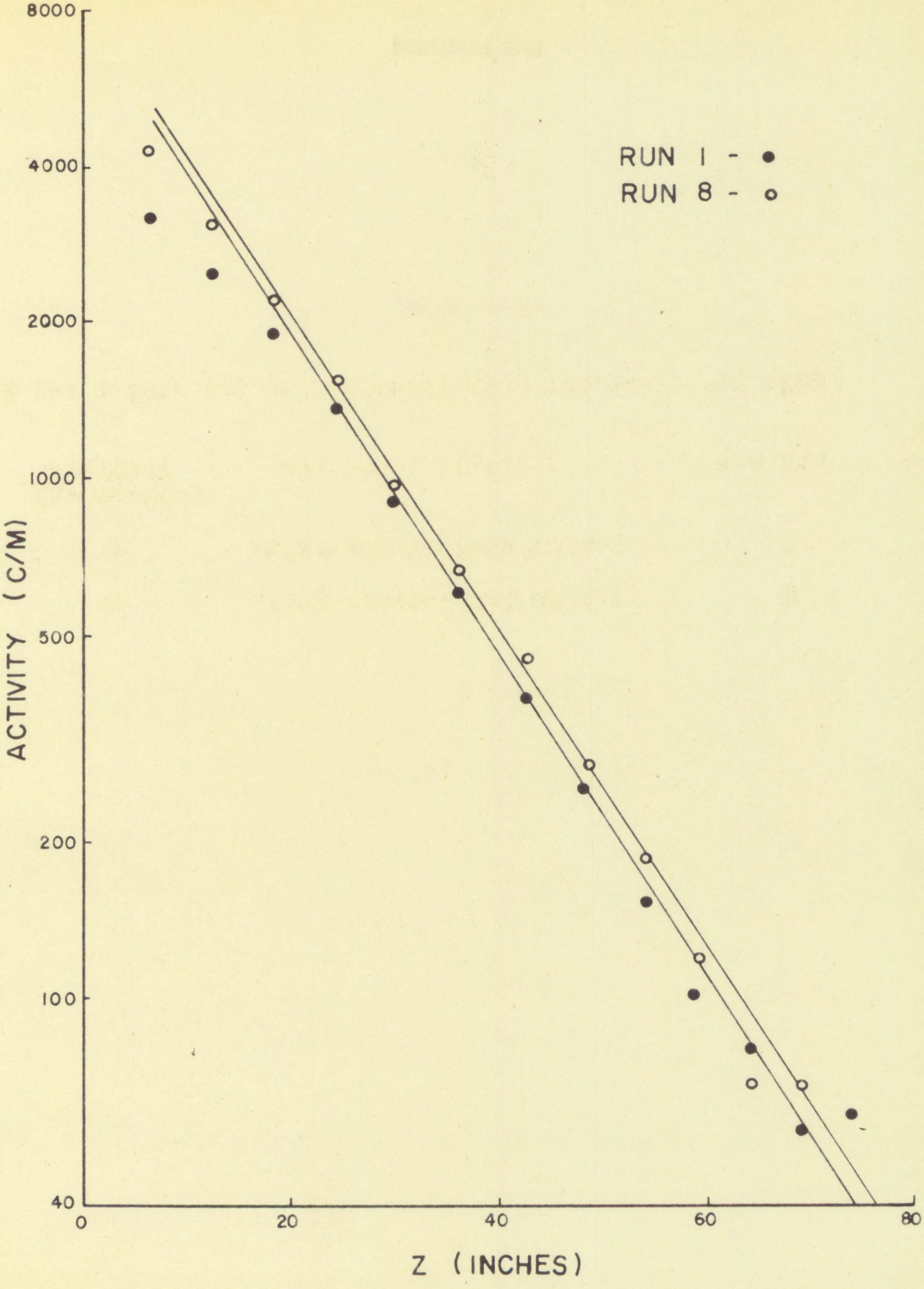

RUN 4 - ●
RUN 5 - o
8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)

Fig. 16. Vertical flux distribution for Runs 1 and 8

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
1	Source compartment empty	31
8	Source compartment full	64


8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
RUN 1 - ●
RUN 8 - ○
0
20
40
60
80
Z (INCHES)

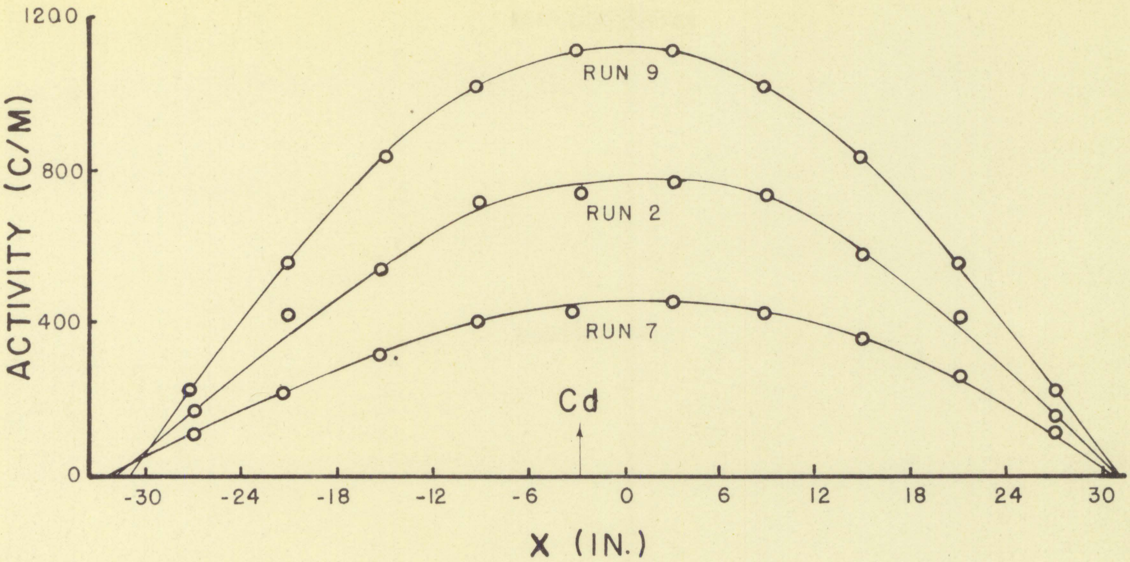


Fig. 17. Horizontal flux distribution at y = 0, z = 30 in. for Runs 2, 7 and 9

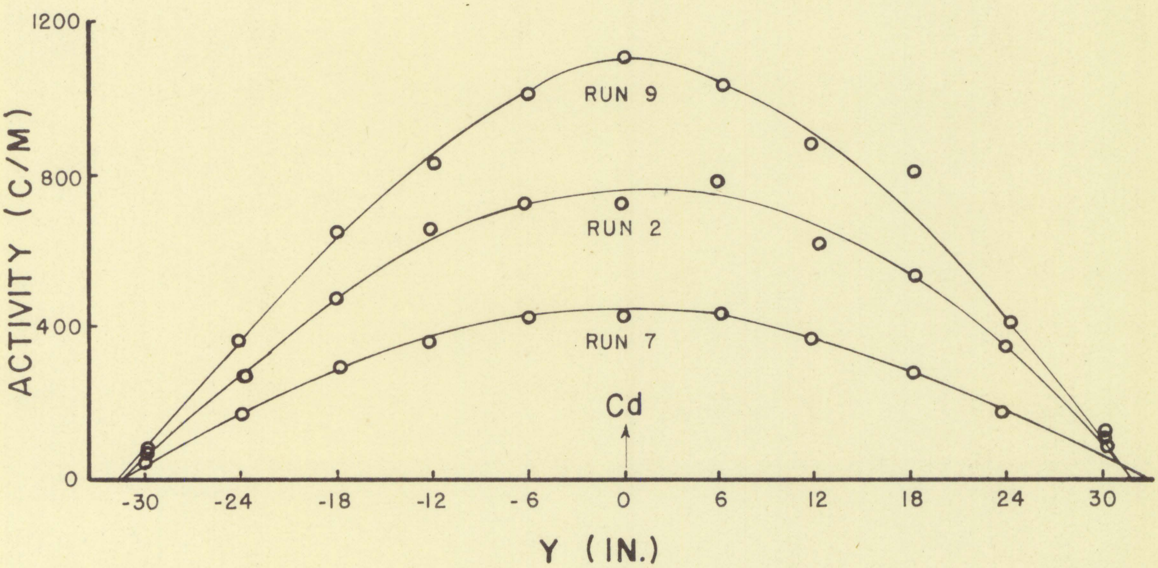


Fig. 18. Horizontal flux distribution at x = -3 in., z = 30 in. for Runs 2, 7 and 9

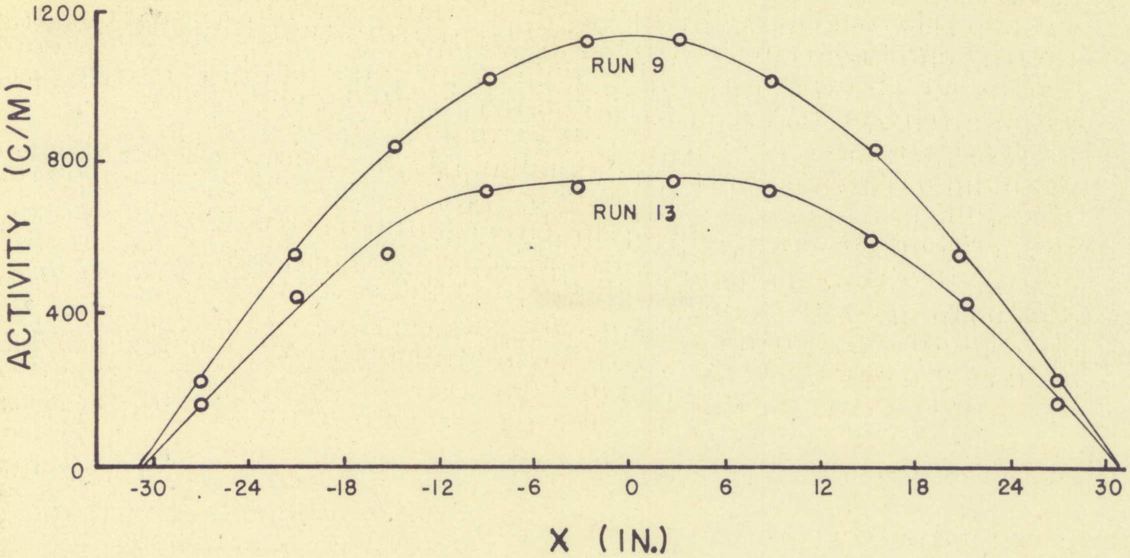


Fig. 19. Horizontal flux distribution at y = 0, z = 30 in. for Runs 9 and 13

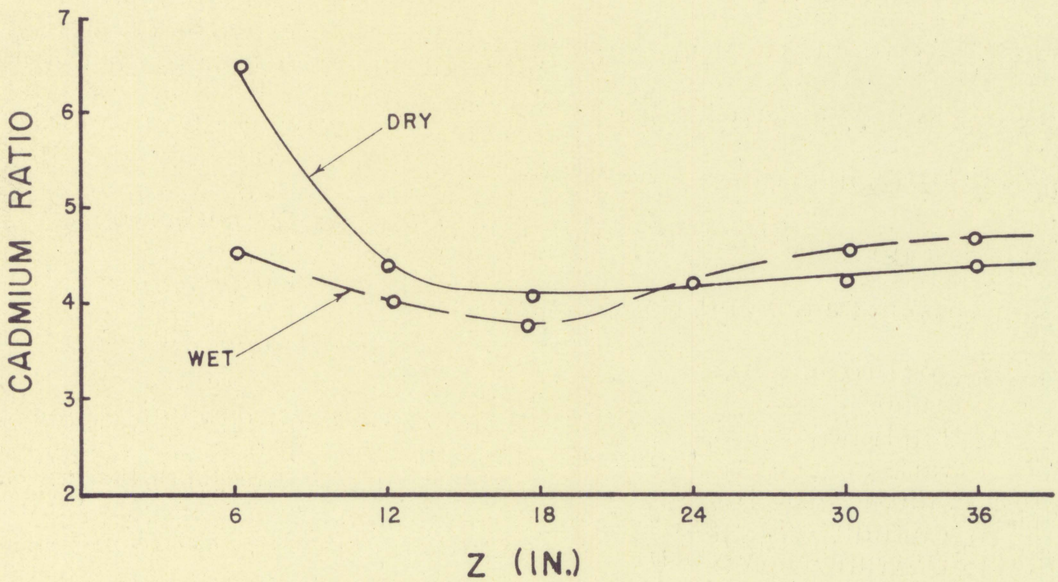


Fig. 20. Cadmium ratios for averages of Runs 1, 2, 3 and 4 and Runs 5, 6 and 7

Fig. 21. Vertical flux distribution for Runs 8, 9 and 10

Run no.	Variable parameter	Buckling ($x10^{-6}cm^{-2}$)
8	Cadmium covered foils present	64
9	No cadmium in assembly	81
10	No cadmium in assembly	74

(The straight line plots of Runs 9 and 10 are so close that they appear as a single line on a graph of this scale.)

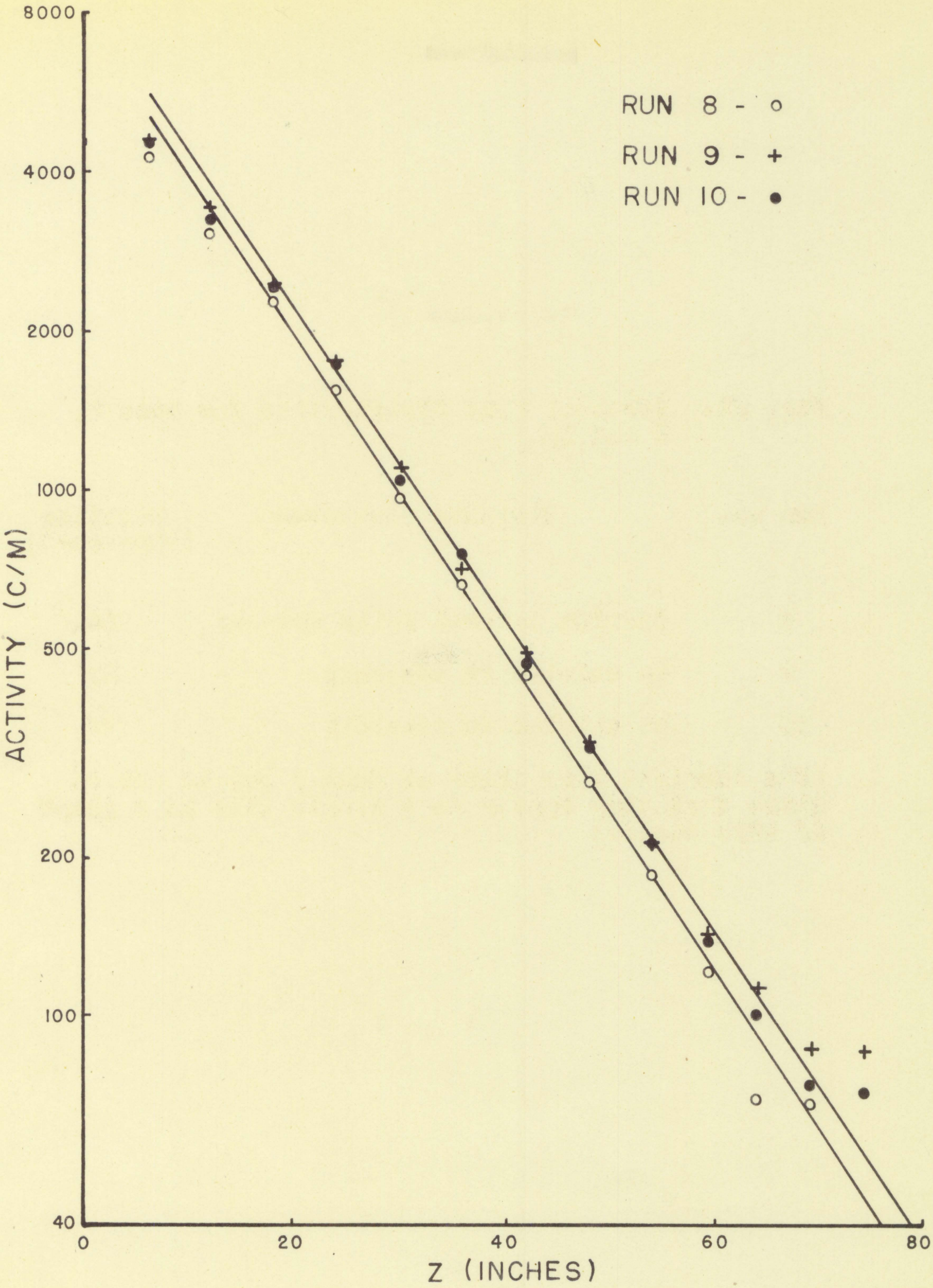

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)
RUN 8 - ○
RUN 9 - +
RUN 10 - ●

Fig. 22. Vertical flux distribution for Runs 10 and 11

Run no.	Varisble parameter	Buckling ($\times 10^{-6} cm^{-2}$)
10	All tanks full	74
11	All tanks empty	112

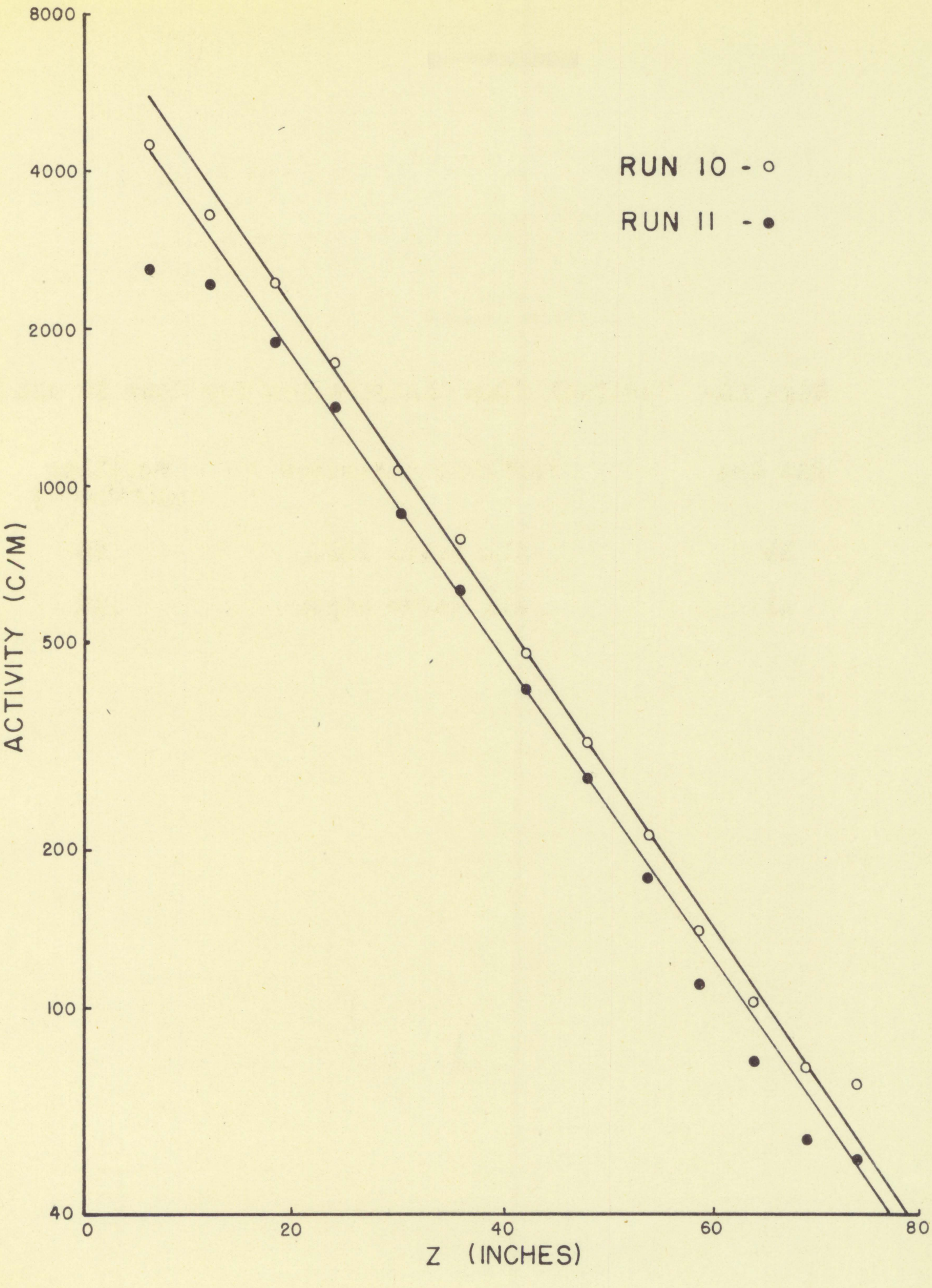
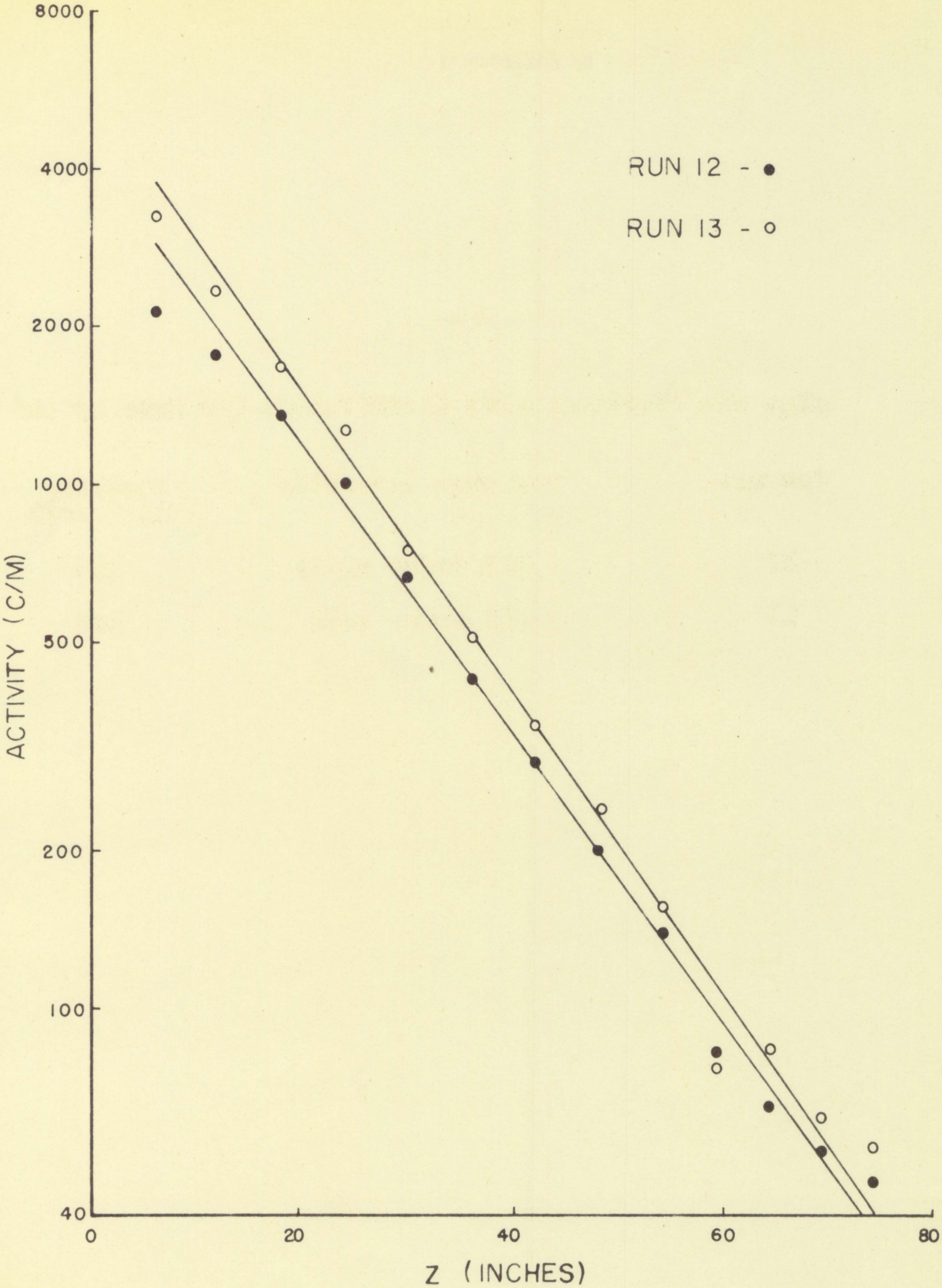

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
RUN 10 - o
RUN 11 - •
0
20
40
60
80
Z (INCHES)

Fig. 23. Vertical flux distribution for Runs 12 and 13

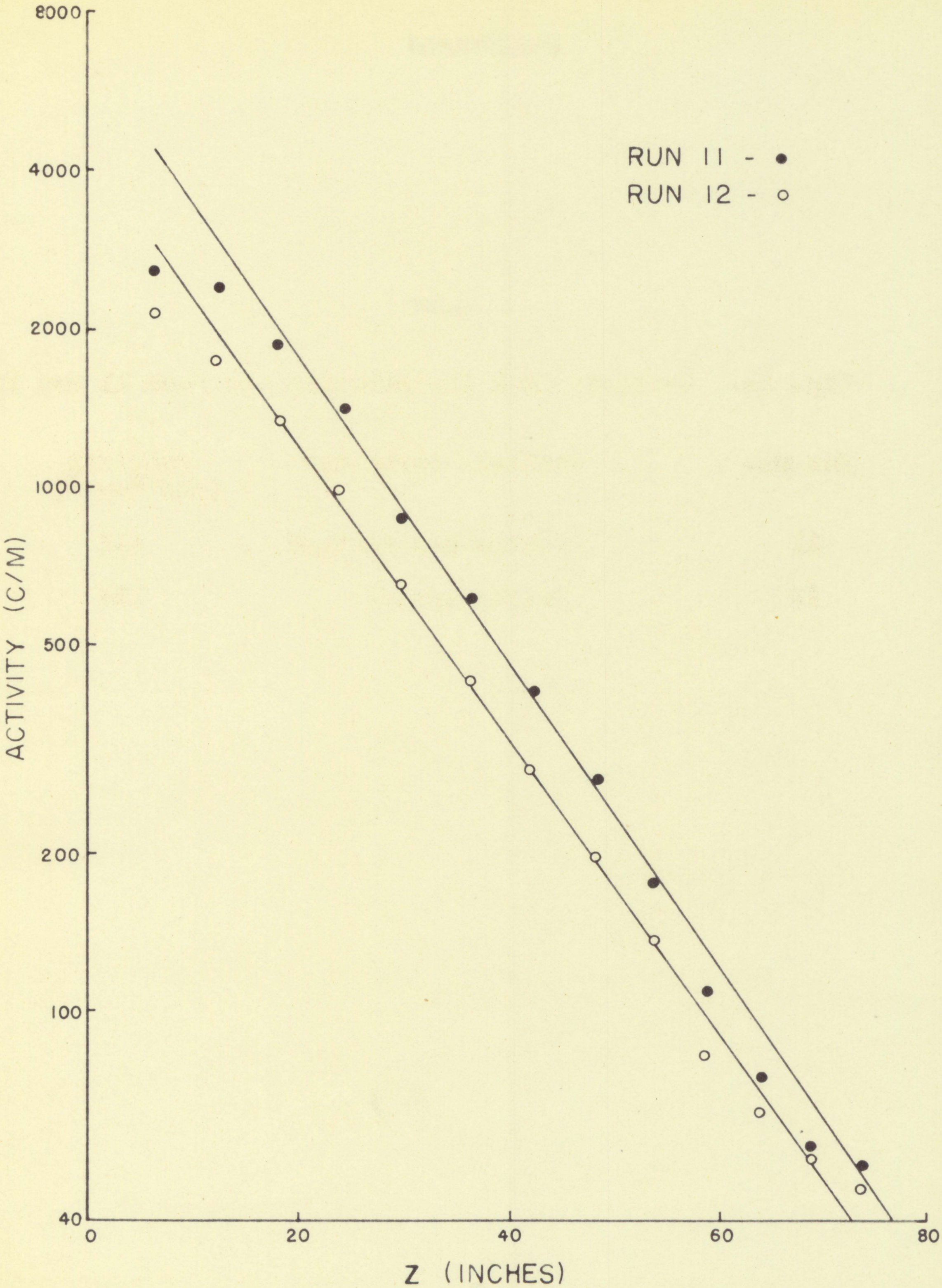
Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
12	All tanks empty	154
13	All tanks full	108



8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)
RUN 12 - ●
RUN 13 - ○

Fig. 24. Vertical flux distribution for Runs 11 and 12

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
11	Source centralized	112
12	Source spread	154



8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
RUN 11 - ●
RUN 12 - ○
0
20
40
60
80
Z (INCHES)

Fig. 25. Vertical flux distribution for Runs 10 and 13

Run no.	Variable parameter	Buckling ($\times 10^{-6} cm^{-2}$)
10	Source centralized	74
13	Source spread	108

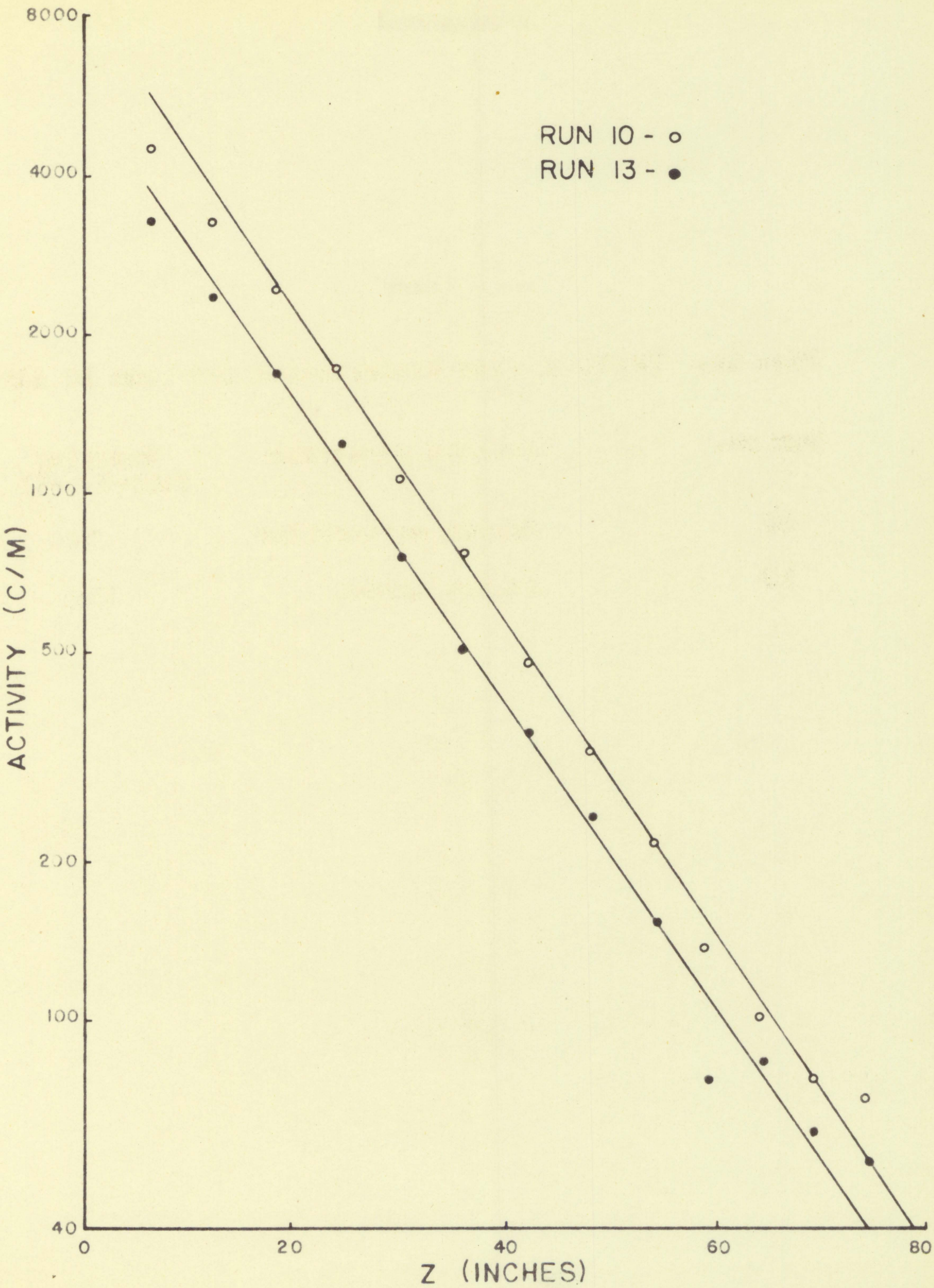

RUN 10 - o
RUN 13 - ●
8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)

Fig. 26. Vertical flux distribution for corrected and uncorrected activities of Run 1

Run no.	Buckling ($\times 10^{-6} cm^{-2}$)
1 (corrected)	31
1 (uncorrected)	-81

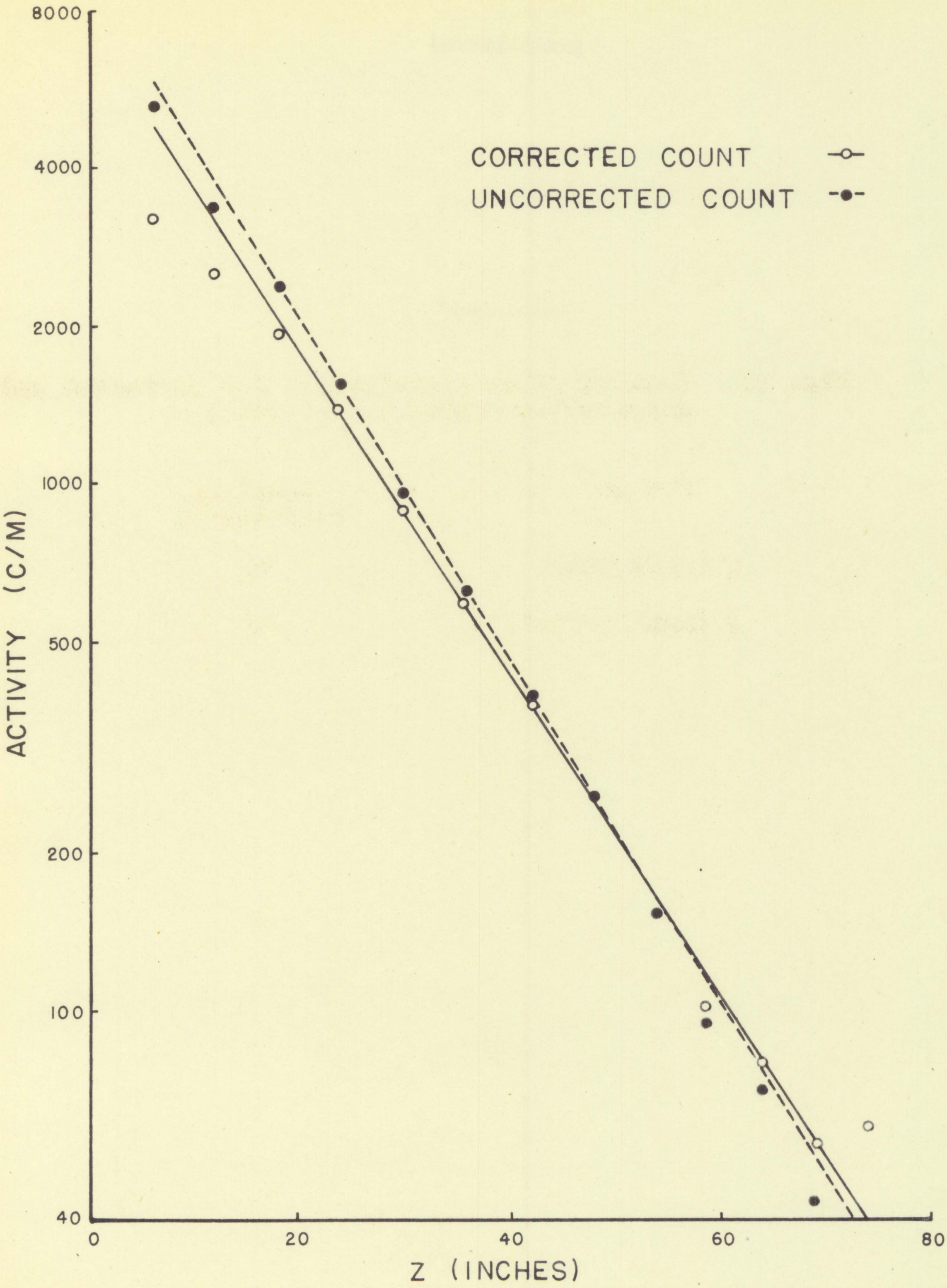
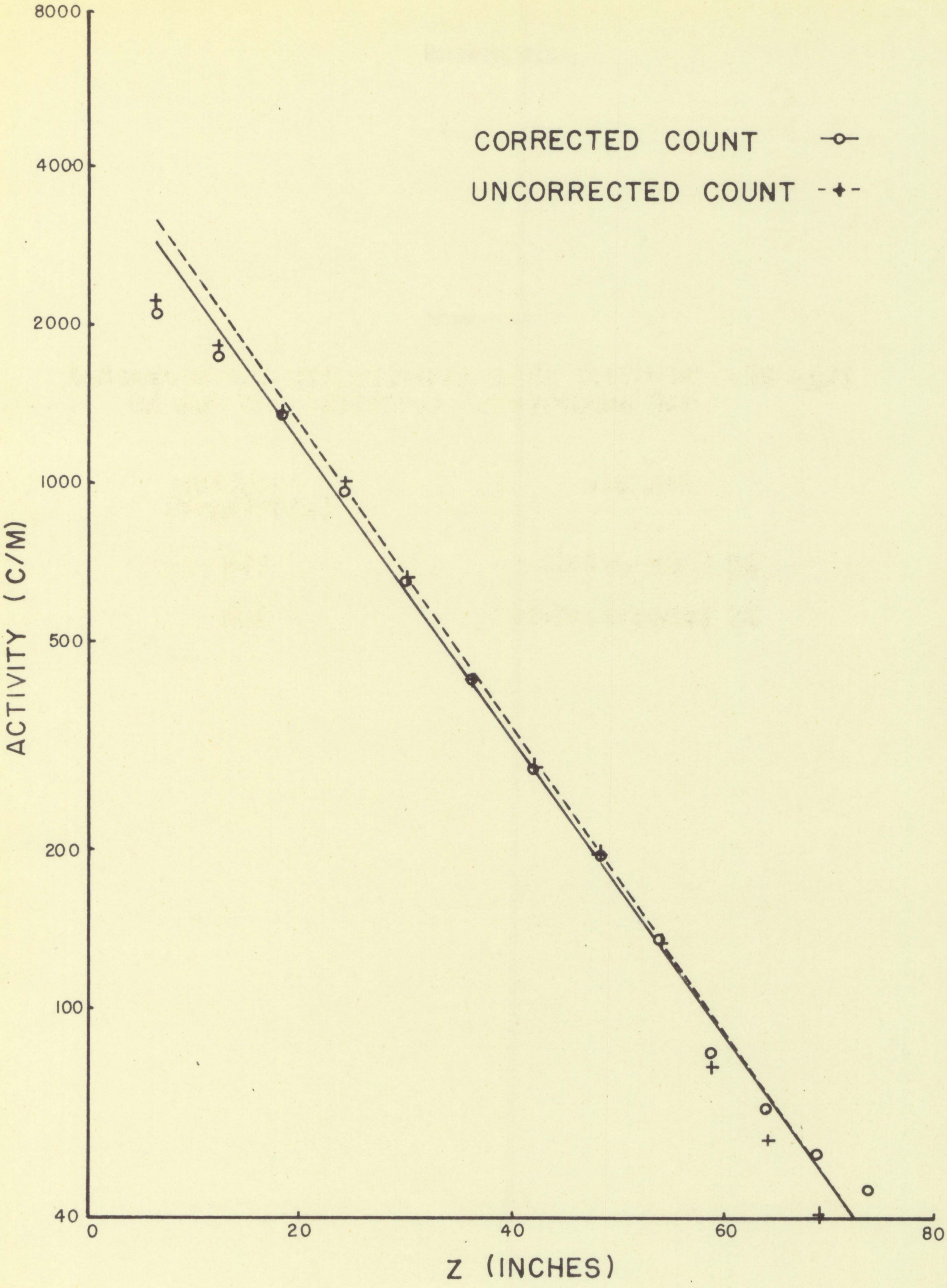

8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)
CORRECTED COUNT
UNCORRECTED COUNT

Fig. 27. Vertical flux distribution for corrected and uncorrected activities of Run 12

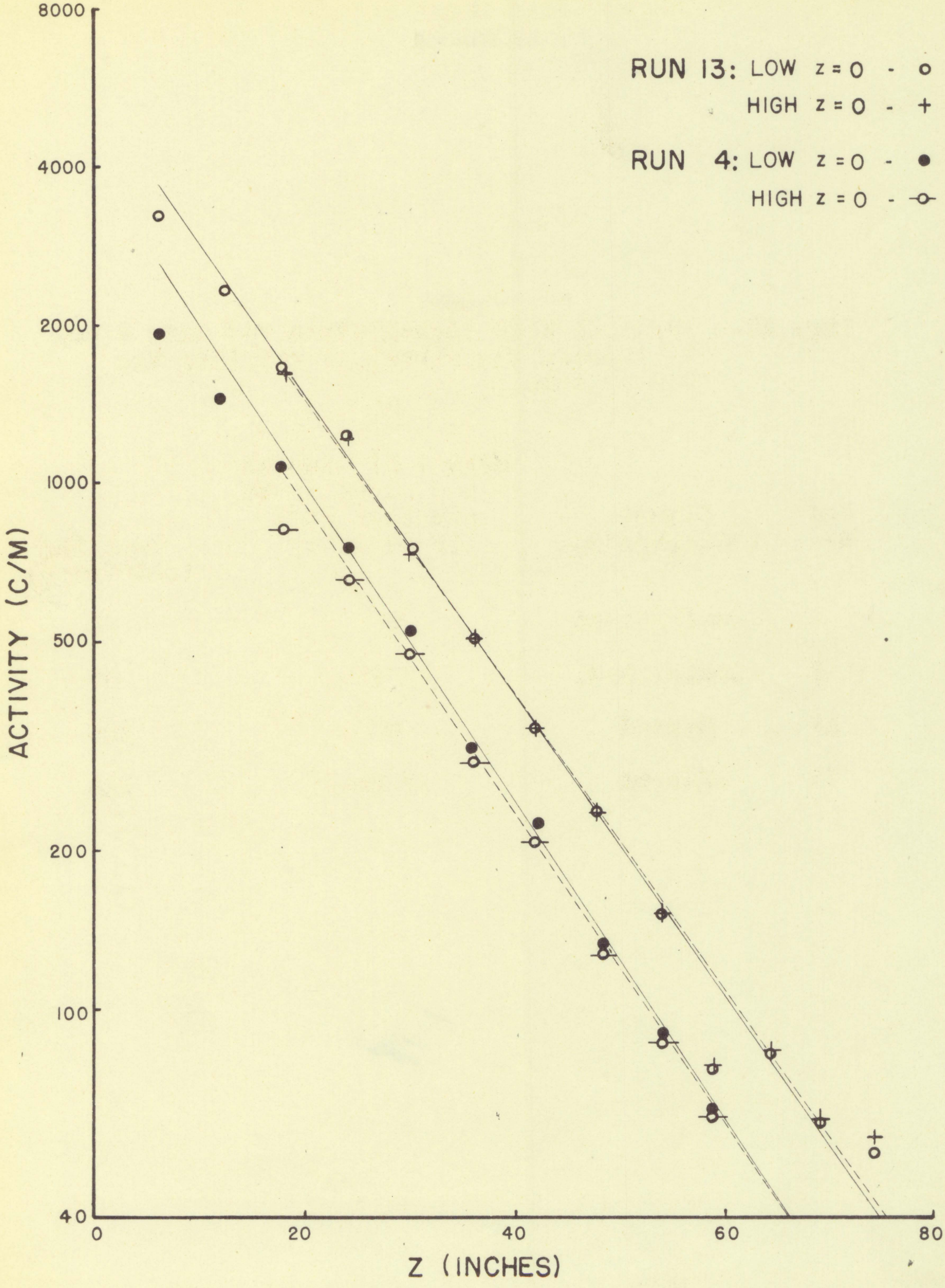
Run no.	Buckling ($\times 10^{-6} cm^{-2}$)
12 (corrected)	154
12 (uncorrected)	127



CORRECTED COUNT
UNCORRECTED COUNT
8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)

Fig. 28. Vertical flux distribution for Runs 4 and 13 showing the effect of shifting the z = 0 plane

Run no.	Source configuration	Height of assumed z = 0 plane above physical base of assembly	Buckling ($\times 10^{-6} cm^{-2}$)
4	Centralized	0	20
4	Centralized	12 in.	87
13	Spread	0	108
13	Spread	12 in.	118


RUN 13: LOW z = 0 - o
HIGH z = 0 - +
RUN 4: LOW z = 0 - ●
HIGH z = 0 - -o-
8000
4000
2000
1000
500
200
100
40
ACTIVITY (C/M)
0
20
40
60
80
Z (INCHES)