KINETICS OF COUPLED REACTOR CORES USING SIX GROUPS OF DELAYED NEUTRONS

by

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INTRODUCTION

Reactor kinetics is a study of the time behavior of a reactor system. Control of a nuclear reactor is the most important aspect. The purpose of this study is to determine as accurately as possible some of the characteristics involved in control of the UTR-10 reactor. One method which is used to give an accurate determination of the responses of the reactor to step input of reactivity is six delayed neutron group series using six groups of delayed neutrons. The main advantage of this method over the one-delayed-neutron group approximation is that the six-group method is applicable to either small or large changes in K_{eff} whereas one-delayed-neutron group theory is applicable to small changes in the effective multiplication factor, K_{eff} and gives only a qualitative picture to the transient response of the reactor. Since the UTR-10 is made up of two reactor cores, the space and time variables are inseparable. Therefore, the reactor kinetic equations must be derived assuming that space and time variations of neutron flux are inseparable. Seven equations must be solved, one for the neutron flux and six equations in six unknowns for the delayed neutron precursors.

The UTR-10 was chosen for this study for several reasons. First, preliminary work has already been done using an analog computer and one-delayed-group theory.

In order to find out more about the response of the UTR-10 to criticality conditions without danger to the reactor or repeated scramming, more analysis of a theoretical nature is required. This report takes up sinusoidal input of reactivity which was one of the several areas explored in the preliminary work done with an analog computer here at Iowa State University.*

Theoretical calculations will be made to determine the effect which oscillating the regulating rod will have upon the neutron population in the two core UTR-10 reactor. Since the regulating rod will be oscillated at various frequencies, it is desired to show the effect of frequency oscillation of the regulating rod on the neutron level in the two core reactor. In order to do this, the kinetic equations will be modified by taking the Laplace Transform of the kinetic equations and introducing a new variable, $j\omega$ into the equations for the general Laplace Transform variable s. The equations will then relate frequency of oscillation of the regulating rod and neutron level population in the reactor. From the results which are obtained, a Bode Diagram of neutron level versus frequency can be plotted.

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*Danofsky, Richard A. Kinetic behavior of coupled reactor cores. Unpublished M.S. Thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1960.

Stability and regulating rod frequency can then be established by a determination of the flat portion of the open loop response curve as discussed in the results section of this thesis.

REVIEW OF LITERATURE

Baldwin (1) investigated the kinetics of the two-core argonaut reactor. The kinetic equations for a two-core reactor were derived from the general kinetic equations for one region by including a coupling interaction term for each region.

R. W. Bussard and R. D. DeLauer (2) give a section on the kinetics of nuclear reactors as applied to reactor control which is general enough to be of help in the understanding of reactor kinetics as applied to reactor control.

Danofsky (3) derives the transfer function of a two-core reactor using modified two group theory and solves the equations on an analog computer to give a complete qualitative description of reactor kinetics of the UTR-10 reactor at Iowa State University. In this work, an average value of the constants for delayed neutrons is used in order to facilitate the study.

Glasstone (4) has a chapter devoted to reactor kinetics and is of use as a reference textbook on reactor kinetics.

Harrer *et al.* (5) have made a study of the experimental and theoretical frequency responses of the CP-2 reactor. The validity of the reactor transfer function was proved.

Keppin et al. (6) present the latest delayed neutron values for the six groups. The value of β was changed from 0.0075 to 0.0065 due to the results of their work.

Murray (7) has a discussion of the fundamentals of reactor kinetics including a derivation of the reactor kinetic equations from the basic diffusion equations.

Schultz (8) gives curves of frequency response using six groups of delayed neutrons for a one core reactor. Reactor control and natural frequency of oscillation are also discussed in general and specific terms so that using the proper transfer function as given by Schultz, one could design a complete control system. Schultz also discusses the stability of dynamic systems to include the steady state response of a system to a sinusoidal input. The phase and amplitude relationships between input and output variables as a function of frequency are described by the transfer function for the system with $j\omega$ substituted for the Laplace operator.

Weinberg and Wigner (9) present the development of the reactor kinetic equations normally used in time behavior studies. A discussion of the importance of the delayed neutrons on reactor control and the mechanism by which these neutrons affect reactor operation and control is presented. Also presented are complete studies of the poisoning effect, and the negative temperature coefficient effect on reactivity. Equations have been derived which would prove of use when introducing these effects into the transfer function as a negative feedback effect on the reactivity.

DEVELOPMENT OF THE REACTOR KINETIC EQUATIONS

For purposes of analysis it may be assumed that the two separate fuel regions of the UTR-10 reactor can be treated individually with reactivity coupling between cores. Each region is subcritical if it exists alone, but the coupling effect of the other region allows the system of two slabs to be critical or supercritical. Each core region is treated as a black box having an individual reactivity and average neutron density. It is assumed that the spatial distribution of neutron flux within a slab does not affect the behavior of that region. The reactivity coupling effect is due to the interaction exchange of neutrons between the two regions and it is assumed that the coupling effect of region 2 on region 1 is proportional to the average neutron flux in region 2 and the coupling effect of region 1 on 2 is proportional to the average neutron flux in region 1. It is necessary to use a time dependent neutron level equation of the form

$$\frac{dn}{dt} = \frac{n\,\delta K}{\ell} - \frac{\beta n}{\ell} + \sum_{i=1}^{6} \lambda_i C_i + f(n) \qquad 1$$

where n = average neutron density in a fuel region, $\frac{\text{neutrons}}{\text{cm}^3}$

$\delta K = K_{eff} - 1$

ℓ = neutron lifetime in a finite medium, sec.

β = total delayed neutron fraction

λ_i = decay constant for i_{th} group of delayed neutron precursors, sec^{-1}

C_i = concentration of i_{th} group of delayed neutron precursors, $\frac{\text{precursors}}{\text{cm}^3}$

$f(n)$ = coupling effect, function of other core region.

The preceding equation is applied to each of the fuel regions of the reactor core. A differential equation of this form, with the exception of the coupling term, is normally used to describe the kinetic behavior of a single region reactor. The differential equation for the delayed neutron precursor concentration in each slab is given by $\frac{dC_i}{dt} = \frac{\beta_i n}{\ell} - \lambda_i C_i$. 2

Since some delay time is required before a change in neutron density in one core is reflected in the behavior of the opposite region, the coupling effect is assumed to be proportional to the neutron level at a time $(t - \tau)$ previously. The symbol represents the delay time. The coupling function between cores is assumed to be

$$f(n_1) = \frac{\alpha\, n_2(t - \tau)}{\ell} \qquad 3$$

where $n_2(t - \tau)$ = average neutron density in opposite slab a time $(t - \tau)$ previously, neutrons /cm^3

α = coupling coefficient

ℓ = neutron lifetime, sec.

The coupling coefficient, α, can be thought of as an equivalent reactivity since it is determined in terms of an equivalent δk for a region.

The reactor equation for region one then becomes

$$\frac{dn_1}{dt} = \frac{(\delta K)_1 n_1}{\ell} - \frac{\beta n_1}{\ell} + \sum_{i=1}^{6} \lambda_i C_{i_1} + \frac{\alpha n_2}{\ell}(t - \tau). \qquad 4$$

It was assumed that β, λ_i, α and τ were the same for each reaction. The equation for region two is identical to the preceding equation with the subscripts interchanged.

An expression for $n(t - \tau)$ was derived as follows. It was assumed that $n(t - \tau)$ could be expanded in a series, the first two terms being

$$n(t - \tau) = n(t) - \frac{dn}{dt}\tau \qquad 5$$

$n(t)$ is the neutron density at time t, $n(t - \tau)$ is the neutron density at time $(t - \tau)$, and $\frac{dn}{dt}$ is the rate of change of neutron density with time which was assumed to be constant over the time interval τ. Substitution of 5 into 4 yields the final modified reactor kinetic equations for the two regions

$$\frac{dn_1}{dt} = \frac{(\delta K)_1 n_1}{\ell} - \frac{\beta n_1}{\ell} + \sum_{i=1}^{6} \lambda_i C_i + \frac{\alpha n_2}{\ell} - \frac{\alpha\, dn_2}{\ell\, dt}\tau \qquad 6$$

and

$$\frac{dn_2}{dt} = \frac{(\delta K)_2 n_2}{\ell} - \frac{\beta n_2}{\ell} + \sum_{i=1}^{6} \lambda_i C_{i_2} + \frac{\alpha n_1}{\ell} - \frac{\alpha\, dn_1}{\ell\, dt}\tau \qquad 7$$

The differential equations for the delayed neutron precursors for the two regions are

$$\frac{dC_{i_1}}{dt} = \frac{\beta_i n_1}{\ell} - \lambda_i C_{i_1} \qquad 8$$

and $$\frac{dC_{i_2}}{dt} = \frac{\beta_i n_2}{\ell} - \lambda_i C_{i_2}. \qquad 9$$

ANALYSIS OF DATA AND PRESENTATION OF RESULTS

Equations

The final equations which were used to plot frequency vs. neutron level variation have been developed in Appendix D and are given below:

$$\bar{a}s\, N_1(s) = \frac{n_{10}K_1(s)}{\ell} + \frac{\alpha_0 e^{-\tau s}}{\ell} \qquad \frac{\alpha_0 N_1(s) e^{-\tau s}}{\bar{a}s\, \ell}$$

$$N_1(s) = \frac{n_{10}\bar{a}s\ell^2 K_1(s)}{\ell(\bar{a}^2 s^2 \ell^2 - \alpha_0^2 e^{-2\tau s})}$$

$$\frac{n_{10}\bar{a}s\ell K_1(s)}{(\bar{a}^2 s^2 \ell^2 - \alpha_0^2 e^{-2\tau s})} \qquad \text{Eq. } D_{22}$$

$$\frac{N_2(s)/n_{20}}{K_1(s)} = \frac{1.2\alpha_0 e^{-\tau s}}{\bar{a}^2 s^2 \ell^2 - \alpha_0^2 e^{-2\tau s}} \qquad \text{Eq. } D_{26}$$

The equation for core I includes the angular frequency (0-100 cps). The range 0-100 cps has been chosen because it is in this range that control is possible (8). However, control is only possible on the flat portion of this range. The equations for core I also include the lifetime (1.35×10^{-4}) which is referenced in Table 1, and the quantity denoted by $\bar{a}$, where

$$\bar{a} = 1 + \sum_{i=1}^{6} \frac{\beta_i}{\ell(j\omega + \lambda_i)} .$$

Now $\bar{a}$ may vary from about 1.006 -.01j to 1000 - 500j depending on the frequency. As can be seen, when the frequency is low, $\bar{a}$ is a large quantity, and when the frequency is high, $\bar{a}$ is a small quantity. As the frequency goes to 0, $\bar{a}$ goes to infinity. The numerator of the transfer function thus includes $\bar{a}$ and the angular frequency, while the denominator includes $\omega^2 \bar{a}^2$. However, since ω is the dominant factor at most frequencies the amplitude and thus the neutron level variation depends on $\frac{1}{\omega}$. At low frequencies the neutron level variation is greater than at high frequencies.

These equations are relatively accurate and give a quantitative analysis of reactor control. Factors which have been taken into account which are usually left out of a qualitative reactor analysis include: the lag time, which is the time for neutrons born in one core to affect the reactivity in the other, all six groups of delayed neutrons and their constants, and the coupling between the two cores. Usually analogue computer methods, or experimental methods using an average value for six groups of delayed neutrons are attempted in order to study reactor kinetics qualitatively. The average values simplify the calculations and make permissible the use of analogue computer techniques. Using all six groups of delayed neutrons, a matrix array of amplifiers is required. The lag time is so small that it is usually left out.

However, the lag time is extremely important in core two as this is the only term in the numerator. From the equation for core two, this term is $e^{-j\omega\tau}$ which can be represented by Cos $\omega\tau$ - j Sin $\omega\tau$. τ is the delay time and is about 10^{-3} seconds. This value is referenced in Table 1. Therefore, the term $e^{-j\omega\tau}$ could be represented by $1-j\omega 10^{-3}$ for small frequencies.

Graphs

Using the equations developed, graphs were plotted for Core I and Core II. It was assumed that the regulating rod is oscillated at different frequencies. Graphs of neutron level variation vs. frequency were obtained from the data, which has been normalized to a frequency of 1 cycle per second. The neutron level amplitude has been plotted in Figure 1 in terms of decibels which is given by 20 $\log_{10}$ amplitude.

Comparing the results to what was predicted by the equations, one sees that with coupling present the neutron level variation for Core I agrees completely with the predicted behavior. The level approaches negative infinity at 0 cycles per second and rises to a positive level below the arbitrary normalization frequency 1 cycle per second. When the coupling term is dropped in the denominator, the curve obtained is slightly higher and appears to follow the frequency variation more closely. Substituting the constants into the equations shows that the coupling term is often more significant than the frequency or the effect of the delayed neutrons, and since the coupling term is in the denominator it tends to lower the amplitude and serve as a constant damping term on the curve. From physical considerations this means that the second core should dampen the frequency response of the first core since neutrons must go to change the neutron level in the second core also.

However, a change in the reactivity in core 1 will change the neutron level variation in core 2 only slightly as neutrons from core 2 will diffuse back into core 1 while neutrons from core 1 are diffusing to core 2. The curve for core 2 does rise at lower frequencies but the rise is not so great as is the case for core 1. Core 2 does not rise at the same points as core 1 but instead maintains a nearly flat neutron level variation with frequency. From a control point of view, this flat characteristic is desirable and as will be shown later eliminates core 2 from being a hazard from a frequency of oscillation point of view.

In striving for accuracy, the coupling coefficient has been introduced which in the equation for core 2 is the most significant term, thereby making the solution nearly a constant for all frequencies. It is felt therefore that core 2 does have a greater frequency response than is indicated by the Equation D_{26} for core 2.

Comparison with Published Results

M. S. Schultz (8), in Control of Nuclear Reactors and Power Plants, gives a graph of neutron level variation vs. frequency for one core, which would correspond to the graph plotted from these equations for no coupling term. There is nearly a one to one correspondence for $\ell = 1.35 \times 10^{-4}$ seconds. (See core 1, no coupling).

The Bode Diagram for the phase shift of core I (Figure 2) is also of the same shape as the published results given by Schultz (8). The Bode Diagram has a slightly different phase relationship to frequency for the two core reactor than does the Bode Diagram for a single core reactor. However, the difference is not enough to cause concern as only slight variation is noted.

The Bode Diagram for phase shift is used in conjunction with the neutron level variation diagram to establish the natural frequency of oscillation of the regulating rod.

Analysis of Final Results

The curves for core I and for core II plotted in Figure 1 exhibit behavior which is not found for a single core reactor.

Examining the data for core I, it is seen that the amplitude increases with increasing frequency instead of decreasing as the data for a single core reactor predicts. Upon examining the equation for core I, one sees that the denominator is nearly a constant term while the numerator contains the terms $\bar{a}s\ell$ which are usually present in the denominator of a single core reactor. Since $\bar{a}$ and s increase with frequency, the amplitude for a one core reactor decreases with increasing frequency. The denominator for core I of the coupled reactor cores contains the terms $\bar{a}^2s^2\ell^2 - \alpha_0^2e^{-\tau s}$. However, $\bar{a}^2s^2\ell^2$ is less than $\alpha_0^2e^{-\tau s}$ which leads to the constant term $\alpha_0^2e^{-\tau s}$. Therefore, while $\bar{a}s\ell$ increases with increasing frequency, the denominator remains nearly a constant.

Therefore, it is suggested that the coupling coefficient is too large or coupled reactors should be designed with a much smaller coupling coefficient if automatic oscillatory control of the regulating rod is desired. A suggested value is .001 instead of .01 for α_0.

When the coupling coefficient is dropped, and the equation for core I is plotted, Figure 1 is obtained which exhibits normal behavior.

Figure 1. Amplitude of the reactor transfer function

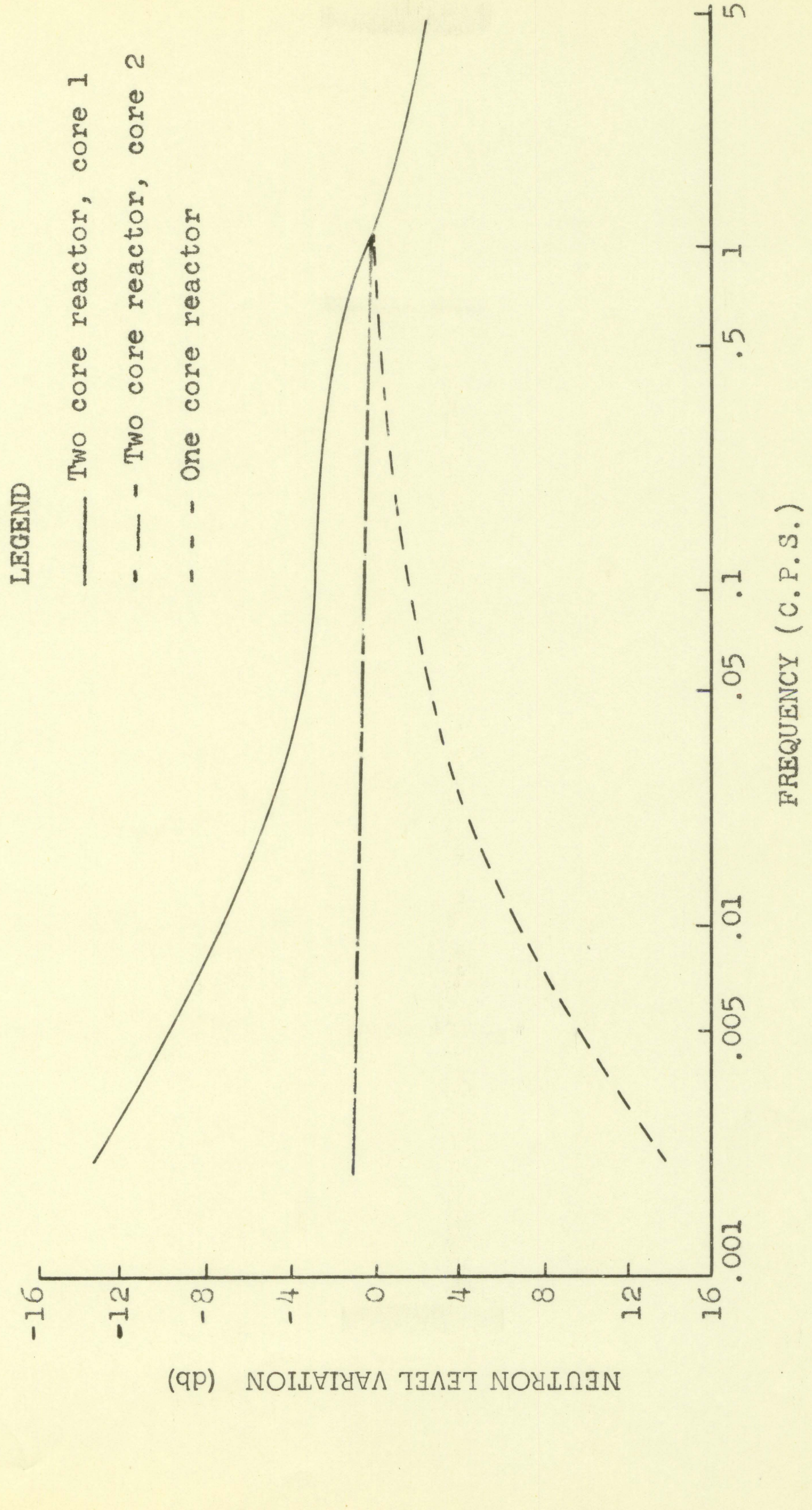
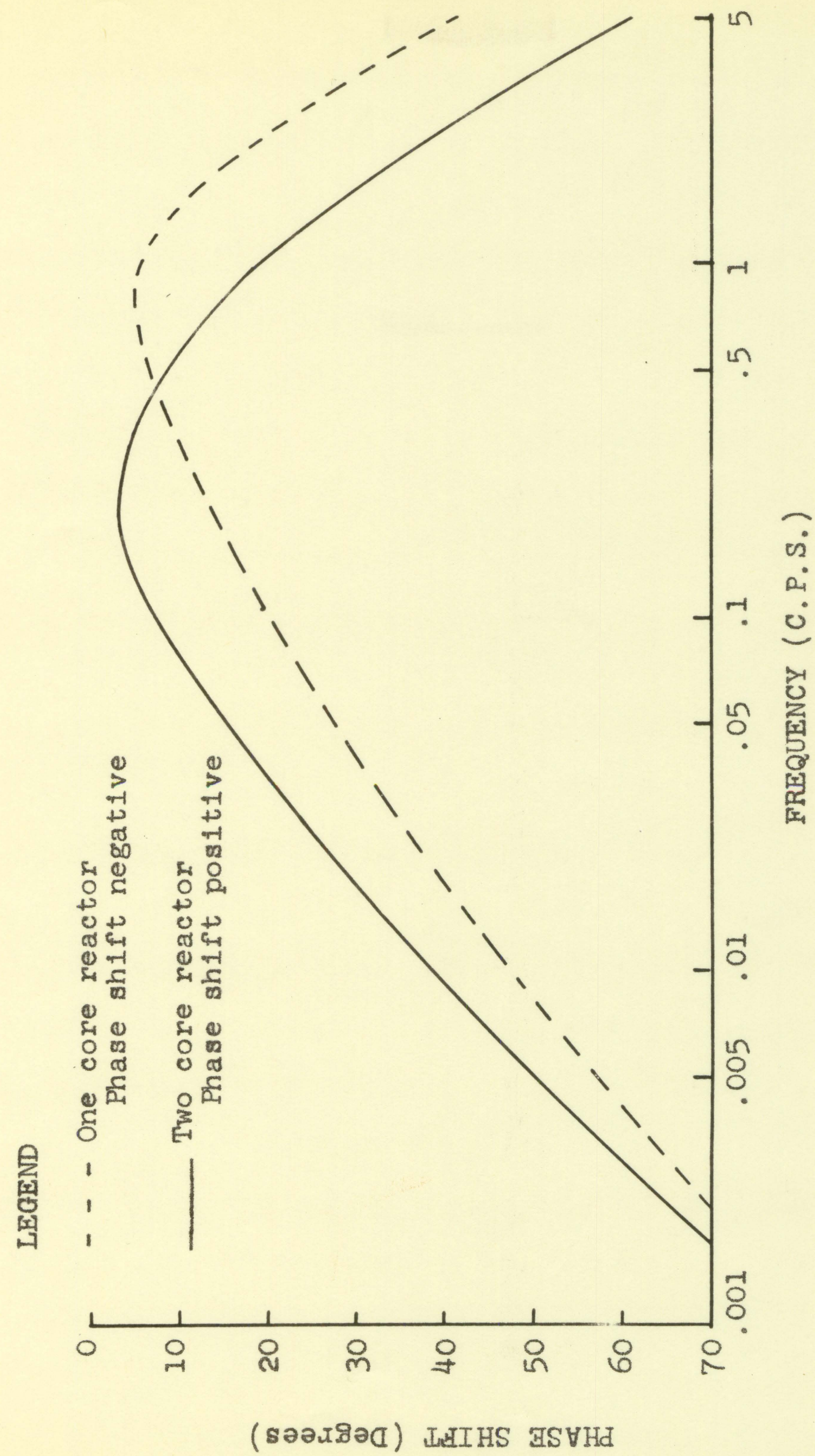

LEGEND
Two core reactor, core 1
Two core reactor, core 2
One core reactor
NEUTRON LEVEL VARIATION (db)
-16
-12
-8
-4
0
4
8
12
16
FREQUENCY (C.P.S.)
.001
.005
.01
.05
.1
.5
1
5

Figure 2. Phase of the reactor transfer function


LEGEND
One core reactor
Phase shift negative
Two core reactor
Phase shift positive
PHASE SHIFT (Degrees)
0
10
20
30
40
50
60
70
.001
.005
.01
.05
.1
.5
1
5
FREQUENCY (C.P.S.)

DISCUSSION AND CONCLUSIONS

A control system is designed to operate on the flat portion of the open-loop response curve. From these calculations one sees that one may oscillate the regulating rod at a frequency between (.05 cps - .5 cps) for core 1 and between (.005 cps - 1 cps) for core 2. However, over-all safety philosophy dictates against fast control systems with high frequencies. First, the components which are used in these fast systems are not reliable as those used in slower systems. Second, a failure in a fast acting system can in itself inject a fast transient disturbance. The faster the capabilities of the system, the more severe the transient. From a safety point of view one should then design any needed control system to be as slow as possible and still satisfy the over-all performance requirements.

Taking into consideration the nuclear capabilities of automatic control systems, a natural frequency for safe operation can be estimated. The complete loop response including temperature effects and poisoning effects has not been considered, as well as the delay in what might be termed "reporting time". Reporting time would be the time between an excursion and the response of the regulating system to this excursion.

If one were to design a control system for a reactor, he should take the preceding things into account by providing a feedback loop for negative temperature coefficient and poison. He should also take into account the phase shift of the reactor transfer function and determine if continuous oscillation at the natural frequency is possible and desirable. However, he should consider, that, while it is quite possible to keep a reactor under control with a continuously oscillating system, prudence dictates that wear on the control-system components should be minimized. Although the area of reactor control is beyond the scope of this thesis, it is possible to pick a range of frequency operation at which safe oscillation of a regulating rod should be possible without instability or large power excursions as the maximum worth of the regulating rod is usually kept well below .006 in reactivity which is in the range of the delayed neutrons.

SUGGESTED TOPICS FOR FURTHER WORK

Using Two Group Theory and the Results of this thesis, one could make a preliminary design of a control system for a two core reactor. By using the proper transfer function one could also study the effects of the negative temperature coefficient and poisoning in the reactor. To do this, it would be necessary to include feedback loops in the reactor transfer function equations.

Further study of reactor control using a quantitative approach is recommended. Experimental studies of frequency response to reactivity oscillation in a safe range of oscillation and at constant power level will give much information about the kinetics of coupled reactor cores.

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APPENDIX A: SAMPLE CALCULATION

$$\frac{N_1(s)/n_{10}}{K_1(s)} = \frac{s\,\bar{a}\,\ell}{s^2\ell^2\bar{a}^2 - \alpha_0{}^2 e^{-2\tau s}}$$

Let s be $j\omega$ so that a plot of neutron level density versus frequency of oscillation can be obtained. Also substitute in constants from Tables 1 and 2

$$\frac{N_1(s)/n_{10}}{K_1(s)} = \frac{j\omega(1.35 \times 10^{-4})\,\bar{a}}{-\omega^2(1.35 \times 10^{-4})^2\,\bar{a}^2 - 2.402 \times 10^{-4}(1-2\times10^{-3}j\ )}$$

where $e^{-2\tau s}$ can be written as $\cos 2\tau\omega - j\sin 2\tau\omega = 1-2j\tau\omega$

and

$$\bar{a} = 1 + \frac{1.83\,(.0127-j\omega)}{\omega^2+(.0127)^2} + \frac{10.25\,(.0317-j\omega)}{\omega^2+(.0317)^2} + \frac{9.03\,(.115-j\omega)}{\omega^2+(.115)^2}$$

$$+ \frac{1.937\,(.311-j\omega)}{\omega^2+(.311)^2} + \frac{6.16\,(1.4-j\omega)}{\omega^2+(1.4)^2} + \frac{1.25\,(3.87-j\omega)}{\omega^2+(3.87)^2}$$

The equation for $\bar{a}$ was obtained from the definition of $\bar{a}$ and by rationalizing the denominators.

$\bar{a}$ was defined to be

$$\bar{a} = 1 + \sum_{i=1}^{6} \frac{\beta_i}{\ell(s+\lambda_i)}$$

Let $\omega = .005$

Substituting in $\omega = .005$, an equation for the amplitude is obtained

$$\frac{N_1(s)/n_{10}}{K_1(s)} = \frac{6.75 \times 10^{-7}\,j\bar{a}}{-4.56 \times 10^{-14}\bar{a}^2 - 2.402 \times 10^{-4} + 2.402 \times 10^{-9}j}$$

$\bar{a}$ comes out to be

$$\bar{a} = 1141.75 - 112.29j$$

$$\bar{a}^2 = 1.29198 \times 10^6 - 2.564 \times 10^5 j$$

$$\frac{N_1(s)/n_{10}}{K_1(s)} = \frac{(7.708j + 7.58 \times 10^{-1})}{1.409 \times 10^{-4}j - 2.403}$$

$$A = \text{amplitude}$$

$$= \sqrt{\frac{(7.708)^2 + (.758)^2}{(2.403)^2 + (1.409 \times 10^{-4})^2}}$$

$$A = 3.223$$

$$\phi = \tan^{-1} \frac{7.708}{.758}$$

$$= +84^\circ\ 24'$$

For 1 cycle per second,

$$A = 18.45$$

Normalize A to 1 cycle per second

$$A_{normalized} = .1747$$

Express A in decibels

$$\text{decibels} = 20 \log_{10} A$$

$$= -15.154 \text{ for } f = .0008$$

where $f = \frac{\omega}{2\pi}$

$$= \frac{.005}{2\pi}$$

For $f = 1$ cycle/sec

$$A = 0 \text{ db}$$

APPENDIX B: TABLES OF CONSTANTS

Table 1. Values of constants to be used in reactor equations[a]

ℓ	α_o	τ	R
1.35×10^{-4}	.0155	10^{-3}	1.2

[a]Source: Danofsky, Richard A. (3, pp. 17,20,30,53). Kinetic behavior of coupled reactor cores. Unpublished M.S. Thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1960.

Table 2. Delayed neutron values to be used in the kinetic equations[a]

Half-life (sec)	λ_i (sec^{-1})	β_i	Total
54	.0127	.000247	.0065
22	.0317	.001384	
5.6	.115	.00122	
2.12	.311	.002646	
.45	1.40	.000832	
.15	3.87	.000169	

[a]Source: Keppin, G. R., and Wimett, T. F. (6, p. 89). Reactor kinetic functions: a new evaluation. Nucleonics. 16, 10: 86-90. October, 1958.

APPENDIX C: TABLES OF RESULTS

Table 3. Values calculated for core I of the coupled reactor cores

Frequency cps	Phase angle	Amplitude	Normalized amplitude	Decibels
.0008	+84°24'	3.223	.1747	-15.154
7.2667	+74°01'	27.894	1.5119	3.590
1.0000	+18°14'	18.450	1.0000	0.000
.0063	+49°31'	6.291	.3410	-9.345
.6159		16.251	.8808	-1.102
.0183	+29°22'	10.450	.5664	-4.938
.0495	+17°57'	12.883	.6983	-3.119
.0020	+66°38'	4.137	.2242	-12.987
15.9200	+95°01'	33.620	1.8220	5.211
1.5920	+27°02'	18.570	1.0065	.056
.7760	+19°07'	15.410	.8350	-1.566
.3184		14.660	.7940	-2.004
.4776		15.060	.8163	-1.763

Table 4. Values calculated for core II of the coupled reactor core

Frequency	Amplitude	Normalized amplitude	Decibels
1.0000	5.319×10^3	1.000	0.000
.0063	4.990×10^3	.938	-.556
.2229	5.254×10^3	.988	-.105
.0183	5.068×10^3	.953	-.418
.0495	5.150×10^3	.968	-.282
.0020	4.986×10^3	.937	-.562

Table 5. Values calculated for one core reactor

Frequency	Amplitude	Normalized amplitude	Decibels
1.0000	2.390×10^2	1.000	0
.0063	6.613×10^2	2.767	8.8402
.6159	2.423×10^2	1.014	0.1208
.2229	2.794×10^2	1.169	1.3563
.0183	4.038×10^2	1.690	4.5577
.0495	3.335×10^2	1.400	2.9226
.0020	1.214×10	5.080	14.1173

APPENDIX D: THEORY AND DEVELOPMENT OF FINAL FORM OF KINETIC EQUATIONS FROM REACTOR KINETIC EQUATIONS MODIFIED FOR A TWO CORE REGION

$$\frac{dn_1(t)}{dt} = \frac{n_1 \delta K_1(t)}{\ell} - \frac{\alpha n_1(t)}{\ell} + \sum_{i=1}^{6} \lambda_i C_i(t) + \frac{\alpha n_2(t-\tau)}{\ell} \qquad \text{Eq. } D_1$$

$$\frac{dC_{i1}(t)}{dt} = \frac{\beta_i n_1(t)}{\ell} - \lambda_i C_{i1}(t) \qquad \text{Eq. } D_2$$

Substituting 2 into 1

$$\frac{dn_1(t)}{dt} = \frac{n_1 \delta K_1(t)}{\ell} - \frac{\beta n_1(t)}{\ell} + \sum_{i=1}^{6} \frac{\beta_i n_1(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_{i1}(t)}{dt} + \frac{\alpha n_2(t-\tau)}{\ell} \qquad \text{Eq. } D_3$$

From $\sum \beta_i = \beta$, we get

$$\frac{dn_1(t)}{dt} = \frac{n_1 \delta K_1(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_{i1}}{dt} + \frac{\alpha n_2(t-\tau)}{\ell} \qquad \text{Eq. } D_4$$

Since the transient response to a small change in reactivity is desired, the neutron density, delayed neutron precursor density, and reactivity may be represented at any time t by their initial values plus a small perturbation which is a function of the time.

If $n_1(t)$ is the neutron density at any time t, this neutron density may be represented by

$$n_1(t) = n_{10} + N_1(t)$$

where n_{10} is the initial neutron density and $N_1(t)$ is the change in neutron density due to a change in reactivity.

Let

$$n_1(t) = n_{10} + N_1(t)$$

$$\delta K_1(t) = \Delta K_{10} + K_1(t)$$

$$\frac{dC_{i1}(t)}{dt} = \frac{dC_{i10}}{dt} + \frac{dC_{i1}(t)}{dt} \qquad \text{Eq. } D_5$$

Substituting Eqns. D_5 into D_4 and dropping 2nd order terms, we get

$$\frac{dN_1(t)}{dt} = \frac{N_{10}\Delta K_{10} + \Delta K_{10}N_1(t) + n_{10}K_1(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_{i10}}{dt}$$

$$- \sum_{i=1}^{6} \frac{dC_{i1}(t)}{dt} + \frac{\alpha_0 n_{20}}{\ell} + \frac{\alpha_0 N_2(t-\tau)}{\ell} \qquad \text{Eq. } D_6$$

Now from steady state values

$$\frac{dC_{i10}}{dt} = 0$$

$$\frac{\alpha_0 n_{20}}{\ell} + \frac{n_{10}\Delta K_{10}}{\ell} = 0$$

$$\frac{\beta n_{10}}{\ell} + \lambda_i C_{i10} = 0 \qquad \text{Eq. } D_7$$

$$\frac{dN_1(t)}{dt} = \frac{\Delta K_{10} N_1(t)}{\ell} + \frac{n_{10} K_1(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_{i1}(t)}{dt}$$

$$+ \frac{\alpha_0 N_2(t - \tau)}{\ell} \qquad \text{Eq. } D_8$$

$\Delta K_{10} N_1(t)$ is a product of two Δ terms (ie ΔK_{10} is small)

$$\frac{dN_1(t)}{dt} = \frac{\alpha_0 N_2(t - \tau)}{\ell} + \frac{n_{10} K_1(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_{i1}(t)}{dt} \qquad \text{Eq. } D_9$$

$$\frac{dC_{i1}(t)}{dt} = \frac{\beta_i n_{10}}{\ell} + \frac{\beta_i N_1(t)}{\ell} - \lambda_i C_{i10} - \lambda_i C_{i1} \qquad \text{Eq. } D_{10}$$

From Eq. D_7 and Eq. D_{10}, we get

$$\frac{dC_{i1}(t)}{dt} = \frac{\beta_i N_1(t)}{\ell} - \lambda_i C_{i1}(t) \qquad \text{Eq. } D_{11}$$

Taking the Laplace Transform of Eq. D_{11} gives

$$s\,C_{i1}(s) = \frac{\beta_{i1}}{\ell} N_1(s) - \lambda_{i1} C_{i1}(s) + C_{i1}(0) \qquad \text{Eq. } D_{12}$$

Solving for $s\,C_{i1}(s)$ gives

$$s\,C_{i1}(s) = \frac{s\,\beta_{i1} N_1(s)}{\ell(s + \lambda_{i1})} + \frac{s\,C_{i1}(0)}{(s + \lambda_i)} \qquad \text{Eq. } D_{13}$$

The initial condition transforms may be dropped since we define a transfer function in terms of the steady state response. Taking the Laplace Transform of Eq. D_9, and using Eq. D_{13} gives

$$s\,N_1(s) = \frac{\alpha_0 N_2(s)e^{-\tau s}}{\ell} + \frac{n_{10}K_1(s)}{\ell} - \sum_{i=1}^{6} \frac{s\,\beta_{i1}N_1(s)}{\ell(s+\lambda_{i1})} \qquad \text{Eq. } D_{14}$$

Rearranging Eq. D_{14} gives

$$s\,N_1(s)\left[1 + \sum_{i=1}^{6} \frac{\beta_{i1}}{\ell(s+\lambda_{i})}\right] = \frac{n_{10}K_1(s)}{\ell} + \frac{\alpha_0 N_2(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{15}$$

Since the reactivity will only effect one core, core 2 will have the same equation as core 1 with the subscripts interchanged and no reactivity.

$$s\,N_2(s) = \frac{\alpha_0 N_1(s)e^{-\tau s}}{\ell} - \sum_{i=1}^{6} \frac{s\,\beta_i N_2(s)}{\ell(s+\lambda_i)} \qquad \text{Eq. } D_{16}$$

Rearranging Eq. D_{16} gives

$$s\,N_2(s)\left[1 + \sum_{i=1}^{6} \frac{\beta_i}{\ell(s+\lambda_i)}\right] = \frac{\alpha_0 N_1(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{17}$$

Let

$$1 + \sum_{i=1} \frac{\beta_i}{\ell(s+\lambda_i)} = \bar{a}$$

Eq. D_{17} becomes

$$\bar{a}s\,N_2(s) = \frac{\alpha_0 N_1(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{18}$$

Eq. D_{15} becomes

$$\bar{a}s\,N_1(s) = \frac{n_{10}K_1(s)}{\ell} + \frac{\alpha_0 N_2(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{19}$$

Eq. D_{18} becomes

$$\bar{a}s\ N_2(s) = \frac{\alpha_0 N_1(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{20}$$

Eq. D_{19} becomes

$$\bar{a}s\ N_1(s) = \frac{n_{10}K_1(s)}{\ell} + \frac{\alpha_0 N_2(s)e^{-\tau s}}{\ell} \qquad \text{Eq. } D_{21}$$

Substituting Eq. D_{20} into Eq. D_{21} gives

$$\bar{a}s\ N_1(s) = \frac{n_{10}K_1(s)}{\ell} + \frac{\alpha_0 e^{-\tau s}}{\ell} \quad \frac{\alpha_0 N_1(s)e^{-\tau s}}{\bar{a}s\ell}$$

$$N_1(s) = \frac{n_{10}\bar{a}s\ \ell^2 K_1(s)}{\ell(\bar{a}^2 s^2 \ell^2 - \alpha_0{}^2 e^{-2\tau s})}$$

$$\frac{n_{10}\bar{a}s\ \ell K_1(s)}{(\bar{a}^2 s^2 \ell^2 - \alpha_0{}^2 e^{-2\tau s})} \qquad \text{Eq. } D_{22}$$

$$N_2(s) = \frac{\alpha_0 e^{-\tau s}}{\bar{a}s\ell} \quad \frac{n_{10}\bar{a}s\ \ell K_1(s)}{(\bar{a}^2 s^2 \ell^2 - \alpha_0{}^2 e^{-2\tau s})} \qquad \text{Eq. } D_{23}$$

$$= \frac{n_{10}e^{-\tau s}K_1(s)\,\alpha_0}{(\bar{a}^2 s^2 \ell^2 - \alpha_0{}^2 e^{-2\tau s})} \qquad \text{Eq. } D_{24}$$

$$\frac{n_{10}}{n_{20}} = 1.2 = \text{Flux tilting} \qquad \text{Eq. } D_{25}$$

$\frac{n_{10}}{n_{20}}$ is given in Appendix B by the symbol R.

$$\frac{N_2(s)/n_{20}}{K_1(s)} = \frac{1.2\,\alpha_0 e^{-\tau s}}{\bar{a}^2 s^2 \ell^2 - \alpha_0{}^2 e^{-2\tau s}} \qquad \text{Eq. } D_{26}$$