A REACTOR USING UO_2-NaK SLURRY

by

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I. INTRODUCTION

The use of a slurry as a nuclear reactor fuel was suggested early in the development of nuclear reactor types. At that time, the problems associated with the handling of solids suspended in a fluid and the mechanical stability of such suspensions caused the investigators to concentrate their efforts on other types of reactors.

Modern technology has solved or simplified many of the problems, and now the advantages of circulating fuels coupled with the development of new suspensions has aroused new interest in slurry reactors. The reactor suggested here, although not new in mechanical design, does involve a new system, and one of the purposes of this thesis is to investigate the characteristics of liquid-metal slurries and to study a particular slurry with regard to its use as a reactor fuel.

The choice of the UO_2-NaK system was made and a criticality analysis was made to determine the effect of changing some of the many variables on the critical mass and the dimensions of the reactor core, particularly the effect of placing a breeder blanket around the bare reactor. In addition, a reactor cycle was proposed for a reactor using the UO_2-NaK system, and the problems associated with the use of the system as a reactor fuel were discussed.

II. SURVEY OF TYPES OF LIQUID-METAL FUEL REACTORS

The use of a circulating fuel in a nuclear reactor offers many desirable advantages over solid fuel elements, such as simplified structural design, easier fuel handling, simplified fuel processing, and continuous removal of fission products. The circulating fuel in many reactor designs is a solution of the fissionable material in a fluid, or a suspension of the solid fuel in a fluid, referred to as a slurry. Slurry reactors have been studied, and several designs have been presented, some of which will be summarized here.

Original consideration was given to suspensions of the uranium oxides in water, and later to suspensions of intermetallic compounds in liquid metals. Other systems using a gas as the fluid and other forms of the fuel as the solid phase were also considered. As yet, construction of a slurry reactor of any type has not been attempted. The research groups at Brookhaven National Laboratory (BNL) have shown the greatest interest in liquid-metal fuels, and although they are primarily interested in a solution of uranium in liquid bismuth, they have suggested a number of intermetallic-compound dispersions which they consider worthwhile, both as reactor fuels and breeder materials (1). Some of the intermetallic-compound dispersions considered by BNL are

Table 1. Possible intermetallic-compound dispersions in liquid-metal systems[a]

Solid compound	Density (gm/cm^3)	Liquid compound (wt. %)	Density (gm/cm^3)	Freezing point (°C)
U_3Bi_5	12.4	Bi	10.0	273
U_3Bi_5	12.4	55.5 Bi, 44.5 Pb	10.3	125
UBi	12.5	85 Pb, 15 Bi	10.5	300
USn_3	10.0	5 Sn, 35 Bi, 60 Pb	10.0	160
USn_3	10.0	6 Sn, 56.2 Bi, 37.8 Pb	9.8	115
Th_3Bi_5	10.5	Bi	10.0	273
Th_3Bi_5	10.5	55.5 Bi, 44.5 Pb	10.3	125

[a]Reproduced from Reference 1

listed in Table 1.

It will be noted that bismuth, lead, tin, and their alloys are considered by BNL to be the most important liquid phases. A more extensive table listing other dispersions may be found in Volume 2 of the Reactor Handbook (2).

It has been found at BNL that the compound U_3Bi_5 can be dispersed in bismuth or lead-bismuth alloys containing up to 80 per cent lead (1). The addition of lead to the bismuth decreases the solubility of the uranium. If a higher uranium concentration than that possible with U_3Bi_5 is desired, the use of the UBi compound is suggested. This compound is in equilibrium with bismuth-lead alloys containing 90-98 per cent lead below 600 C (1). Both these compounds are

pyrophoric, and both have a higher density than their respective suspending fluids, which might result in some settling in dilute dispersions.

The addition of tin to lead-bismuth alloys provides another possible suspension. The compound USn_3 is "apparently stable" in many bismuth-lead-tin alloys (1). The density of USn_3 is matched by liquid-metal alloys containing 3-5 per cent tin, 35-40 per cent bismuth, and the remainder lead.

Thorium is considered for use in a breeder blanket. The existence of $ThBi_3$ is doubtful, but it is probable that the compound Th_3Bi_5 is in equilibrium with bismuth and lead-bismuth alloys containing up to at least 92 per cent by weight lead at temperatures up to 450 C. A compound, possibly $ThPb_3$, is in equilibrium with liquid lead containing up to one per cent bismuth (1).

A conceptual design of an externally-cooled, power-breeder, thermal reactor (LMFR) has been presented by BNL (1). This reactor uses U^{233} dissolved in bismuth as the fuel and the intermetallic compound Th_3Bi_5 suspended in bismuth as the breeder blanket. The fuel concentration in this reactor is limited by the low solubility of uranium in bismuth (600 ppm at 400 C). The reactor was designed to produce 450 megawatts of heat, uses a graphite moderator, has a cylindrical core five feet in diameter and five feet high, and is designed for a thermal efficiency of over 35 per cent. Some integral

processing systems which are characteristic of this reactor are removal of gaseous fission products by passing argon through the fuel, and separation of fission products from the fuel by a fused-salt extraction process.

BNL has also conceived a design for an internally-cooled, power-breeder reactor (3). The fuel in this reactor consists of 1.77 per cent by weight USn_3 (U^{233}) dispersed in a liquid containing 56.2 per cent bismuth, 6 per cent tin, and 37.8 per cent lead. The breeder blanket is a dispersion of Th_3Bi_5 in bismuth which also serves as the coolant. The purpose of internal cooling is to reduce the holdup of fuel slurry in the external circuit.

Another possibility for a liquid-metal slurry is a dispersion of a compound such as UO_2, U_3O_8, or UF_4. Early attempts to disperse these compounds in liquid metals were unsuccessful, but an investigation of the UO_2-Bi system by Knolls Atomic Power Laboratory (KAPL) and engineering studies of the UO_2-NaK system have been made by Argonne National Laboratory (ANL) (4, 5). A study group at KAPL investigating the UO_2-Bi system has come to the preliminary conclusions that a reactor using the system can operate continuously using natural uranium, would employ a simple fuel processing cycle, and would have a high conversion ratio. These conclusions were based on the assumptions that UO_2 can be dispersed in molten bismuth and that the system can be contained in

beryllium tubes (4). The group at ANL believes that the UO_2-NaK system would be satisfactory as a nuclear reactor fuel (5), but has not made any preliminary criticality calculations using this system in a reactor. The findings of this group will be discussed in detail in Section III.

III. CHARACTERISTICS OF LIQUID-METAL SLURRIES

A. Mechanical Stability

The mechanical stability of a reactor slurry requires that the particles be uniformly distributed throughout the fluid when the slurry is in motion, and that the particles be easily redispersed if allowed to settle out for an arbitrary length of time. The mechanical stability of a liquid-metal slurry requires that the solid phase be wetted by the liquid, and that the density of the solid be matched with that of the liquid. The liquid must completely wet the solid to ensure that the particles will stay in suspension and not be floated out by the presence of gases. The density requirement eliminates the need for outside influences, such as forced agitation with mixers, to maintain the suspension. The use of small particles (1-10 microns) with low settling velocities is also helpful in maintaining mechanical stability. The stability of fine particle suspensions comes primarily from Brownian movement.

It is possible to satisfy these requirements with intermetallic compounds such as U_3Bi_5, USn_3, etc., but it is not always possible with dispersions of ionic compounds such as UO_2, UF_4, etc. Early attempts to produce stable dispersions

of ionic compounds in liquid metals proved difficult because of insufficient wetting.

The best condition for the wetting of a solid by a liquid exists if the contact angle for the liquid phase is 0°. Contact angle measurements for ionic compounds were made at BNL, and it was found that UO_2 and UF_4 are wetted by sodium and that the wetting of UO_2 and UF_4 by bismuth is improved by the addition of a small amount of sodium (2). No information is available on the ionic compounds of thorium suspended in liquid metals.

In some preliminary tests, a group at ANL found that a 10 volume-per cent UO_2-NaK slurry remains uniform at "conventional pumping speeds and with modest power consumption" (5). Work is being continued to investigate slurries having a concentration greater than 23 volume-per cent UO_2.

Caking and flocculation are two stability problems which are characteristic of individual dispersions. Caking occurs when the solid compound of the slurry sticks to the containing material. Flocculation occurs when particles in molecular motion collide with each other resulting in agglomerations which may settle out faster than the dispersed particles or may rise to the surface if the agglomerate contains air or other gases. It has been stated that UO_2 that had been suspended in NaK at 500 C could be easily resuspended after settling and did not cake or flocculate even

after prolonged settling (5).

B. Chemical Stability

Chemical stability requires that a solid material remain stable against chemical reaction with the liquid phase. This problem arises in the case of ionic compound dispersions. For the UO_2-Na system, the reaction is

$$UO_2 + 4\ Na \longrightarrow U + 2Na_2O.$$

Calculation of the free energy change for this reaction and a similar one for PuO_2 indicated that very little reduction of the oxide to the metal would occur at temperatures up to 1225 C (5).

Chemical stability does not present serious problems in the case of liquid-metal solutions or intermetallic-compound dispersions, but the physical changes which occur in these systems can create certain problems and solve others. In the example of a solution of uranium in bismuth in a reactor, the accidental cooling of the fuel stream can cause a uranium-bismuth compound to precipitate in the form of platelets which can plug a small channel. In one example of an intermetallic-compound dispersion, when the compound is dissolved and reprecipitated, the compound reappears in the form of uniform cubes. Thus, the problems of particle comminution due to fission, particle growth due to flocculation, and removal of fission products, can sometimes be solved by proper

use of the physical changes that take place in a particular system. These problems and solutions are, of course, specific for each dispersion, and have not been studied extensively.

C. Preparation of Dispersions

Dispersions of intermetallic compounds in liquid metals can be made in a number of ways. The simplest method is that of reacting the finely divided solid material with the liquid. For uranium and thorium this is accomplished in the laboratory by heating the mixture in a graphite crucible at 1200 C for one hour (1). This method has been used to produce dispersions of $U(Sn, Pb)_3$ in lead-tin alloys, resulting in a particle size of about three microns (2). Although uranium powder produced by the decomposition of the hydride is generally used, dispersions of sub-micron size particles are obtained by the direct use of the hydride (2).

Besides the methods of direct preparation and hydride decomposition mentioned above, is the method of exfoliation, in which the metallic chips are added to a molten alloy at a low temperature and allowed to react, after which the product exfoliates away and is collected. A fourth method is electrolysis with a fused salt. This method also results in finely divided dispersions.

Ionic-compound dispersions are produced simply by adding the compound directly to the liquid metal. It is sometimes necessary to do this at a high temperature to ensure complete wetting (5).

D. Slurry Handling

The problems of handling liquid-metal slurries are, except for the specific problems due to the physical properties of the liquid metals, essentially the same as the handling problems of slurries in general. These problems are corrosion and erosion of container materials by the fluid and by the solid particles, prediction of pressure loss in the flow of the slurry, concentration limitations, and pumping and separating of the two phases. These problems will be discussed in some detail.

1. Corrosion and erosion

A container material which is subject to corrosion may protect itself by the formation of an oxide coating. The solid particles present in a slurry may wear away this coating and expose the base material to corrosion by the fluid. In addition to this, there is erosion of the material by particle impingement alone. Erosion by particle impingement is increased with increased turbulence and is greatest in bends, elbows, and valves, where the solid particles strike the

material with the full force of their forward motion.

It has been stated that graphite is satisfactory in contact with bismuth-uranium solutions, and that the mass transfer problems in heat exchanger tubes carrying bismuth-uranium solutions are solved by the use of inhibitors such as magnesium and zirconium (1). It has also been stated that beryllium is believed to be more resistant than graphite to erosion by UO_2 particles in a UO_2-bismuth slurry (4). However, the first statement cannot be assumed to hold true for dispersions because of the erosive nature of the particles, and the second statement has not been verified by experimental tests. In evaluating the resistance of a material to erosion, the results obtained for one dispersion cannot be assumed to be true for another type of dispersion.

Experiments with a dispersion of USn_3 in bismuth against graphite at temperatures above 650 C indicated that no serious carbide deposition should occur (3). This result was obtained with a stationary system.

The corrosive properties of the liquid-metal slurry are probably indistinguishable from those of the pure fluid and in some cases, particularly the UO_2-bismuth and UO_2-lead systems (4), the resistance of the container materials to corrosion does not depend upon a protecting film, so that erosion by particle impingement would be the most serious problem.

The corrosion behavior of liquid metals has been studied by various investigators, and their results are summarized and discussed in the Liquid Metals Handbook (6). It has been found that the corrosion rate is very sensitive to impurities, particularly oxygen, contained in the liquid metal. Since oxygen content of the liquid metal is so important, a system containing UO_2 requires particular consideration. The use of an oxygen "getter" is suggested in this case, and one choice is a slight excess of uranium metal, although beryllium and calcium are also effective.

2. Pressure loss

It has been found at Iowa State College (7) that the pressure loss due to friction of the flow of slurries is dependent upon whether the flow is horizontal or vertical, and whether it is upward or downward. The pressure drop is also dependent upon the solid density, fluid density, fluid viscosity, particle size, concentration, pipe diameter, and velocity of the mixture. However, the work done at Iowa State College is limited to aqueous suspensions and the results cannot be extended to liquid-metal slurries.

It has been found that the pressure drop and power consumption for a UO_2-NaK slurry flowing in a one-inch diameter pipe were practically the same for a 5 volume-per cent

suspension as those for the pure fluid but increased greatly when the solid concentration was increased to 10 volume-per cent (5).

3. Concentration

The maximum concentration of solid particles which can be carried by a reactor slurry is of importance when the choice of slurries is to be made. The highest possible concentration of uranium is usually preferred, because of the higher breeding gain attainable with higher concentrations. The maximum volume concentration carried by a slurry is limited by the decrease in mobility with increasing concentration, or the condition at which the effective viscosity of the slurry becomes too great to allow pumping. The size and shape of the solid particles also affect the maximum concentration which can be carried by a fluid.

Because of the viscosity limitations, a concentration of 25 per cent by volume is the maximum allowable concentration for dispersions in the lead-bismuth-tin alloys (1). This corresponds to a concentration of about 10 per cent by weight of thorium or uranium for the intermetallic compound dispersions in the lead-bismuth-tin alloys. The maximum concentration of UO_2 in the UO_2-NaK system is unknown, but it is believed that a slurry having a concentration of 10 per cent by volume can be easily handled with moderate power consumption (5).

4. Pumping and separating

Liquid-metal slurries may be pumped by centrifugal or electromagnetic pumps. The design problems of centrifugal pumps are essentially the same for all reactor slurries, i. e., suitable seals to prevent escape of the highly radioactive slurry, erosion of the moving parts of the pump, and protection of the bearings from the chemical action of the slurry. It has been found at ANL that an Allis-Chalmers General Purpose canned-rotor pump is satisfactory for use in laboratory tests of the handling characteristics of the UO_2-NaK system (5).

Separation of the solid fuel particles from the liquid is sometimes necessary for fuel reprocessing or slurry dilution. One common method is filtration. Filtration of liquid-metal slurries has been found to be successful on a laboratory scale with stainless steel or graphite filters (3).

E. Radiation and Fission Effects

The effects of the exposure of liquid metals to a neutron flux and to gamma radiation, and the effects of fission on the solid fuel particles present in a reactor slurry present certain problems and provide solutions for others. Neutron capture by liquid metals frequently leads to the formation of radioactive isotopes. Two significant examples are bismuth

and sodium. Bismuth undergoes a (n, γ) reaction to form bismuth-210 (half-life, 5 days), which decays by beta-emission to yield polonium-210. Polonium-210 is a hazard because it is toxic and difficult to contain. Sodium, upon the capture of a neutron, forms sodium-24 (half-life, 15 hours), which emits beta particles and two gamma-rays of fairly high energy (1.38 and 2.75 Mev).

The solid fuel particles will necessarily undergo fission, and as a result, may undergo comminution. A disadvantage of this reduction in particle size is an increase in the apparent viscosity of the slurry, while an advantage is the separation of the fission products from the fuel resulting in easier chemical processing of the fuel.

The study of radiation and fission effects requires in-pile tests and the results depend upon the specific constituents of the slurry.

F. Heat Transfer

Little is known about the heat transfer from slurries. Liquid-metal slurries, in particular, are deficient in experimental heat transfer study. So far, it has been considered satisfactory to evaluate film-heat-transfer coefficients for forced circulation from the Lyon-Martinelli equations (6). Heat exchanger calculations for circulating

slurries may be made if the heat carried by the solid material is allowed for along with the heat carried by the fluid.

In a reactor using solid fuel elements, the limitation to heat transfer inside the core is the film-coefficient for heat transfer across the surface of the fuel elements. In a liquid-fuel, externally-cooled reactor, the factors that limit the amount of heat that can be removed from the core are the heat capacity of the fluid, the mass rate of flow of the fuel, and the permissible temperature rise of the fuel, since the rate of heat removal is given by

$$Q = m\, C_p(T_{out} - T_{in})$$

where Q is the total heat rate, m is the mass rate of flow, C_p is the specific heat of the fluid, T_{out} is the outlet temperature of the fluid, and T_{in} is the inlet temperature of the fluid.

G. Fuel Processing

In a circulating-fuel reactor, the reasons for continuous fuel processing are to prevent the precipitation of fission products, to reduce the poisoning by the fission products, to be able to continuously add the fuel make-up, and to remove the radioactive byproducts, such as polonium-210 (from bismuth).

One processing method suggested for use in the liquid metal fuel reactor (LMFR) designed by BNL is that of fused-salt extraction (1, 3). This process takes advantage of the

fact that fission products may be separated from a liquid-metal-fuel solution by contacting the fused salt with the liquid metal. For the uranium-bismuth solution in the LMFR, the fused salt suggested is the LiF-KF eutectic (1).

It is possible to treat dispersions in the same manner, if advantage is taken of the separation of the fission products from the solid phase when the dispersion is dissolved in the liquid phase at a high temperature. The fuel is then precipitated and the liquid phase containing the fission products is removed to the processing loop by filtering out the solid phase with a steel filter. This method was suggested for the internally-cooled LMFR (3) to eliminate removal of the fuel from the reactor.

The removal of gaseous fission products might be made possible by sparging the fuel with an inert gas such as argon or helium. This method has been suggested to remove polonium and xenon from the bismuth-uranium solution in the LMFR (1).

IV. CRITICALITY OF A REACTOR USING THE UO_2-NaK SYSTEM

The choice of a liquid-metal slurry as the fuel for a nuclear reactor was made because of the higher possible uranium concentrations obtainable in a slurry as compared to a solution of uranium in a liquid metal. It was observed that the UO_2-NaK system offers certain advantages which make the system worthy of investigation. These advantages are a fluid with a low melting point (12 F for the eutectic mixture), which would eliminate the need for a heating system to melt the metal before starting the reactor; a fluid with a low density, which would considerably lower the pumping power required (compared to the heavier metals); and the use of a fluid with a high heat capacity compared to that of the heavier metals, which would reduce the mass rate of flow required to remove the heat. The most apparent disadvantages of this system are the use of a fluid with a fairly high neutron absorption cross-section, which would reduce the number of neutrons available for breeding, and the use of a solid material with a density considerably greater than that of the fluid, which presents the problem of settling out of the particles if the movement of the slurry is stopped.

It was assumed that the slurry would be in the form of

small particles of UO_2 suspended in the sodium-potassium (NaK) eutectic alloy (22 per cent sodium by weight). On the basis of the engineering studies at ANL (5), the concentration was limited to 10 per cent UO_2 by volume, and the velocity was limited to 20 feet per second.

In the investigation of the criticality of the reactor, it was initially assumed that the reactor would be thermal, would be cooled by circulating the slurry through external heat exchangers, would use graphite as the moderating material, and would use U^{233} as the fuel. The core would be an upright, circular cylinder, with the slurry flowing upward through a heterogeneous lattice of fuel channels.

An initial study of criticality was made using the Fermi Age theory applied to a bare, homogeneous reactor. This was followed by a study using the one-group diffusion theory applied to a reactor with a breeder blanket surrounding the core.

A. Fermi-Age Theory for Bare Reactor

An initial study of criticality was made using the Fermi-Age approximation to determine the critical radius of a bare reactor as a function of the volume ratio of moderator to fuel slurry.

The critical equation for a bare, homogeneous reactor is

$$\frac{k_\infty e^{-B_c^2 \tau}}{1 + L^2 B_c^2} = 1 \qquad (1)$$

where B_c^2 is the critical buckling of the core, and the other factors are defined in Appendix A.

For a critical reactor the critical core buckling must be equal to the geometric buckling B_g^2, which is given by

$$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2 \qquad (2)$$

for a cylindrical, homogeneous reactor.

For minimum critical volume of a cylindrical reactor H = 1.847 R. Substitution of this into Equation 2 gives

$$B_g^2 = \frac{8.68}{R^2} \qquad (3)$$

and therefore the critical radius is given by

$$R_c^2 = \frac{8.68}{B_c^2} \qquad (4)$$

where B_c^2 is obtained from Equation 1. It is necessary to evaluate the remaining factors in Equation 1 in terms of the composition of the core and the properties of the core materials. The three factors are the infinite multiplication factor, the diffusion length for thermal neutrons, and the Fermi Age. It was decided that the factors would be developed in terms of the volume ratio of fluid to solid (V_{NaK}/V_{UO_2}) and the volume ratio of moderator to fuel (V_c/V_{UO_2}). The

details of the evaluation of each factor are given in Appendix B, but the development will be summarized here.

The infinite multiplication factor was evaluated from the four-factor formula ($k = \eta f \epsilon p$), and it was assumed that p and ϵ were equal to one. The thermal utilization was calculated from

$$f = \frac{\Sigma_{a,\ fuel}}{\Sigma_{a,\ core}} = \frac{N_{U23}\ \sigma_{a,\ U23}}{N_{U23}\sigma_{a,U23} + N_c \sigma_{a,c} + N_{NaK}\ \sigma_{a,NaK}} \quad (5)$$

where the N's refer to the atomic densities of each material in the reactor. Equation 5 may be written as

$$f = \frac{\sigma_{a,\ U23}}{\sigma_{a,U23} + \frac{N_c}{N_{U23}}\ \sigma_{a,c} + \frac{N_{NaK}}{N_{U23}}\ \sigma_{a,Na\ K}} \quad (6)$$

and the ratios of the N's may be converted to volume ratios, resulting in

$$f = \frac{\sigma_{a,U23}}{\sigma_{a,U23} + \frac{V_{NaK}}{V_{UO_2}}\ \frac{\Sigma_{a,NaK}}{n_{UO_2}} + \frac{V_c}{V_{UO_2}}\ \frac{\Sigma_{a,c}}{n_{UO_2}}} \quad (7)$$

where Σ_a is the macroscopic cross-section of the individual component, and n_{UO_2} is the molecular density of UO_2. Substitution of the appropriate constants into Equation 7 gives

$$k_\infty = \eta f = \frac{836}{362 + 0.54\ \frac{V_{NaK}}{V_{UO_2}} + .0214\ \frac{V_c}{V_{UO_2}}} \quad (8)$$

Thus, k_∞ may be found as a function of the concentration and the volume ratio of carbon to uranium dioxide (V_c/V_{UO_2}).

The thermal diffusion length is found from the relation

$$L^2 = \frac{D}{\Sigma_a} \tag{9}$$

where the diffusion coefficient of the core is given by

$$D_{core} = \frac{1}{3\Sigma(1-\bar{\mu}_o)\left(1 - \frac{4}{5}\frac{\Sigma_a}{\Sigma} + \frac{\Sigma_a}{\Sigma}\frac{\bar{\mu}_o}{1-\bar{\mu}_o} - \cdots\right)} \tag{10}$$

and the average absorption cross-section of the core is found from

$$\Sigma_{a,core} = N_{U23}\,\sigma_{a,U23} + N_c\,\sigma_{a,c} + N_{NaK}\,\sigma_{a,NaK}. \tag{11}$$

After performing operations similar to those for the infinite multiplication factor, the formulas for D_{core} and Σ_a become

$$D_{core} = \frac{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}}{5.805 + .0909\frac{V_{NaK}}{V_{UO_2}} + 1.093\frac{V_c}{V_{UO_2}}} \tag{12}$$

and

$$\Sigma_{a,core} = \frac{8.87 + .01323\frac{V_{NaK}}{V_{UO_2}} + .000524\frac{V_c}{V_{UO_2}}}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \tag{13}$$

The Fermi Age is given by

$$\tau(E) = \int_{E_{th}}^{E_f} \frac{D}{\xi\Sigma_s}\cdot\frac{dE}{E} \tag{14}$$

and the assumption was made that the factors were independent of energy, resulting in

$$\tau = \frac{D}{\xi\Sigma_s}\ln\frac{E_f}{E_{th}}. \tag{15}$$

Evaluation of the slowing-down power is made from

$$\overline{\xi \Sigma_s} = (\xi N \sigma_s)_c + (\xi N \sigma_s)_{NaK} + (\xi N \sigma_s)_{U23} + (\xi N \sigma_s)_o \qquad (16)$$

which may be written as

$$\overline{\xi \Sigma_s} = \frac{.00174 \frac{V_{NaK}}{V_{UO_2}} + .0609 \frac{V_c}{V_{UO_2}} + .0224}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \qquad (17)$$

The use of Equations 12, 15, and 17 gives the Fermi Age as a function of the concentration and V_c/V_{UO_2}.

Evaluation of the factors was made for a 10 per cent by volume suspension and evaluation of B_c^2 was made for different values of V_c/V_s using Equation 1. The critical radius of a bare cylindrical core was then calculated using Equation 4. The results of this study are shown in Figure 3 for a system using U^{233}. The individual factors k_∞, L^2, and τ, are shown in Figure 1 as a function of V_c/V_s.

B. One-Group Theory for Reactor with Breeder Blanket

The diffusion equations for a two-region reactor with reproduction in the core are

$$D_c \nabla^2 \phi_c - \Sigma_{a,c} \phi_c + k \Sigma_{a,c} \phi_c = 0 \qquad (18)$$

for the core, and

$$D_R \nabla^2 \phi_R - \Sigma_{a,R} \phi_R = 0 \qquad (19)$$

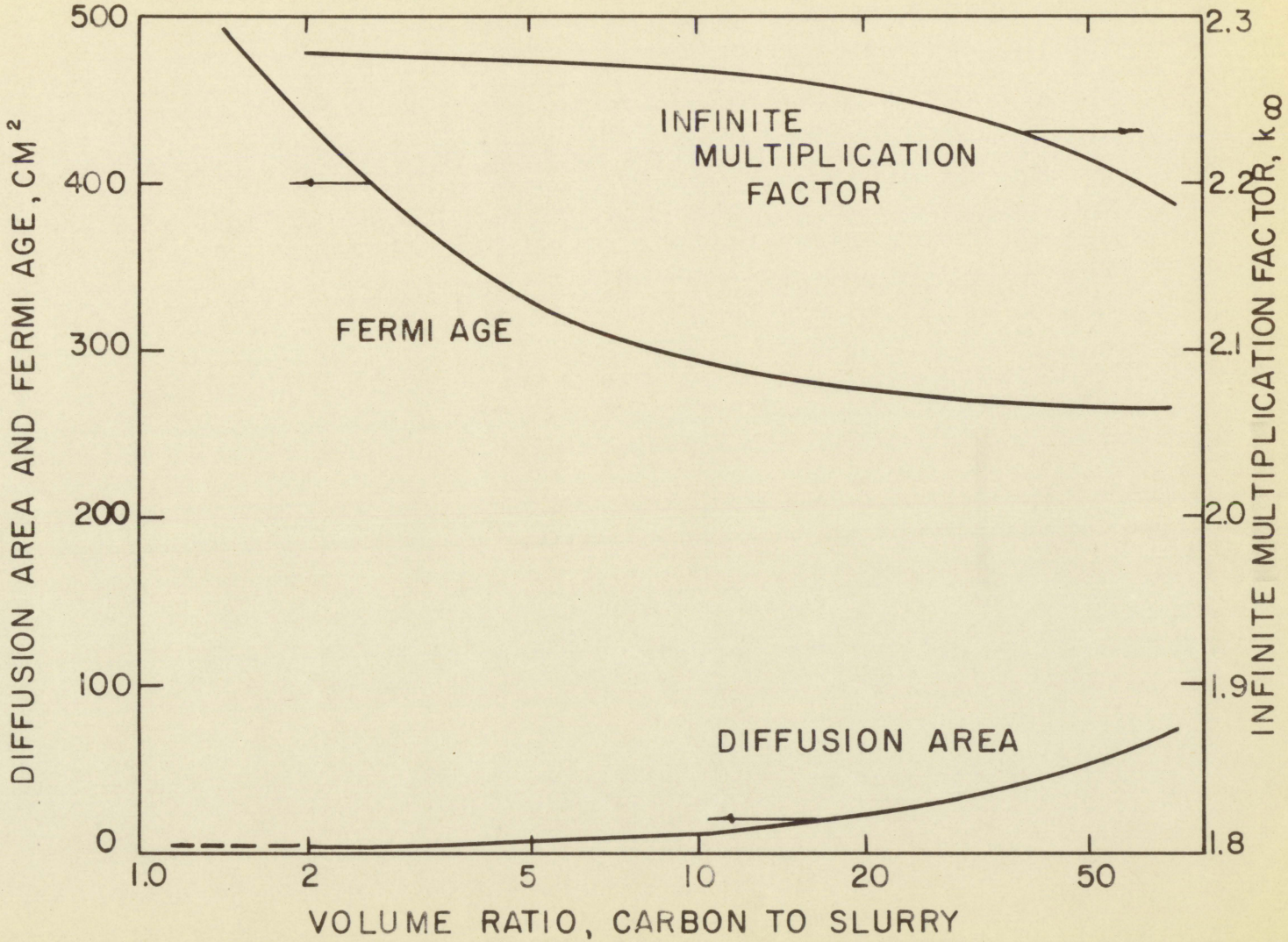

Figure 1. Nuclear properties of UO_2-NaK reactor

for the reflector.

Solution of these equations, and application of appropriate boundary conditions gives the one-group critical equation which, for a cylindrical reactor, is

$$D_R \kappa_R J_o(B_c R_o) \left[K_1(\kappa_R R_o) + \frac{K_o(\kappa_R R_1)}{I_o(\kappa_R R_1)} I_1(\kappa_R R_o) \right]$$

$$= D_c B_c J_1(B_c R_o) \left[K_o(\kappa_R R_o) - \frac{K_o(\kappa_R R_1)}{I_o(\kappa_R R_1)} I_o(\kappa_R R_o) \right] \quad (20)$$

where R_o = radius of the core,

R_1 = radius of the core plus thickness of the reflector plus the extrapolation distance,

κ_R = the reciprocal of the diffusion length of the reflector,

and B_c^2 is equal to $k_\infty - 1/M^2$, where M^2 is the migration area and is equal to the sum of the diffusion area and the Fermi Age.

For a blanket containing a 10 per cent by weight Th_3Bi_5 suspension in bismuth and having a carbon to slurry volume ratio of two,

$$D_R = 0.99 \text{ cm},$$

and $$\kappa_R = 0.067 \text{ cm}^{-1}.$$

For the blanket thicknesses of interest (greater than two feet), the thickness can be assumed to be essentially infinite, and Equation 20 reduces to

$$D_c B_c \frac{J_1(B_c R_o)}{J_o(B_c R_o)} = D_R \kappa_R \frac{K_1(\kappa_R R_o)}{K_o(\kappa_R R_o)}, \qquad (21)$$

which is solved by trial and error to obtain the critical radius (R_o) for a reactor with an infinite blanket. The results of this study are shown in Figures 4 and 5.

C. Investigation of Heat Transfer Requirements

The rate of heat removal in a circulating fuel reactor is given by

$$Q = m C_p \Delta T_c \qquad (22)$$

where m is the mass rate of flow of the slurry, and ΔT_c is the temperature difference of the slurry from the reactor inlet to the reactor outlet.

If the mass rate of flow is replaced by its equivalent, $v \rho A_s$, Equation 22 becomes

$$Q = v \rho A_s \Delta T_c \qquad (23)$$

or

$$A_s = \frac{Q}{v \rho C_p \Delta T_c} \qquad (24)$$

The cross-sectional area for slurry flow A_s is seen to be a function of the heat removal rate, the fluid velocity, and the temperature rise for a specific mixture. The relationship between the flow area and the core radius is found by the following development.

The core volume is the sum of the carbon volume and

slurry volume,

$$V_{core} = V_{carbon} + V_{slurry} \tag{25}$$

or

$$V_{core} = \left(\frac{V_c}{V_s} + 1\right) V_s \tag{25a}$$

which may be written as

$$\pi R^2 H = \left(\frac{V_c}{V_s} + 1\right) A_s H \tag{25b}$$

which reduces to

$$R^2 = \frac{A_s}{\pi}\left(\frac{V_c}{V_s} + 1\right) \tag{25c}$$

The volumetric heat capacity ρC_p is evaluated for the slurry with the following relation,

$$(\rho C_p)_{mix} = e_v(\rho C_p)_{UO_2} + (1 - e_v)(\rho C_p)_{NaK} \tag{26}$$

where e_v is the volume concentration of the solid material.

The specific heat of UO_2 was calculated from the following equation, obtained from Reference 8.

$$C_{p,UO_2} = 19.20 + 1.62 \times 10^{-3}T - 3.96 \times 10^{5}T^{-2} \tag{27}$$

The calculation resulted in a value of 0.0740 Btu/lb ^{o}F for the specific heat of UO_2.

Substitution of these values into Equation 26 gives

$$(\rho C_p)_{mix} = 40.2\ e_v + 10.23\ \text{Btu/ft}^3\ ^{o}\text{F}. \tag{28}$$

If the slurry flows at a velocity of 20 feet per second, and the heat removal rate is

$$Q = 3.412 \times 10^{6}\ P \quad \text{(Btu/hr)} \tag{29}$$

where P is the reactor power in megawatts, Equation 24 may be

written as

$$A_s = \frac{3.412 \times 10^6}{(20)(3600)(40.2\ e_v + 10.23)\ \Delta T_c} \qquad (30)$$

and Equation 25c becomes

$$R_c^2 = \frac{14.6\ P}{(40.2\ e_v + 10.23)\ \Delta T_c} \left(\frac{V_c}{V_s} + 1\right) . \qquad (31)$$

For a concentration of 10 per cent,

$$R_c^2 = \frac{1.057\ P}{\Delta T_c} \left(\frac{V_c}{V_s} + 1\right) , \qquad (32)$$

from which the core radius for adequate heat removal can be plotted versus the volume ratio of carbon to fuel slurry (V_c/V_s) for a specific power and specific temperature rise. The results are shown in Figure 3 for the bare reactor, and in Figure 5 for the reactor with breeder blanket.

V. DESCRIPTION OF THE REACTOR CYCLE

The UO_2-NaK reactor uses small particles of UO_2 suspended in NaK as the fuel slurry and graphite as the moderator. The flow diagram for the proposed reactor cycle is shown in Figure 2. The reactor cycle would include the reactor, composed of a core, a breeder blanket, and a containing vessel, a degasser to remove the gaseous fission products, piping from the reactor to the heat exchangers and back to the reactor, centrifugal pumps, and a bypass system for fuel processing. Additional heat exchangers and processing systems would be required for the blanket slurry, composed of Th_3Bi_5 particles suspended in bismuth. The fuel slurry would transfer its heat to a secondary coolant.

The following is a description of the proposed cycle and refers to the flow diagram in Figure 2. It was previously assumed that the core would be an upright, circular cylinder of graphite with a heterogeneous lattice of fuel tubes. The slurry leaves the top of the reactor and enters a degassing chamber which uses argon to remove the xenon and other gaseous fission products. The slurry then passes to the secondary heat exchangers where the heat is transferred to the secondary coolant. The cooled slurry is then pumped back to the bottom

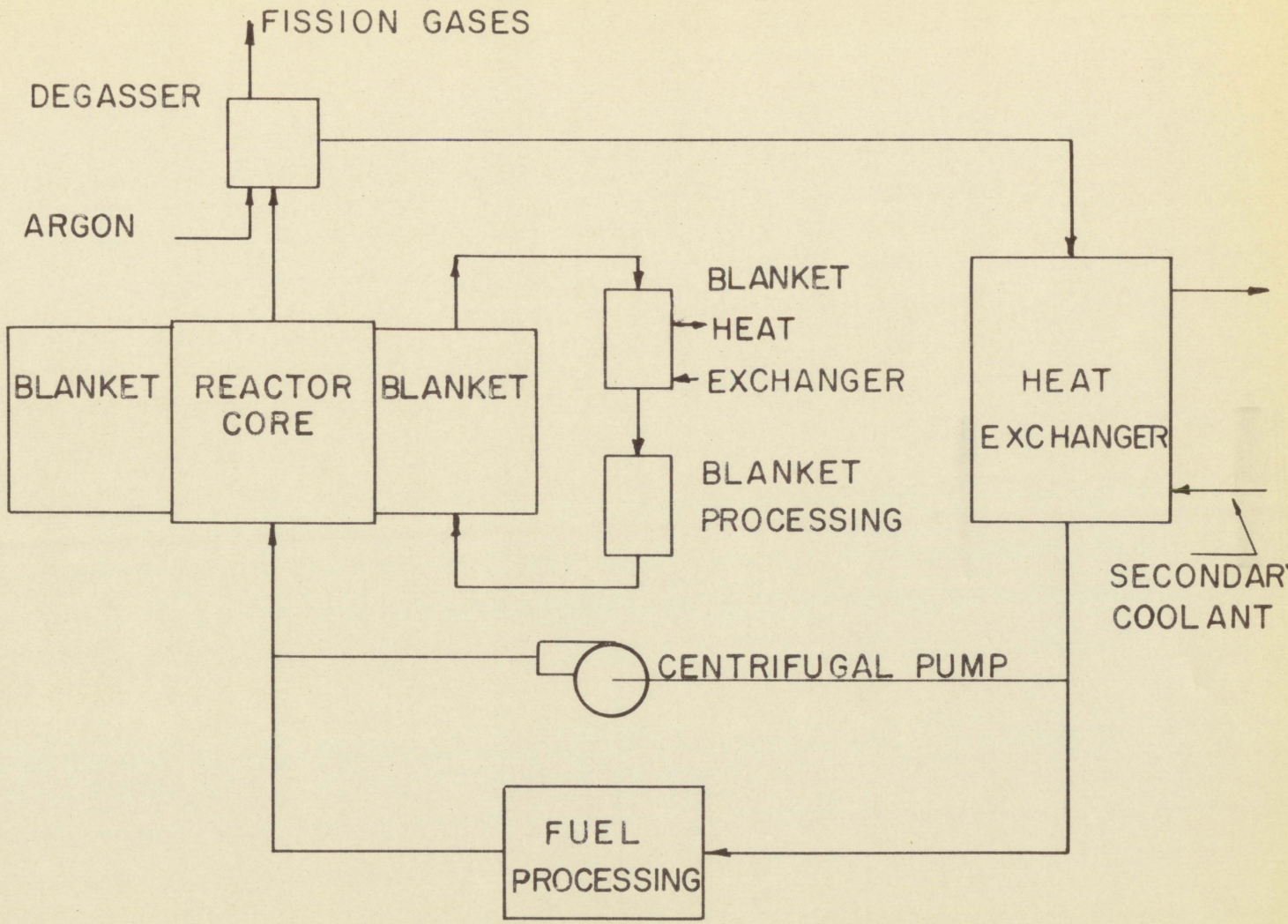

Figure 2. UO_2-NaK reactor cycle

header and through the core, in which the slurry is again heated by the fission process. Criticality in the external circuits is prevented by the absence of moderator and by divided flow.

The large density difference between the solid (11 gm/cm^3) and the fluid (0.77 gm/cm^3) presents the problem of settling in parts of the system where the velocity is reduced. The high velocity and the use of vertical flow would be sufficient to eliminate this problem in the core and heat exchangers, but settling could occur in the headers or the external piping system. This problem could be reduced considerably by the use of very small particles and suspensions in the colloidal range (sub-micron size) are suggested. The density of the Th_3Bi_5 breeding material (10.5 gm/cm^3) is almost matched by that of the bismuth (10.0 gm/cm^3) used as the suspending fluid, so the problem of settling would not be serious in the blanket system.

The pumping power required for a slurry is increased by an increase in concentration. Engineering studies of the UO_2-NaK slurry by ANL (5) indicated that the power requirement for a 10 per cent by volume suspension is approximately 80 per cent greater than that for the pure fluid. An estimate of the power required to pump a slurry through a bare reactor producing 500 megawatts resulted in a value of 200 kilowatts. This power requirement would be proportionately reduced in

the blanketed reactors operating at lower power levels.

A portion of the fuel slurry leaving the heat exchanger would be removed for reprocessing. It would be necessary to remove the solid fuel particles from the fluid. Since the density of the solid is considerably greater than that of the fluid, this could be done by allowing the particles to settle out in settling tanks. A series of filters could also be used if the settling rate is too low. The replacement of fuel and adjustment of concentration would be done prior to the entrance of the slurry into the core.

The blanket system would be a secondary source of heat, since there would be a certain amount of fission of the U^{233} atoms produced from the thorium, and also some absorption of gamma-radiation from the core. The relative amount of power obtainable from the blanket is a function of the U^{233} concentration in the blanket slurry which in turn is a function of the processing rate of the blanket slurry. It is conceivable that an additional 10 per cent of the reactor power could be produced in the blanket slurry. This heat could be used in a reheater or feedwater heater for the steam power cycle.

For an economical reactor, it would be necessary to use cheap and abundant materials. The materials used in the construction of the reactor depend considerably on the results

of tests of the handling characteristics of the UO_2-NaK slurry. The use of graphite or beryllium in contact with the slurry as suggested for other designs (3, 4) is unrealistic because of the unknown erosive properties of the slurry. The fuel channels in the core and the reactor vessel would be fabricated from one of the stainless steels. This would increase the absorption cross-section of the core.

The fuel (U^{233}) can be obtained from a reactor that burns U^{235} and produces U^{233}. The conversion of the uranium metal to the dioxide would not involve a very great additional expense. The NaK used as the suspending fluid is available in sufficient quantities so that the price would not be prohibitive. The cost per pound of NaK is about the same as bismuth (6).

The procedure for starting the reactor would be the circulation of pure NaK through the core and addition of the UO_2 particles until the desired concentration is reached. Reactor shutdown could be accomplished by a reduction of the concentration by filtering out the solid fuel particles. Emergency shutdown in case of a power failure could be provided for with allowance for particle settling into the inlet header or the addition of a series of tanks into which the slurry could be dumped. Another possibility is purging of the core with an inert gas.

Control of the reactor would be provided for with control rods. The effect of delayed neutrons is unknown since the probability of a delayed neutron being released inside the core was not estimated.

VI. DISCUSSION AND CONCLUSIONS

The survey of liquid-metal fuel reactors presented in Section II indicates that this type of reactor has not received notable attention until the past few years. Brookhaven National Laboratory is the principal contributor to the knowledge of liquid-metal fuels and the groups at BNL have concentrated their efforts on systems using bismuth, lead, and tin. The only type of reactor to reach an advanced design stage is the externally-cooled LMFR designed by BNL which uses a solution of uranium in bismuth as the fuel and a suspension of Th_3Bi_5 in bismuth as the breeder material. Another type which has reached a conceptual design stage is the internally-cooled LMFR, also being investigated by BNL.

The number of possible suspensions of intermetallic compounds in liquid metals (see Table 1) illustrates many of the other possibilities for types of nuclear reactor fuels and the ionic-compound dispersions present another possibility. The UO_2-NaK slurry investigated in this thesis represents only one of the many possible types of liquid-metal slurry fuels.

The limited information on the characteristics of liquid-metal slurries indicates a need for further

investigation of the mechanical, chemical, and nuclear characteristics of suspensions. It is evident from the discussion in Section III that no definite predictions can be made about the mechanical stability, heat transfer characteristics, and handling characteristics of liquid-metal slurries. The most significant advances have been made in the region of fuel processing of liquid-metal fuels, but the question of processing liquid-metal slurries is still open for investigation. In evaluating the characteristics of liquid-metal slurries it is then only possible to make general statements about the advantages of one system over another.

The UO_2-NaK system does not satisfy some of the general rules used in the evaluation of a nuclear reactor fuel slurry, one example being the use of a fluid with a fairly high neutron absorption cross-section which would affect the neutron conservation desirable in breeder reactors. Another example is the use of a solid-fluid combination in which the density of the solid is considerably greater than the density of the fluid, which would result in settling out of the particles if the reactor were shut down, or if the velocity were reduced in some part of the system. However, the effect of the high concentration of fuel is to reduce the quantity of NaK necessary in the reactor, and the use of very small

particles should reduce the tendency for the particles to settle out.

A preliminary criticality analysis was made using the Fermi-Age approximation for a bare reactor. The results are given in the form of critical radius and mass corresponding to a given volume ratio of carbon to slurry and are shown in Figure 3. The core radius required to remove heat at the rate of 500 megawatts is also shown in Figure 3 and the point at which the curve for critical radius crosses the curve for adequate heat removal is the point at which the reactor would be operating. For a power of 500 megawatts and a temperature rise of 300 F, this point occurs at a core radius of 2.26 feet, a volume ratio of 1.9, and a critical mass of 635 kilograms. It is useful to compare the UO_2-NaK bare reactor with the LMFR to draw some preliminary conclusions about the system. Table 2 gives corresponding values for the important operating characteristics of each reactor. The following general conclusions can be drawn from the results of the Fermi-Age approximation.

1. A higher potential breeding ratio results from an increase in fuel concentration. Therefore the fuel concentration should be kept as high as possible if breeding is to be employed. For the same reason, the volume ratio of carbon to slurry should be kept as low as possible.

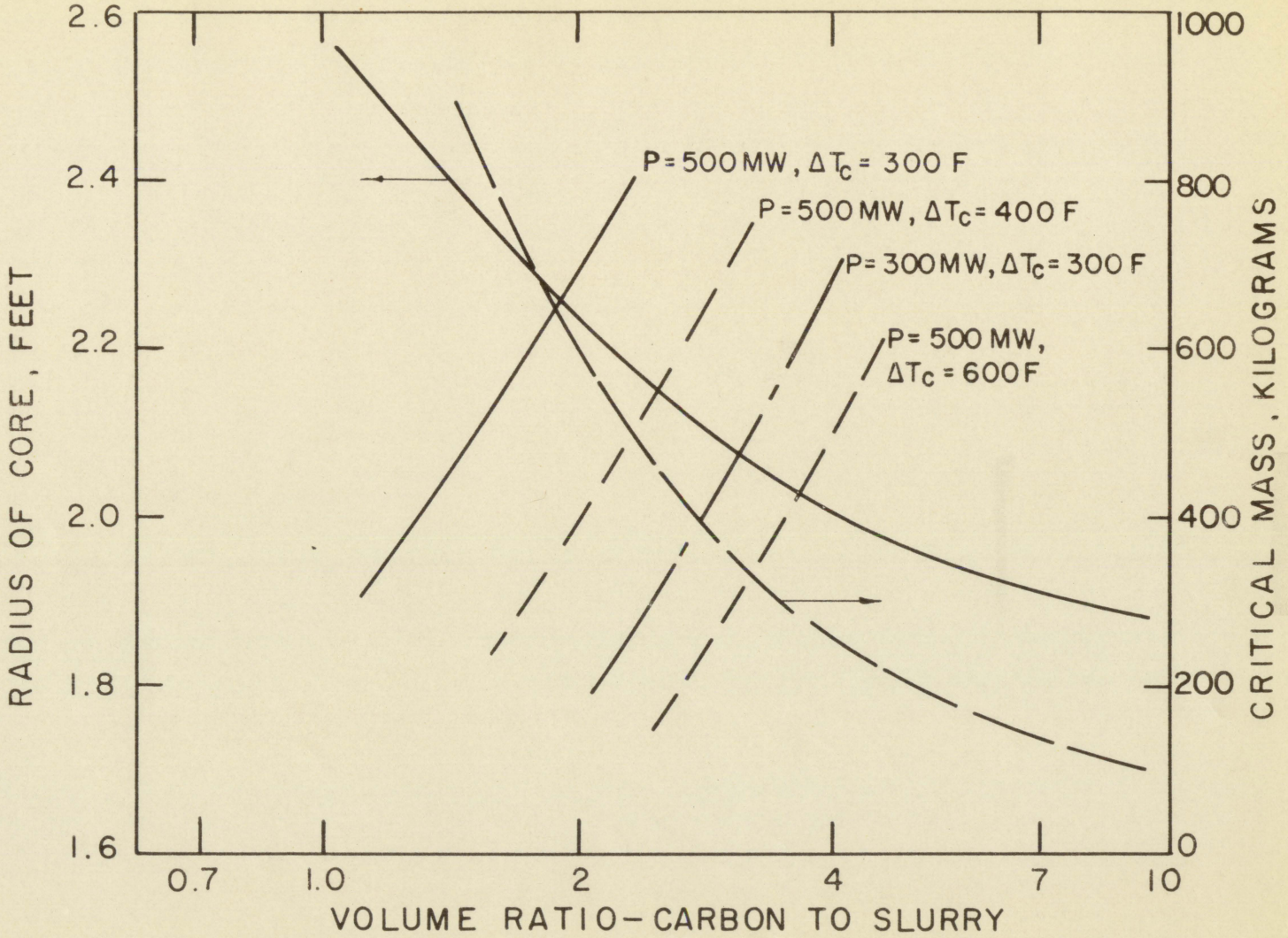

Figure 3. Determination of critical radius of bare reactor

Table 2. Comparison of LMFR and UO_2-NaK bare reactor

	LMFR	UO_2-NaK
Reactor Power (megawatts)	500	500
Radius (ft)	2.5	2.26
Height (ft)	5.0	4.17
Temperature Rise of Fuel (^oF)	270	300
Critical Mass (kg)	10.6	635
Average Thermal Neutron Flux (n/cm^2 sec)	1.7×10^{15}	2.88×10^{13}
Core Holdup (ft^3)	57	23.1
Graphite to Fuel Volume Ratio	0.820	1.9
Flow Area (ft^2)	11.2	5.48
Number of 2-inch Dia. Tubes	511	251
Fuel Concentration	669 ppm	10% by volume
Power Density (kw/liter)	310	765
Specific Power (kw/kg)	47,100	788
Mass Rate of Flow of Fuel (lb/hr)	1.76×10^8	2.89×10^7
Potential Breeding Ratio	0.964	1.166

2. An increase in the volume ratio of carbon to slurry results in a decrease in critical radius and mass, but also results in a decrease in the quantity of heat that can be removed from the core.

3. The UO_2-NaK bare reactor corresponding to a power level of 500 megawatts has approximately the same dimensions

as the LMFR, but has a power density which is twice that of the LMFR. This is to be expected, since the volume of UO_2-NaK slurry required is about half the volume of the U-Bi solution required for the same power level.

4. The significant difference in the two reactors is the critical mass. The critical mass for the bare, UO_2-NaK reactor is about 60 times the critical mass of the reflected U-Bi reactor. This is due, for the most part, to the greater amount of neutron leakage from the bare reactor.

5. The potential breeding ratio of the UO_2-NaK reactor is significantly greater than one, which suggests the use of a breeder blanket surrounding the core. The basis for the breeding gain was a blanket having an efficiency of 0.9.

The conclusions drawn from the preliminary study indicate that it would be desirable to place a breeder blanket about the core to utilize the neutrons leaking out of the core. The selection of appropriate blanket materials is important since the breeding ratio attainable is dependent upon the blanket efficiency. Since the average number of neutrons produced from fission for every neutron absorbed in the fuel is 2.31 and one neutron is required to maintain the chain reaction, the maximum number of neutrons available for breeding is 1.31. Of these, a portion will be absorbed by poisons in the core, leaving about 1.275 neutrons available for breeding. If a breeding gain is to be realized, then the blanket

efficiency cannot be lower than $\frac{1.000}{1.275} = 0.785$, and in fact must be larger. Calculation of the efficiency of a blanket using a 10 per cent by volume ThO_2 suspension in NaK for different volume ratios of carbon to slurry gave a value of about 0.5, which is considerably lower than that desired. Therefore, it was decided that a 10 per cent by weight dispersion of Th_3Bi_5 in bismuth would be used. The efficiency of a blanket having a volume ratio of carbon to slurry equal to two is 0.893.

The calculations were made using a one-group diffusion theory for a reactor with a reflector, and the results are given in the form of critical radius and mass as a function of the volume ratio of carbon to slurry. The results were plotted on Figure 4, for comparison with the results of the bare reactor, and on Figure 5, for comparison with the heat transfer requirements for power levels of 100 and 200 megawatts. The conditions for each power level were taken from Figure 5, calculations were made, and the important characteristics for each reactor listed in Table 3. The following conclusions were drawn from the results of the one-group approximation for a reflected reactor applied to a reactor using the UO_2-NaK system.

1. The critical radius and mass are considerably reduced when the neutron leakage is reduced by the addition of a breeder blanket.

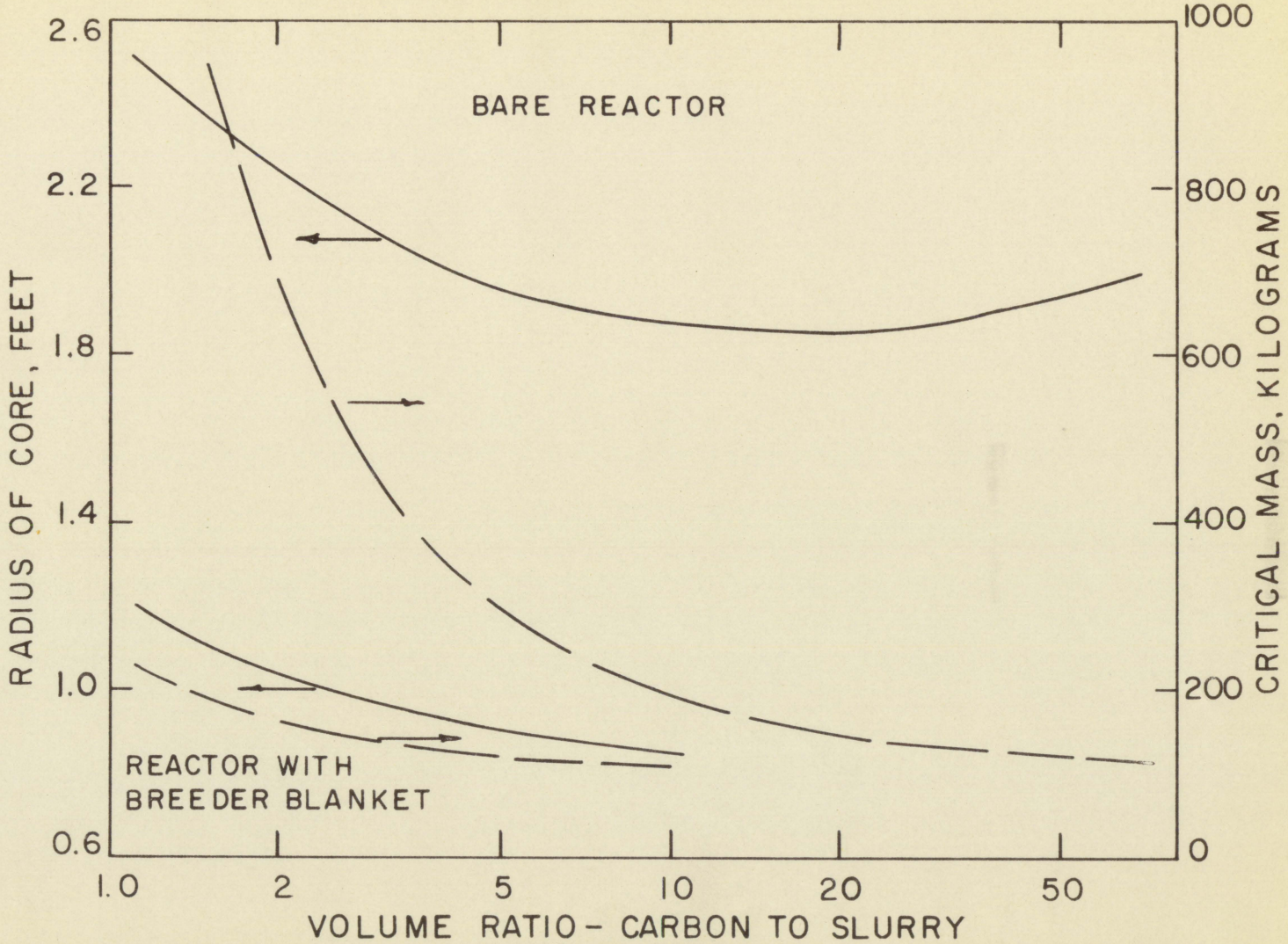

Figure 4. Effect of breeder blanket on critical radius and mass

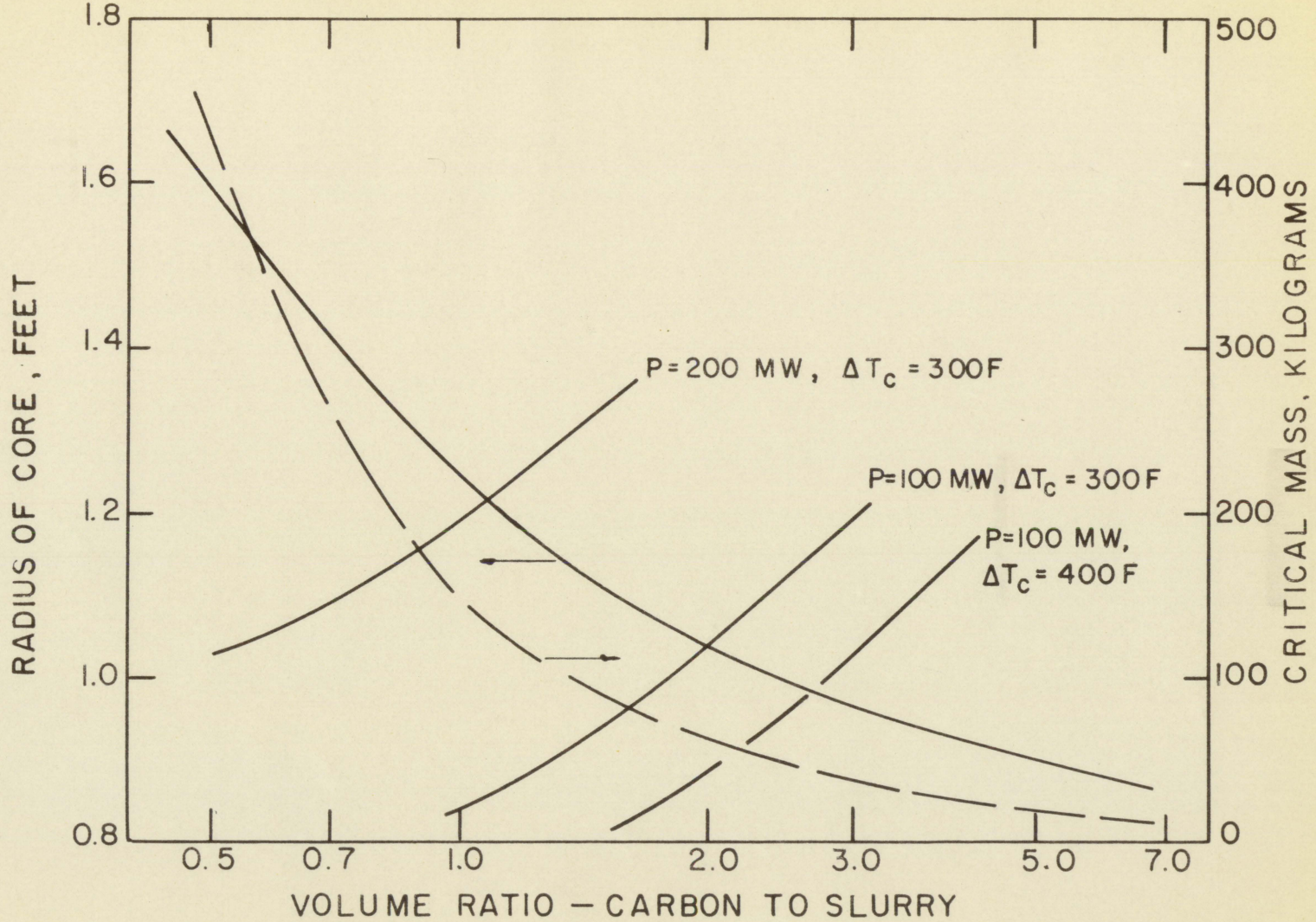

Figure 5. Determination of critical radius of breeder reactor

Table 3. Comparison of two possible breeder reactors using the UO_2-NaK system

	I	II
Reactor Power (megawatts)	100	200
Volume Ratio of Carbon to Slurry	2.0	1.0
Core Radius (ft)	1.03	1.24
Core Height (ft)	1.903	2.29
Critical Mass of U^{233} (kg)	60	150
Average Thermal Flux in Core (n/cm^2sec)	6.08×10^{13}	4.86×10^{13}
Flux at Core-Blanket Interface (n/cm^2sec)	5.67×10^{13}	4.34×10^{13}
Leakage Flux (n/cm^2sec)	1.537×10^{10}	1.57×10^{10}
Volume of Slurry in Core (ft^3)	2.11	5.53
Specific Power (kw/kg)	1667	1333
Power Density (kw/liter)	1675	1278
Breeding Ratio	1.1985	1.2000

2. The amount of heat that can be removed from the reflected reactor is considerably less than that removable from the bare reactor, but the amount of heat produced per kilogram of fuel (specific power) is greater for the reflected reactor. The increase in the amount of heat produced per unit volume of fluid (power density) is greater for lower power levels because of the decreased height of the reactor.

3. The breeding ratios obtainable from each reactor are almost identical, indicating that there is no difference in this respect.

No attempt was made to optimize any of the variables of the design (except breeding ratio) since the exact purpose of the reactor is not known. The results are given in a form such that the operating conditions chosen (power level and temperature rise) may be found and the reactor size and critical mass determined. The possible designs presented here are based on a maximum possible breeding ratio. If it is desirable to increase the power level or increase the specific power, the concentration of the uranium dioxide in the slurry could be reduced. The effect of the decrease in concentration on the breeding ratio would determine how much the concentration could be reduced.

The advantages of the use of the UO_2-NaK system as a liquid-metal fuel are as follows:

1. The use of a slurry permits a higher concentration of fissionable material in the core than that obtained with a solution of uranium in the liquid metals. This is desirable because of the corresponding increase in the breeding ratio.

2. The use of a liquid metal (NaK) with a low density considerably reduces the pumping power required compared to the heavier liquid metals (Bi, Pb, Sn) proposed by the groups at BNL.

3. The holdup of fuel slurry in the reactor cycle is reduced by the use of a fluid with a higher heat capacity compared to the heat capacity of the heavier metals proposed by the groups at BNL.

4. The breeding ratio possible with the UO_2-NaK system is considerably higher than that of the LMFR proposed by the groups at BNL.

The disadvantages of the use of the UO_2-NaK system as a liquid-metal fuel are as follows:

1. The use of a fluid (NaK) with a high neutron absorption cross-section affects the neutron conservation desirable in a breeder reactor.

2. The critical mass of a UO_2-NaK reactor using a 10 per cent suspension of UO_2 is higher than that of comparable reactors. The mass of fissionable material required is considerably increased by the holdups in the external piping, heat exchangers, and headers.

3. The power level of UO_2-NaK reactors is considerably reduced when breeding is employed because of the smaller amount of slurry needed for criticality.

4. The choice of a fused-salt compatible with the NaK in a fused-salt extraction process for removal of fission products is difficult because of the high reactivity of the potassium.

The following general conclusions can be drawn from the investigation of the UO_2-NaK system as a reactor fuel.

1. The mechanical, chemical, and nuclear characteristics of UO_2-NaK slurries have received limited investigation but the problems are relative to liquid-metal slurries in general, and their investigation does not appear to present any major difficulties. The chemical processing of the UO_2-NaK system presents a major problem.

2. A reactor using this system can be made critical, can operate at a power level commensurate to the requirements of a central-station power reactor, and would have a breeding ratio greater than one, or a breeding gain, which satisfies the requirement for a combination power-breeder reactor.

3. If the primary purpose of the reactor is breeding, reactors operating in the 100-200 megawatt range with low volume ratios of carbon to slurry would be used.

4. If the primary purpose of the reactor is power, the concentration could be lowered and the core enlarged to provide for the larger flow rate of the slurry. Another possibility is the use of U^{235} (enriched) as the fuel.

VII. RECOMMENDATIONS FOR FURTHER STUDY

The use of the UO_2-NaK system in a nuclear reactor depends upon the stability of UO_2 suspended in NaK and circulated through a reactor. It is therefore suggested that studies be made of the mechanical and nuclear properties of the system. This would involve mechanical stability tests, head-loss tests, and in-pile tests of the resistance of the suspension to nuclear radiations. A method of processing the UO_2-NaK system to remove fission products is required. The possibility of reducing the holdup of slurry in the external heat-transfer system should be investigated.

Further study of the criticality of the UO_2-NaK reactor should point in the following directions:

1. A reactor with a smaller critical mass. This could be done by reducing the concentration of UO_2 in NaK. This in turn would increase the size of the reactor and allow higher power levels, but may decrease the breeding ratio. The purpose of this study might be to find the minimum concentration which would allow a breeding gain. The approach would essentially be the same as that of this thesis.

2. More accurate results for the proposed reactor. This would involve application of the two-group theory, or if

possible, a multi-group theory.

3. A reactor using enriched U^{235} as the fuel. Preliminary calculations for this system indicated that a larger core would be required if U^{235} enriched to 90 per cent were used, but with a core containing U^{238}, the factor of internal breeding might prove to be advantageous.

4. The possibility of a fast reactor using the PuO_2-NaK system.

5. The use of sodium in place of NaK as the suspending fluid. Two advantages of sodium are a lower neutron absorption cross-section and the better possibility of finding a fused salt which could be used for processing a sodium system. One disadvantage of sodium is the necessity for a heating system to melt the metal when the reactor is started.

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IX. ACKNOLWEDGMENT

The author wishes to thank Dr. Glenn Murphy for suggesting the possibility of a slurry reactor and for his helpful suggestions during this investigation. Appreciation is also expressed to the members of the Engineering Development Group for their suggestions.

X. APPENDIX

A. Nomenclature

1. Physical

A	- area	ft^2, cm^2
C_p	- specific heat	Btu/lb ^oF
e_v	- concentration by volume	--
H	- height	ft, cm
m	- mass rate of flow	lb/hr
P	- power	megawatts
Q	- total heat rate	Btu/hr
R	- radius	ft, cm
ρ	- density	pcf
T	- temperature	^oF, ^oC
v	- velocity	fps
V	- volume	ft^3, cm^3

2. Nuclear

A	- atomic mass	gm/mole
B^2	- buckling	cm^{-2}
C	- constant	neutrons/cm^2 sec
D	- diffusion coefficient	cm
E	- neutron energy	ev, Mev
ϵ	- fast fission factor	--
η	- average number of neutrons produced per absorption of one thermal neutron in the fuel	--

f	- thermal utilization	--
F	- constant	neutrons/cm^2 sec
k	- multiplication factor	--
κ	- reciprocal diffusion length	cm^{-1}
L	- diffusion length	cm
m	- mass	kilograms
M	- migration length	cm
$\bar{\mu}_o$	- average cosine of the scattering angle per collision	--
n	- atomic density of pure material	atoms/cm^3
N_i	- atomic density of component i in reactor	atoms/cm^3
ν	- average number of neutrons produced per fission	--
p	- resonance escape probability	--
ϕ	- neutron flux	neutrons/cm^2 sec
r	- radial coordinate	--
σ	- microscopic cross-section	barns, cm^2
Σ	- macroscopic cross-section	cm^{-1}
τ	- Fermi Age	cm^2
ξ	- average logarithmic energy decrement per collision	--
z	- longitudinal coordinate	--

3. Subscripts

a - for absorption
bl - for blanket
c - for carbon
f - for fission
g - for geometric
l - for leakage
mix - for mixture
o - for oxygen
R - for reflector
s - for scattering
t - for total
th - for thermal
U23 - for uranium-233

B. Nuclear Properties

In the evaluation of the nuclear properties of the reactor, the core was assumed to be at an average temperature of 850 F (455 C), which corresponds to a neutron energy of 0.0627 ev. Table 4 gives the values of the thermal neutron cross-sections and Table 5 gives the physical properties of the core materials used in the evaluation of the nuclear properties. The values of the absorption cross-sections given at energies other than 0.0627 ev were corrected assuming

Table 4. Nuclear properties of elements at 850 F (455 C)[a]

Element	σ_f (barns)	σ_a (barns)	σ_s (barns)	ν	η
Pu^{239}	530[b]	750[b]	9.6	2.88	2.03
U^{233}	330	362	8.2	2.54	2.31
U^{235}	335	420	10.0	2.51	2.12
Th^{232}		7.5	12.5		
C		0.0045[b]	4.8		
Na		0.54[b]	4.0		
K		1.97[b]	1.5		
Bi		0.032[b]	9.0		
O		--	3.8		

[a]Values taken from Reference 9.

[b]Values listed at 0.025 ev

Table 5. Physical properties of materials at 850 F (455 C)

Material	A	ξ	ρ (gm/cm^3)	C_p (Btu/lb °F)
Carbon	12	0.158	1.62	--
NaK	33.8	0.058	0.77	0.21
Oxygen	16	0.120	--	--
$UO_2(U^{233})$	265	--	10.97	0.074

that σ_a varies as the reciprocal of the neutron velocity. It should also be noted that the absorption cross-sections used in the evaluation of the diffusion length must be averaged over the Maxwellian distribution of thermal neutrons. The average thermal neutron velocity is 1.128 times greater than the most probable velocity, and therefore, the effective thermal absorption cross-sections are 0.866-times the cross sections corresponding to the thermal neutron energy.

1. Infinite multiplication factor

The infinite multiplication factor is given by the four-factor formula

$$k_\infty = \eta f \epsilon p \qquad (33)$$

and for the system under consideration, the fast fission factor and resonance escape probability were both assumed to be equal to one.

The average number of fast neutron produced per thermal neutron absorbed in U^{233} is equal to 2.31, and the thermal utilization is given by

$$f = \frac{\Sigma_{a,\ fuel}}{\Sigma_{a,\ core}} = \frac{N_{U23}\ \sigma_{a,\ U23}}{N_{U23}\ \sigma_{a,U23} + N_c\ \sigma_{a,c} + N_{NaK}\sigma_{a,NaK}} \qquad (34)$$

therefore,

$$k_\infty = \eta f = \frac{2.31\ \ \sigma_{a,\ U23}}{\sigma_{a,U23} + \frac{N_c}{N_{U23}}\sigma_{a,c} + \frac{N_{NaK}}{N_{U23}}\sigma_{a,NaK}} \qquad (35)$$

where N represents the atomic densities of the components, expressed in atoms per cubic centimeter of reactor. Equation 35 can be rewritten as

$$\eta f = \frac{2.31\ \sigma_{a,U23}}{\sigma_{a,U23} + \frac{V_{NaK}}{V_{UO_2}} \frac{\Sigma_{a,NaK}}{n_{UO_2}} + \frac{V_c}{V_{UO_2}} \frac{\Sigma_{a,c}}{n_{UO_2}}} \qquad (36)$$

since

$$\frac{N_c}{N_{U23}} \sigma_{a,c} = \frac{V_c}{V_{UO_2}} \frac{n_c}{n_{UO_2}} \sigma_{a,c} = \frac{V_c}{V_{UO_2}} \frac{\Sigma_{a,c}}{n_{UO_2}} \qquad (37)$$

and similarly

$$\frac{N_{NaK}}{N_{U23}} = \frac{V_{NaK}}{V_{UO_2}} \frac{\Sigma_{a,NaK}}{n_{UO_2}} \qquad (37a)$$

where Σ_a is the macroscopic cross-section of the individual components and n_{uo_2} is the molecular density of uranium dioxide. Substitution of the appropriate values of cross-sections and n_{UO_2} into Equation 36 gives

$$\eta f = \frac{(2.31)(362)}{362 + \frac{.01323}{.0245} \frac{V_{NaK}}{V_{UO_2}} + \frac{.000524}{.0245} \frac{V_c}{V_{UO_2}}} \qquad (38)$$

or

$$k_\infty = \frac{836}{362 + 0.54 \frac{V_{NaK}}{V_{UO_2}} + .0214 \frac{V_c}{V_{UO_2}}} \qquad (39)$$

2. Thermal diffusion length

The thermal diffusion length is found from the relation

$$L^2 = \frac{D}{\Sigma_a} \tag{40}$$

The average absorption cross-section of the core is given by

$$\Sigma_{a,\ core} = N_{U23}\ \sigma_{a,U23} + N_c\ \sigma_{a,c} + N_{NaK}\ \sigma_{a,NaK} \tag{41}$$

which may be rewritten as

$$\frac{\Sigma_{a,core}}{N_{UO_2}} = \frac{N_{U23}}{N_{UO_2}}\ \sigma_{a,U23} + \frac{N_c}{N_{UO_2}}\ \sigma_{a,c} + \frac{N_{NaK}}{N_{UO_2}}\ \sigma_{a,NaK} \tag{42}$$

or

$$\frac{\Sigma_{a,core}}{N_{UO_2}} = \sigma_{a,U23} + \frac{V_{NaK}}{V_{UO_2}}\ \frac{\Sigma_{a,NaK}}{\eta_{UO_2}} + \frac{V_c}{V_{UO_2}}\ \frac{\Sigma_{a,c}}{\eta_{UO_2}} \tag{43}$$

using Equations 37 and 37a. Since

$$N_{UO_2} = \eta_{UO_2}\ \frac{V_{UO_2}}{V_{core}} = \frac{\eta_{UO_2}}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \tag{44}$$

Equation 43 becomes

$$\Sigma_{a,core} = \frac{\eta_{UO_2}\ \sigma_{a,U23} + \frac{V_{NaK}}{V_{UO_2}}\ \Sigma_{a,NaK} + \frac{V_c}{V_{UO_2}}\ \Sigma_{a,c}}{1 + \frac{V_c}{V_{UO_2}} + \frac{V_{NaK}}{V_{UO_2}}} \tag{45}$$

Substitution of the appropriate values into Equation 45 gives

$$\Sigma_{a,core} = \frac{8.87 + .01323 \frac{V_{NaK}}{V_{UO_2}} + .000524 \frac{V_c}{V_{UO_2}}}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \qquad (46)$$

The diffusion coefficient corrected by transport theory is given by

$$D = \frac{1}{3\Sigma\ (1 - \bar{\mu}_o)\left(1 - \frac{4}{5}\frac{\Sigma_a}{\Sigma} + \frac{\Sigma_a}{\Sigma}\frac{\bar{\mu}_o}{1-\bar{\mu}_o} - \cdots\right)} \qquad (47)$$

where Σ is the total cross-section.

For the system considered this becomes

$$D_{core} = \frac{1}{2.84 N_c \sigma_{t,c} + 2.025 N_{NaK}\sigma_{t,NaK} + .651 N_{U23}\sigma_{t,U23} + 2.87 N_o \sigma_{t,o}} \qquad (48)$$

or

$$D_{core} \cdot N_{UO_2} = \frac{1}{2.84 \frac{V_c}{V_{UO_2}} \frac{\Sigma_{t,c}}{\eta_{UO_2}} + 2.025 \frac{V_{NaK}}{V_{UO_2}} \frac{\Sigma_{t,NaK}}{\eta_{UO_2}} + .651\,\sigma_{t,U23} + 2.87(2)\sigma_{t,o}} \qquad (49)$$

Substitution of Equation 44 and appropriate constants into Equation 49 gives

$$D_{core} = \frac{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}}{1.093 \frac{V_c}{V_{UO_2}} + .0909 \frac{V_{NaK}}{V_{UO_2}} + 5.805} \qquad (50)$$

3. Fermi Age

The Fermi Age is given by the relation

$$\tau(E) = \int_{E_{th}}^{E_f} \frac{D}{\xi \Sigma_s} \frac{dE}{E} . \qquad (51)$$

If it is assumed that the factors in Equation 51 are independent of energy and all neutrons originate at an average fission energy E_f, then Equation 51 can be integrated to give

$$\tau = \frac{D}{\overline{\xi\Sigma_s}} \ln \frac{E_f}{E_{th}} \tag{52}$$

If the average energy of fission neutrons is 2 Mev, and the thermal energy corresponding to the average temperature of the core is 0.0627 ev, then

$$\tau = \frac{D_{core}}{\overline{\xi\Sigma_s}} \ln \left(\frac{2 \times 10^6}{.0627}\right) = 17.26 \frac{D_{core}}{\overline{\xi\Sigma_s}} \tag{53}$$

where D_{core} is given by Equation 50 and

$$\overline{\xi\Sigma_s} = (\xi N\sigma_s)_c + (\xi N\sigma_s)_{NaK} + (\xi N\sigma_s)_{U23} + (\xi N\sigma_s)_o \tag{54}$$

Since ξ_{U23} is essentially zero,

$$\overline{\xi\Sigma_s} = \frac{(\xi\Sigma_s)_{NaK}\frac{V_{NaK}}{V_{UO_2}} + (\xi\Sigma_s)_c\frac{V_c}{V_{UO_2}} + \xi_o \eta_{UO_2}(2)\sigma_{s,o}}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \tag{55}$$

or

$$\overline{\xi\Sigma_s} = \frac{.00174\frac{V_{NaK}}{V_{UO_2}} + .0609\frac{V_c}{V_{UO_2}} + .0224}{1 + \frac{V_{NaK}}{V_{UO_2}} + \frac{V_c}{V_{UO_2}}} \tag{56}$$

The Fermi Age may then be calculated by the use of Equations 50, 53, and 56.

C. Other Calculations

The critical mass of fissionable material in the core is

$$M_{U23} = \frac{A_{U23}}{A_{UO_2}} \rho_{UO_2} V_{UO_2} \tag{57}$$

where $V_{UO_2} = e_v V_{slurry}$ (58)

$= e_v A_s H = e_v A_s (1.847 R)$ (59)

and $A_s = \frac{\pi R_c^2}{1 + \frac{V_c}{V_s}}$ (60)

Therefore,

$$m_{U23} = \frac{A_{U23}}{A_{UO_2}} \rho_{UO_2} e_v 1.847 \frac{\pi R_c^3}{1 + \frac{V_c}{V_s}} \tag{61}$$

and at a concentration of 10 per cent by volume UO_2,

$$m_{U23} = \frac{158.5 R_c^3}{1 + \frac{V_c}{V_s}} \quad \text{(kg)} \tag{62}$$

where R_c and V_c/V_s are determined from the critical equation.

The potential breeding ratio may be approximated by the following relation, given in Reference 10.

$$B. R. = \eta \left(\frac{k_\infty - 1}{k_\infty} \right) f_{Bl} \tag{63}$$

where f_{Bl} is the blanket efficiency, or the atoms of fissionable material produced (net) per neutron leaking from the core.

The breeding ratio for a particular reactor may be

calculated by assuming that the neutrons available for breeding per neutron absorbed in the fuel is equal to the number of neutrons produced minus the number of neutrons used in maintaining the chain reaction, absorbed in waste material, or leaked out of the reactor. Thus

$$B.\ R. = \eta - 1 - a - b - L \qquad (64)$$

where η = average number of neutrons produced,

a = neutrons absorbed in waste material in the core,

b = neutrons absorbed in waste material in the blanket,

and L = neutrons leaked out of the reactor,

where each term is based on one neutron absorbed in the fuel material.

Equation 64 may be written in terms of known quantities in the reactor as

$$B.\ R. = \eta - 1 - \left(\frac{1}{f_c} - 1\right) - (1 - f_{Bl})\frac{\phi_i}{f_c(\phi_{av})_c} - \frac{\phi_l}{f_c(\phi_{av})_c} \qquad (65)$$

which reduces to

$$B.\ R. = \eta - \frac{1}{f_c} - \frac{(1 - f_{Bl})\phi_i + \phi_l}{f_c(\phi_{av})_c} \qquad (66)$$

where f_c = thermal utilization of the core,

f_{Bl} = thermal efficiency of the blanket,

ϕ_i = thermal neutron flux at the core-blanket interface,

ϕ_1 = thermal neutron leakage at the edge of the blanket,

and $(\phi_{av})_c$ = average thermal neutron flux in the core.

The efficiency of a breeder blanket is analagous to the thermal utilization of a reactor core and is found from

$$f_{Bl} = \frac{\Sigma_{a,\ \text{breeding material}}}{\Sigma_{a,\ \text{total blanket}}} \qquad (67)$$

For comparison, calculations were made for a 10 per cent by volume suspension of ThO_2 in NaK, and for a 10 per cent by weight suspension of Th_3Bi_5 in bismuth, which gave the following results.

Table 6. Effect of slurry materials on blanket efficiency

$\left(\frac{V_c}{V_s}\right)_{Bl}$	f_{Bl} (ThO_2-NaK)	f_{Bl} (Th_3Bi_5-Bi)
1.0	0.52	0.917
2.0	0.506	0.893
4.0	0.489	0.817

The thermal neutron flux distribution determined from the one-group approximation for a two-region reactor is

$$\phi_c\ (r,z) = C\ J_o(B_c r)\ \cos\frac{\pi z}{H} \qquad (68)$$

for the core, and

$$\phi_R(r,z) = F\left[K_o(\kappa_R r) - \frac{K_o(\kappa_R R_1)}{I_o(\kappa_R R_1)} I_o(\kappa_R r)\right] \cos\frac{\pi Z}{H} \tag{69}$$

for the reflector, where R_1 is the sum of the core radius, the blanket thickness, and the extrapolation distance. The constants C and F are determined for the particular reactor in question. The average flux is connected with the power level of the reactor by

$$P = 8.3 \times 10^{10}\ g\ \sigma_f\ \phi_{av} \quad \text{(watts)} \tag{70}$$

where g = grams of fissionable material in the reactor,

σ_f = microscopic fission cross-section, in cm^2/nucleus

and C is related to the average flux in the core by the equation

$$\phi_{av} = \frac{1}{V_c}\int_0^{R_o}\int_{-\frac{H}{2}}^{+\frac{H}{2}} \phi_c(r,z)\ 2\pi r\ dr\ dz \tag{71}$$

and Equation 68.

Equation 68 is combined with Equation 71 to give

$$(\phi_{av})_{core} = \frac{1}{\pi R^2 H}\int_0^{R_o}\int_{-\frac{H}{2}}^{+\frac{H}{2}} C\ J_o(B_c r)\cos\frac{\pi z}{H}\ 2\pi r\ dr\ dz \tag{72}$$

which upon integration gives

$$\phi_{av} = \frac{4\ C}{\pi B_c R_o}\ J_1(B_c R_o) \tag{73}$$

or

$$C = \frac{\pi\ \phi_{av} B_c R_o}{4\ J_1(B_c R_o)}\ . \tag{74}$$