MEMBRANE ANALOGY FOR THE SOLUTION OF THE AGE-DIFFUSION EQUATION

by

George Anthony Englesson

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

Major Subject: Nuclear Engineering

Iowa State University
Of Science and Technology
Ames, Iowa

1959

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	1
II.	REVIEW OF LITERATURE	3
III.	REACTOR THEORY	4
IV.	MEMBRANE THEORY	9
	A. Membrane Equation	9
	B. Variation of Depth to Pressure Differential	12
V.	ESTABLISHMENT OF THE ANALOGY	15
VI.	BOUNDARY CONDITIONS	17
VII.	DESCRIPTION OF EQUIPMENT AND PROCEDURE	18
VIII.	EXPERIMENTAL RESULTS	27
IX.	CONCLUSIONS	32
X.	RECOMMENDATIONS FOR FURTHER STUDY	34
XI.	ACKNOWLEDGEMENTS	35
XII.	BIBLIOGRAPHY	36

I. INTRODUCTION

The idea of solving differential equations using dissimilar models or analogs has become a widely accepted technique. If due consideration has been given to keeping the differential equations of the model similar to the differential equations of the physical phenomena, a solution is readily available without recourse to solving the equations mathematically. In this study, the similarity of the differential equations governing the surface of a membrane and that of the steady state diffusion of neutrons in a media gives the basis for establishing the analogy.

In the theory of neutron diffusion, one of the most fruitful approximations for the slowing down of neutrons is the group diffusion method where the continuous neutron spectrum is replaced by a finite number of energy groups, and neutrons in each group are considered to behave according to the diffusion equations. The constants for each group are then determined by averaging the properties for the energy region represented by that group. Although neither the slowing down of neutrons nor the vector nature of the motion of the neutrons are properly taken into account, the group diffusion method yields an acceptable approximation. In this investigation, the neutrons were considered to fall in a single group (thermal group).

The shape and size of a nuclear reactor can be related to the physical characteristics of the materials in the system through an equation which

combines a concept called Fermi Age with the diffusion equation. The size can be determined directly from the solution of the diffusion equation, and by analogy through use of the model which will be described here. In this analogy, flux in the reactor is related to the depth of the membrane, and the size of the reactor is related to specific weight and surface tension of the membrane material.

II. REVIEW OF LITERATURE

Glasstone's "Principles of Nuclear Reactor Engineering" (1) was used for the development of reactor theory; Murphy's "Similitude in Engineering" (4) for model and membrane theory; and Hetényi's "Handbook of Experimental Stress Analysis" (3) for the immiscible fluid membrane treatment.

Piccard and Baes (6) in 1926 were the first to use the separation surface between two fluids as a membrane for solutions of Laplace and Poisson equations. Further developments in the technique were advanced by Sunatani, Matuyama, and Hatamura (8). However, nowhere in the literature was there any mention of using a loaded membrane in the solution of the diffusion equation.

III. REACTOR THEORY

The subject of bare core reactor theory is developed in several standard texts (1)(2)(7)(10); however, only that part which is required to develope the analogous equation will be discussed here. The differential equation for the steady state diffusion of neutrons in the reactor core is developed by considering a differential cube dx dy dz. The neutron current at one face is assumed to be J, and at a parallel face is assumed to be $J + \frac{\partial J}{\partial s} ds$. The neutron current undergoes a change, for inside the element some neutrons are absorbed and some are produced.

In Figure 1,

J = neutron current (net number of neutrons flowing in a given direction in unit time through a unit area normal to the direction of flow)

Σ_a = macroscopic absorption cross section (cm^{-1})

ϕ = neutron flux (neutrons/cm^2 sec.)

S = source (neutrons/cm^3 sec.)

By continuity of neutron flow,

$$J_x\, dy\, dz + J_z\, dx\, dy + J_y\, dx\, dz = S\, dx\, dy\, dz - \Sigma_a \phi\, dx\, dy\, dz$$

$$+ J_x\, dy\, dz + \frac{\partial J_x}{\partial x} dx\, dy\, dz + J_y\, dx\, dz + \frac{\partial J_y}{\partial y} dy\, dx\, dz + J_z\, dx\, dy$$

$$+ \frac{\partial J_z}{\partial z} dx\, dy\, dz.$$

Therefore,

$$\frac{\partial J_y}{\partial y} + \frac{\partial J_x}{\partial x} + \frac{\partial J_z}{\partial z} - \Sigma_a \phi + S = 0 \qquad (1)$$

From Fick's Law of Diffusion

$$J = - D_0 \text{ grad } n \qquad (2)$$

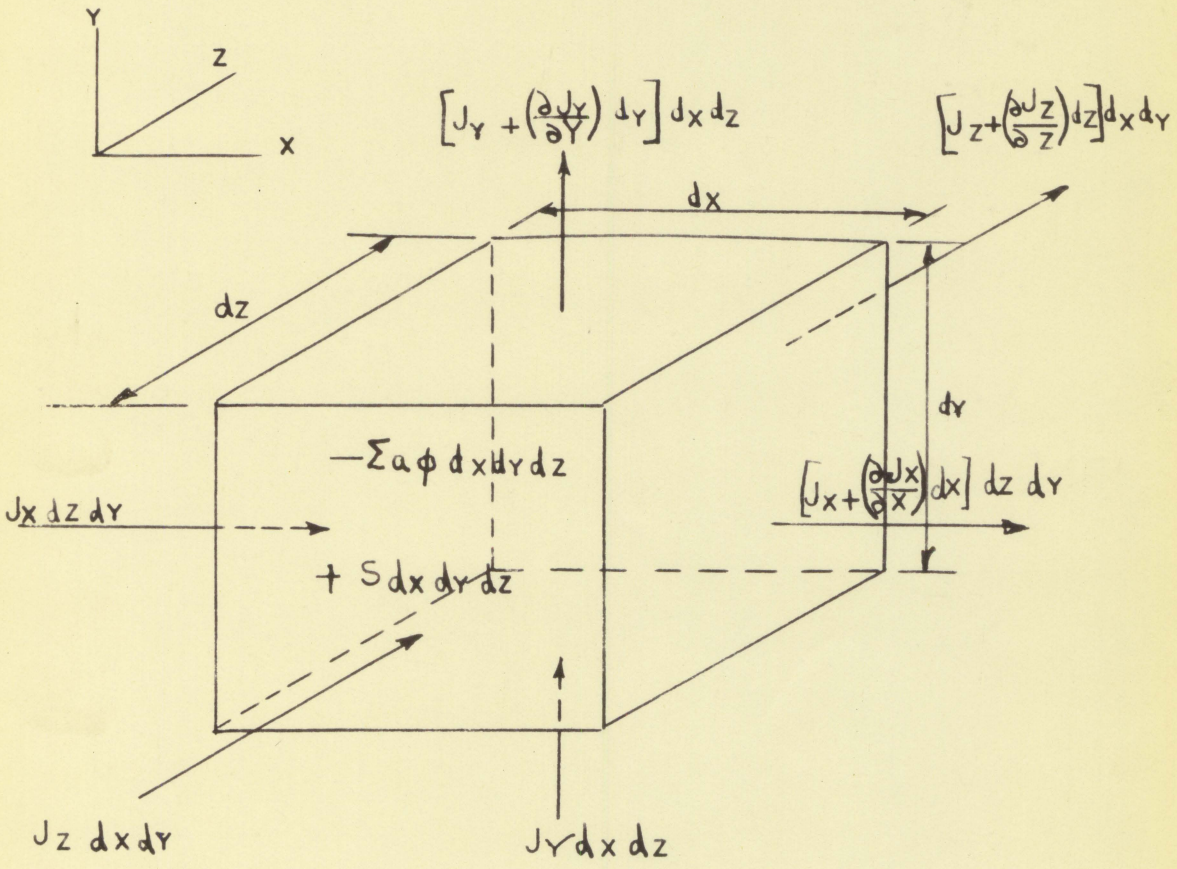

Figure 1. Differential cube for the development of the diffusion equation

where D_0 = diffusion coefficient (cm^2/sec)
n = neutron density (neutrons/cm^3).

and since

$$n = \frac{\phi}{v} \tag{3}$$

where v = neutron velocity (cm/sec) (a constant for each group)

$$J = D \text{ grad } \phi \tag{4}$$

where $D = D_0/v$ (cm)

Therefore, by substitution of equation 4 in equation 1

$$D \frac{\partial^2 \phi}{\partial x^2} + D \frac{\partial^2 \phi}{\partial y^2} + D \frac{\partial^2 \phi}{\partial z^2} - \Sigma a \phi + S = 0 \tag{5}$$

or

$$D \nabla^2 \phi - \Sigma a \phi + S = 0 \tag{6}$$

Equation 6 is the well known diffusion equation for a homogeneous mixture. The developement of the equation is based on the assumption that the neutrons are of one energy group.

The source term S can be determined from the Fermi Age model, wherein the slowing down process is evaluated as a continuous process. In the critical reactor, the source is derived from the fission neutrons which can be represented by the slowing down density at the energy of the group being investigated. Therefore, the source term would be taken as $q(\tau)$ where q is the slowing down density of neutrons at age τ. If some of the neutrons are absorbed during slowing down, the source can be represented as $pq(\tau)$; where p is the resonance escape probability.

The steady state diffusion equation for a critical reactor is then

$$D \nabla^2 \phi - \Sigma a \phi + pq(\tau) = 0 \tag{7}$$

The solution of the age equation $\nabla^2 q(\tau,r) = \frac{\partial q(\tau,r)}{\partial \tau}$, where r is a spatial coordinate, is effected by setting $q(\tau,r) = T(\tau)R(r)$ and separating the variables so that

$$\nabla^2 R(r) + B^2 R(r) = 0 \tag{8}$$

and

$$T(\tau) = Ae^{-B^2\tau} \tag{9}$$

Thus, when $q(\tau,r)$ is evaluated at age 0

$$q(0,r) = \frac{k}{p} \Sigma a \phi \tag{10}$$

where k is the infinite multiplication factor

and then

$$q(\tau,r) = \frac{k}{p} \Sigma a \phi(r) e^{-B^2\tau}. \tag{11}$$

Thus equation 7 becomes

$$D \nabla^2 \phi(r) - \Sigma a \phi(r) + k \Sigma a \phi(r) e^{-B^2\tau} = 0 \tag{12}$$

and since $\frac{\Sigma a}{D} = \frac{1}{L^2}$ where L is defined as the diffusion length,

$$\nabla^2 \phi(r) + \frac{ke^{-B^2\tau} - 1}{L^2} \phi(r) = 0 \tag{13}$$

The relationship between equation 8 and 13 is such that

$$B^2 = \frac{ke^{-B^2\tau} - 1}{L^2} \tag{14}$$

from which the critical equation

$$1 = \frac{ke^{-B^2\tau}}{1 + L^2B^2} \tag{15}$$

for age diffusion theory is developed.

If equation 14 is substituted into equation 13, the characteristic equation for the steady state diffusion of neutrons in the bare core of a reactor is

$$\nabla^2 \phi(r) + B^2 \phi(r) = 0 \qquad (16)$$

In equation 15, the critical equation, B^2, called buckling, is a function of the reactor constants K, L^2, τ, each of which can be calculated from fundamental nuclear constants. Since this buckling is independent of the size and shape of the reactor, it is called materials buckling, and is designated B_m^2.

In equation 16, B^2 is a space dependent quantity, and since it is independent of the materials in the reactor, it is called geometric buckling and designated B_g^2.

When a reactor is critical $B_g^2 = B_m^2$.

Equation 16 is the characteristic differential equation for the reactor system, and it will be matched in the analogy with the membrane equation. In the solution of differential equations, one must also take into account certain boundary conditions which apply to the phenomena under consideration. The boundary conditions, which apply to the reactor system, are discussed in Section VI.

IV. MEMBRANE THEORY

A. Membrane Equation

If a membrane, defined as an elastic member having a uniform surface tension and no shearing resistance, is subjected to a differential pressure, the equation of the surface of the membrane is developed by Murphy (4) as follows:

A differential element dx dy is subjected to a differential pressure (P) and is held in equilibrium by the uniform surface tension (T).

In Figure 2,

$\frac{\partial z}{\partial x}$ = slope

$\frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial x}$ = rate of change of z

$\frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial x}\left(\frac{dx}{2}\right)$ = total amount of change of z

Therefore, the component of the surface tension force in the P direction is (T dy) $\left(\frac{dx}{2}\right)\left(\frac{\partial^2 z}{\partial x^2}\right)$. Since the curvature, concave downward, is taken as negative, the summation of forces in the P direction is

$$P\,dx\,dy - 2T\,dy\left(-\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{dx}{2}\right)\right) - 2T\,dx\left(-\left(\frac{\partial^2 z}{\partial y^2}\right)\left(\frac{dy}{2}\right)\right) = 0$$

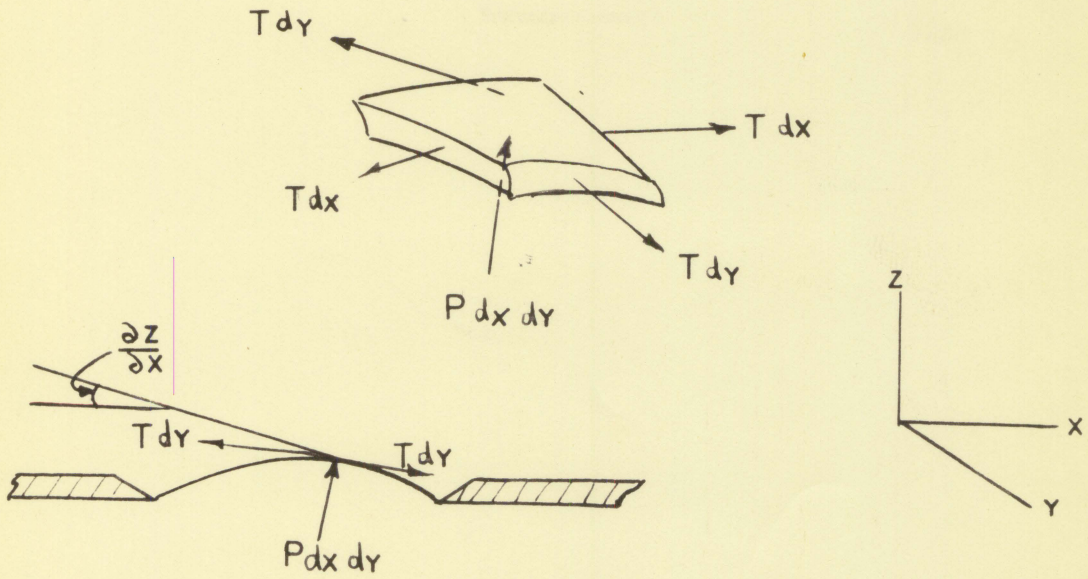

Figure 2. Membrane section

and

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = -\frac{p}{T} \tag{17}$$

which is recognized as a Poisson equation if P and T are held constant.

B. Variation of Depth to Pressure Differential

If the membrane chosen is the boundary between two immiscible fluids, it can be shown that the pressure differential across the membrane is a function of the depth of fluid between the membrane and a reference plane at which the pressure is the same in both fluids.

In Figure 3, let

d, D = specific weight of fluid

p, P = pressure on the membrane

z = distance from the reference plane

The pressures and specific weights of the fluid on one side of the membrane are denoted by small letters and those on the other side by capital letters.

at z_0, $\Delta p_0 = 0 = p_0 - P_0$

at z_1, $\Delta p_1 = p_1 - P_1 = (z_1 d - z_1 D) = z_1(d - D)$

at z_2, $\Delta p_2 = p_2 - P_2$

$$= p_1 + (z_2 - z_1)d - \left[P_1 + (z_2 - z_1)D\right]$$

$$= p_1 - P_1 + (z_2 - z_1)(d - D)$$

$$= z_1(d - D) + z_2(d - D) - z_1(d - D)$$

$$\Delta p_2 = z_2(d - D). \qquad (18)$$

Therefore, the pressure differential on the fluid membrane is everywhere a function of the depth of fluid between the membrane and the zero reference plane, whether the membrane is deflected upward or downward.

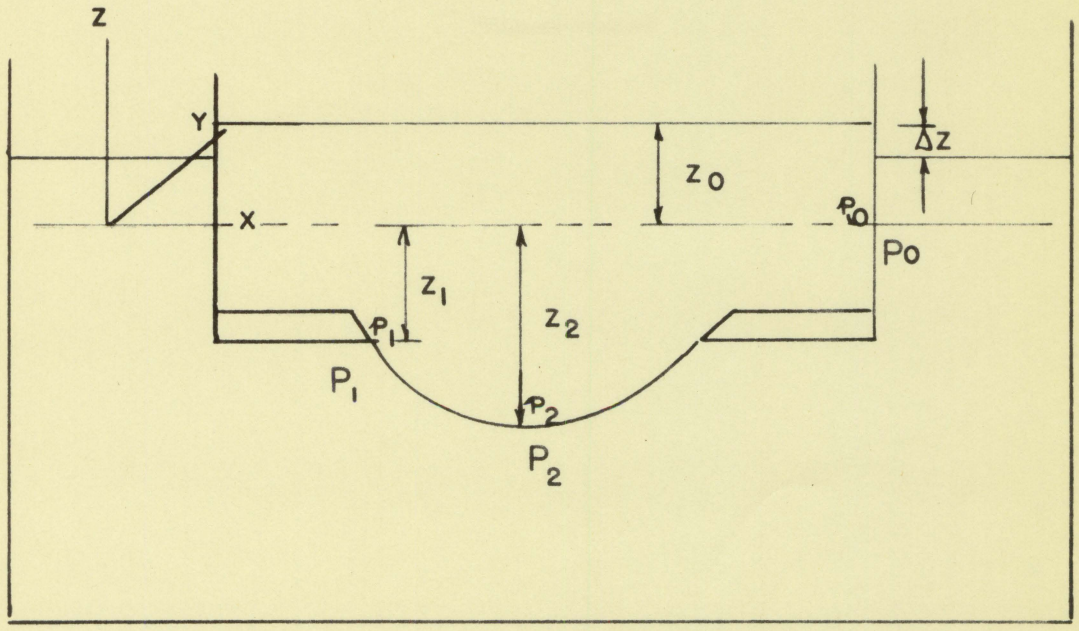

Figure 3. Model cross section

The membrane equation now becomes:

$$\nabla^2 z = \frac{-s(d - D)}{T} \tag{19}$$

where z is the distance from the reference plane to the membrane.

Equation 19, a transformed Poisson equation, is the characteristic differential equation for the loaded membrane and will be matched through an analogy with equation 16, the characteristic differential equation for the neutron flux in a critical reactor.

V. ESTABLISHMENT OF THE ANALOGY

The two characteristic equations can be written as:

$$\nabla^2 \phi + B^2 \phi = 0 \tag{20}$$

(critical equation for neutron flux in a reactor)

$$\nabla^2 z + kz = 0 \tag{21}$$

(membrane equation, where $k = \frac{d - D}{T}$).

It is obvious that the differential equations of the two phenomena are similar; therefore, using the principles of similitude which relate dissimilar models, the systems are said to form an analogy.

To make the analogy effective, corresponding terms in the equations may be related by the following design conditions:

$$x = nx_m \tag{22}$$

$$y = ny_m \tag{23}$$

$$B^2 = n_1 k \tag{24}$$

from which it follows that the prediction equation is

$$\phi = n^2 n_1 z \ . \tag{25}$$

The membrane equation is written in three coordinates, the third coordinate, however, corresponds to the potential function in the diffusion equation; consequently, the analogy can only be used to solve a two dimensional problem, and will be used here for a reactor with two finite dimensions.

The first two design conditions indicate geometric similarity between the membrane and the reactor cross sections. The third condition relates the constant, B^2, to the membrane surface parameters, while the prediction equation relates the flux to the depth of fluid above it.

The analogy as drawn relates B^2 in the reactor to $\frac{(d - D)}{T}$ in the model. Since the specific weight differential and the surface tension are both constants, B^2 is a constant. Therefore, for any membrane cross section, the buckling will be the same. This buckling is, therefore, materials buckling, and the problem becomes that of determining the proper geometric size for an arbitrarily shaped cross section containing prescribed materials.

VI. BOUNDARY CONDITIONS

In addition to the design equations, the boundary conditions which apply to the critical equation must be satisfied in the model system.

The boundary conditions for the critical equation are those which pertain to the diffusion of neutrons through a medium (1).

1. The flux must be continuous at all points including the boundaries.
2. The flux is finite at the boundary, and will become zero at some extropolated distance past the boundary. This distance is equal to 0.71 λt where λt is the transport mean free path.

These conditions are met in the model system by setting the zero pressure differential plane above the membrane and deflecting the membrane downward as shown in Figure 3. The curve of the distended membrane represents the curve associated with the intensity of the neutron flux. If the curve is extropolated from each end with a continuous slope, it intersects the zero plane at an extropolated distance corresponding to the extropolated boundary in the reactor.

VII. DESCRIPTION OF EQUIPMENT AND PROCEDURE

The equipment, shown in Figures 4,5,6 and 7, consists primarily of two tanks, one within the other, and a depth measuring device. The inner tank is constructed of acid-soldered copper sheet with a 0.125 in. brass plate (b) for a base. The cross sections to be investigated are cut into the brass plate, and the membranes are suspended from the plate. The outer tank is constructed of wood with one glass side for observation of the membrane. The depth gage (c) is oriented as shown in Figure 8.

The surfaces (a) and (b) are set parallel to each other and the depth gage (c) is set perpendicular to (a) and (b). The bar (d) is set parallel to a line joining the centers at the two cross sections, and then, with the surface (a) oiled, the base plate (e) can be translated so that the depth gage traverses the line joining the centers of the cross sections.

The membrane is formed from the surface tension of two immiscible fluids, the non-electrolyte para-chlorotoluene (density 1.058) and a sodium chloride solution.

The outer tank is filled with salt water until the fluid meniscus covers the openings in the plate. Then, simultaneously, chlorotoluene is poured into the inner tank and the brine solution in the outer tank at a rate which will keep the membrane formed over the openings in a slightly distended position. Care must be taken to keep the chlorotoluene

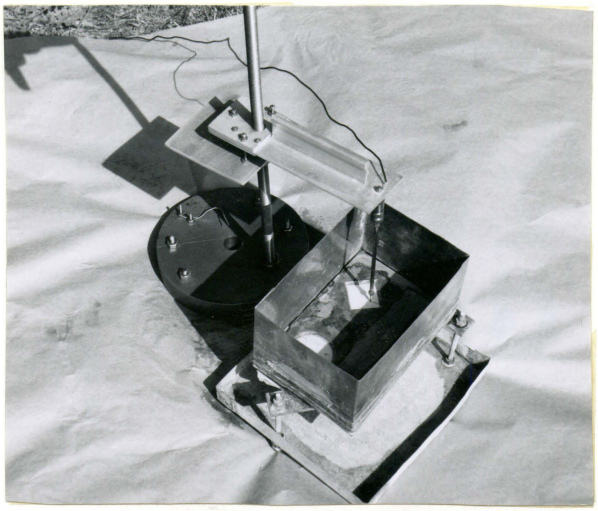

Figure 4. Depth gage and inner tank

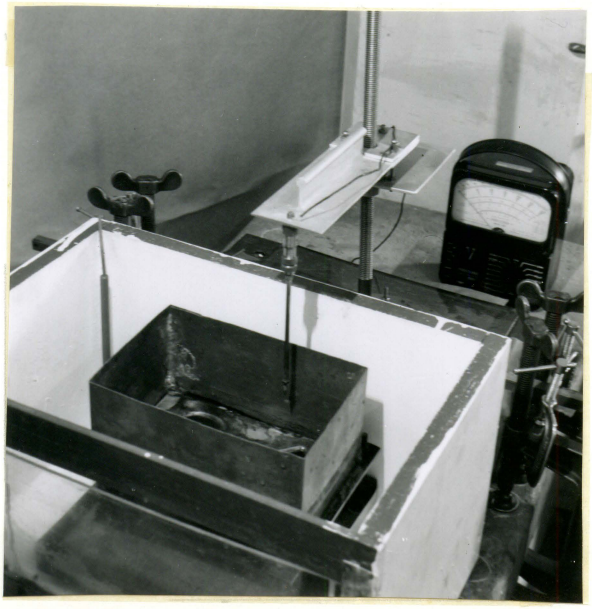

Figure 5. Arrangement of equipment

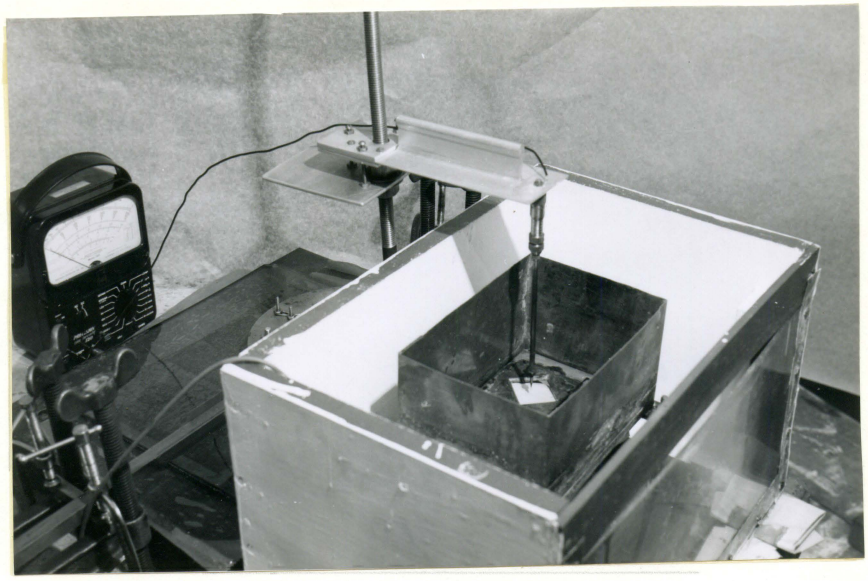

Figure 6. Equipment set up to probe the membrane

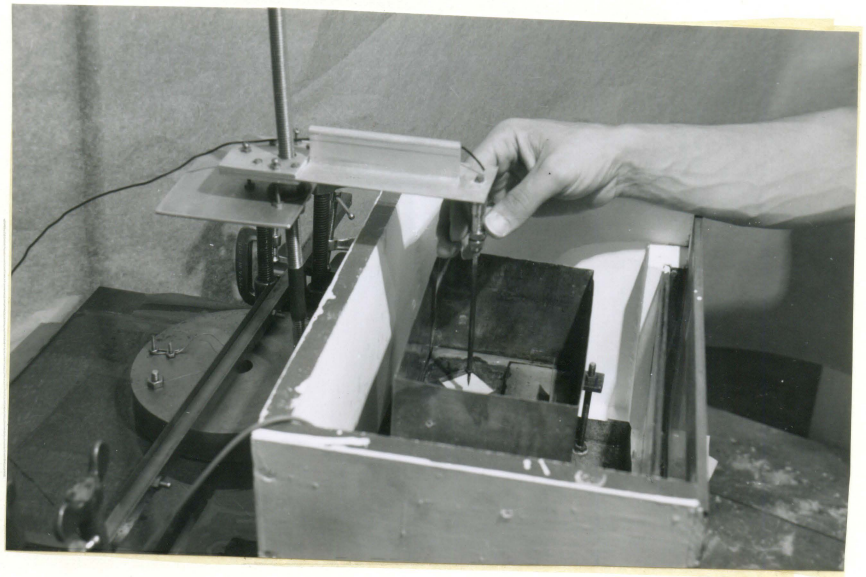

Figure 7. Probing the membrane

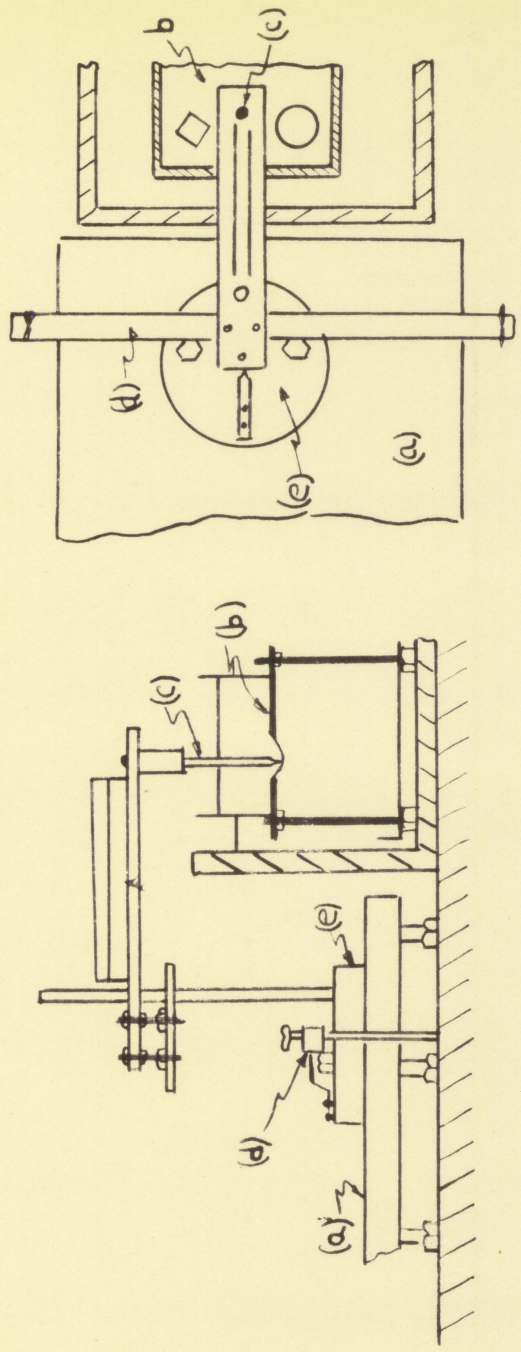

Figure 8. Depth gage alignment

from wetting the under surface of the base plate and to keep the salt water from rising into the inner tank. Sufficient chlorotoluene must be placed in the inner tank so that the reference plane will lie above the membrane if the membrane is distended downward or below the membrane if it is distended upward. The position of the reference plane is determined as follows:

In Figure 3,

Δz = differential heights between the surfaces

z_0 = distance from the chlorotoluene surface to the reference plane.

At the surface of the outside fluid

$$\Delta p = (zd - 0).$$

At z_0

$$\Delta p = 0$$

$$\Delta p = z_0 d - (z_0 - \Delta z)D = 0$$

$$z_0 d = z_0 D - \Delta z D$$

Therefore,

$$z_0 = \frac{\Delta z D}{D - d}. \qquad (26)$$

The levels of the fluids are dependent on the specific weight differential, and whether or not the membrane is distended. The specific weight differential was varied from +0.05 to -0.05; the best results were obtained at a specific weight for the brine of 1.080. If the membrane is to be distended downward, fluid is tapped off from the outer tank. The fluids are held at their new levels by the surface tension of the membrane.

The cross section of the membrane is measured electrically by inserting the depth probe through the non-electrolyte chlorotoluene to the fluid membrane. One lead of an ohmeter is connected to the depth probe and the other inserted into the salt solution; as the depth probe touches the film, continuity occurs, and the value of the depth is read on the micrometer.

If one uses the second boundary condition, the prediction equation and the design conditions, an additional relationship between the two membrane cross sections and, analogously, the two reactor cross sections can be developed as follows:

$$\phi = n^2 n_1 z \quad . \tag{25}$$

By differentiation

$$\frac{\partial \phi}{\partial x} = n^2 n_1 \frac{\partial z}{\partial x}\left(\frac{\partial x_m}{\partial x_m}\right) = n^2 n_1 \left(\frac{\partial z}{\partial x_m}\right)\left(\frac{1}{n}\right)$$

or

$$\frac{\partial \phi}{\partial x} = n n_1 \frac{\partial z}{\partial x_m} \quad . \tag{27}$$

The cosine function can be used as a first approximation for the flux distribution in a reactor, without incurring serious error[1].

$$\phi = A \cos \frac{\pi x}{2R} \tag{28}$$

where A = constant which governs the amplitude of the flux in the reaction

---

[1]See page 215 of Glasstone and Edlund (2).

R = distance from the center of the reactor to the extropolated boundary.

By differentiation

$$\frac{\partial \phi}{\partial x} = \frac{\pi A}{2R} \sin \frac{\pi x}{2R} \tag{29}$$

Evaluated at $x = R$,

$$\frac{\partial \phi}{\partial x} = \frac{\pi A}{2R} \tag{30}$$

If both sides are multiplied by $\frac{R}{A}$

$$\frac{R}{A} \frac{\partial \phi}{\partial x} = \frac{\pi}{2} \tag{31}$$

Therefore, at the extropolated boundary, the slope is a constant in the reactor and analogously in the membrane.

Equation 27 is transformed by multiplying both sides by $\frac{R}{A}$ and substituting for A the analogous value z_{max}.

$$\frac{R}{A} \frac{\partial \phi}{\partial x} = \frac{R}{z_{max}} nn_1 \frac{\partial z}{\partial x_m} \tag{32}$$

Since $R = nR_m$,

$$\frac{R}{A} \frac{\partial \phi}{\partial x} = \text{constant} = \frac{R_m}{z_{max}} n^2 n_1 \frac{\partial z}{\partial x_m} \tag{33}$$

The two cross sections to be investigated are a circle and a square. Therefore,

$$\left[\frac{R_m}{z_{max}} n^2 n_1 \frac{\partial z}{\partial x_m}\right] \text{circle} = \left[\frac{R_m}{z_{max}} n^2 n_1 \frac{\partial z}{\partial x_m}\right] \text{square} \tag{33}$$

The distance R for each cross section is fixed by construction; the buckling scale, n_1, is fixed by the fluid parameters; z_{max} and the $\frac{\partial z}{\partial x_m}$ at the extropolated boundary will be determined experimentally; n in the circle, which will be the control section, will be determined

from equation 22 and the known expression for buckling in an infinite cylindrical reactor by the following method.

When a reactor is critical

$$B_g^2 = B_m^2$$

and

$$B_g^2 = \left(\frac{2.405}{R}\right)^2 \tag{35}$$

Therefore, if the materials in the reactor are specified, R for the reactor can be determined.

In equation 22

$$n = \frac{x}{x_m}$$

and if the radius of the circle in the model is substituted for x_m, while R is substituted for x, the length scale n is determined.

The only unknown left in equation 34 is, therefore, the length scale for the parallelopiped. The scale will also be determined from equation 25 in the following manner. Equation 25 is

$$\phi = n^2 n_1 z \ . \tag{25}$$

The ratio of flux in the square and the cylindrical reactor is

$$\frac{\phi_{\text{circle}}}{\phi_{\text{square}}} = \frac{(n^2 n_1 z)_{\text{circle}}}{(n^2 n_1 z)_{\text{square}}} \ . \tag{36}$$

Since the flux level in a reactor is arbitrary, and depends on the power level of the reactor, the flux levels can be set equal to each other. Therefore, equation 36 can be written

$$(n^2)_{\text{square}} = \frac{(n^2 z)_{\text{circle}}}{(z)_{\text{square}}} \tag{37}$$

A comparison of scale determined from equations 34 and 37 will be made with a scale which will be determined from equation 35 and the known expression for buckling in a square parallelopiped,

$$B^2 = 2\left(\frac{\pi}{a}\right)^2 \qquad (38)$$

where a is the length of the side of the reactor.

For ease in calculation, the length scale of the cylindrical model will be assumed to be equal to unity. This will fix the value of materials buckling in both models and the length (a) in equation 38.

The ratio of $\frac{a}{2}$ from equation 38, divided by the extropolated distance R from the membrane of the square section, will determine the calculated length scale which will be compared to the scale determined from equations 34 and 37.

VIII. EXPERIMENTAL RESULTS

The feasibility of the analogy was investigated by two experiments. In both cases the membrane was distended downward, and the zero pressure differential plane established within the thickness of the base plate. This insured that the cross sections at the intersection of the z surface and the plane were a circle and a square.

The data given in Table 2 and Table 3 are represented in Figure 9 and Figure 10, respectively.

The results obtained from the experiments are summarized in Table 1. The length scale as determined from equation 34 is given in column 2, and that determined from equation 37 is given in column 3. The value calculated from the expressions for buckling is given in column 1. The value of scale given in column 1 is used as a reference to calculate the percentage difference tabulated in columns 4 and 5.

Table 1. Results of the Plot of Membrane Cross Section

	I	II	III	IV	V
	Calculated Length Scale	Experimental Scale from Equation 34	Experimental Scale from Equation 37	Percentage Difference Column II	Percentage Difference Column III
Experiment 1	1.19	1.183	1.035	0.58	13.1
Experiment 2	1.30	1.382	1.054	3.82	16.5

Table 2. Data for Membrane Depth

Experiment 1:

Specific weight chlorotoluene is 1.058 gms/cm^3
Specific weight brine solution is 1.078 gms/cm^3
Position of zero pressure plane is 1.235 in.

Cylinder		Parallelopiped	
Position (in)	Depth (in)	Position (in)	Depth (in)
3.90	1.451	10.45	1.440
3.75	1.452	10.35	1.4405
3.60	1.449	10.25	1.4405
3.50	1.437	10.15	1.436
3.40	1.430	10.05	1.430
3.30	1.412	10.00	1.428
3.25	1.398	9.95	1.412
3.20	1.378	9.90	1.406
3.15	1.357	9.85	1.386
3.10	1.329	9.80	1.366
3.07	1.302	9.75	1.337
3.05	1.285	9.72	1.307
3.02	1.260		
3.00	1.233		

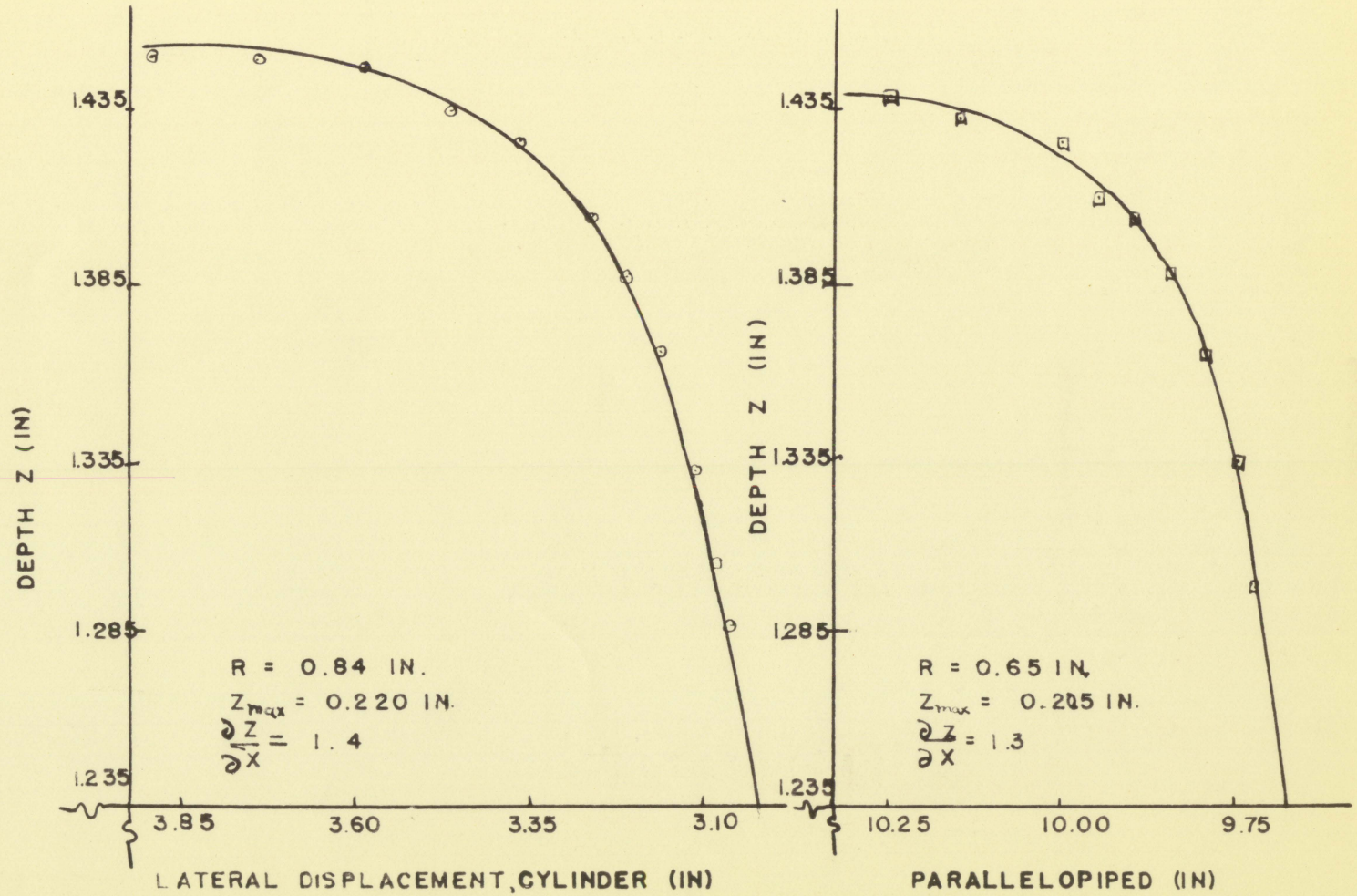

Figure 9. Membrane cross section for data determined in Experiment 1

Table 3. Data for Membrane Depth

Experiment 2:

Zero pressure plane is 1.250

Cylinder		Parallelopiped	
Position (in)	Depth (in)	Position (in)	Depth (in)
4.85	1.474	9.85	1.451
4.75	1.480	9.75	1.454
4.65	1.476	9.65	1.454
4.55	1.475	9.55	1.448
4.45	1.473	9.50	1.4385
4.35	1.469	9.45	1.435
4.30	1.462	9.40	1.431
4.25	1.457	9.35	1.419
4.20	1.455	9.30	1.407
4.15	1.451	9.25	1.384
4.10	1.444	9.20	1.358
4.05	1.430	9.17	1.333
4.00	1.410	9.15	1.287
3.95	1.388		
3.90	1.347		
3.88	1.329		
3.85	1.303		
3.83	1.247		

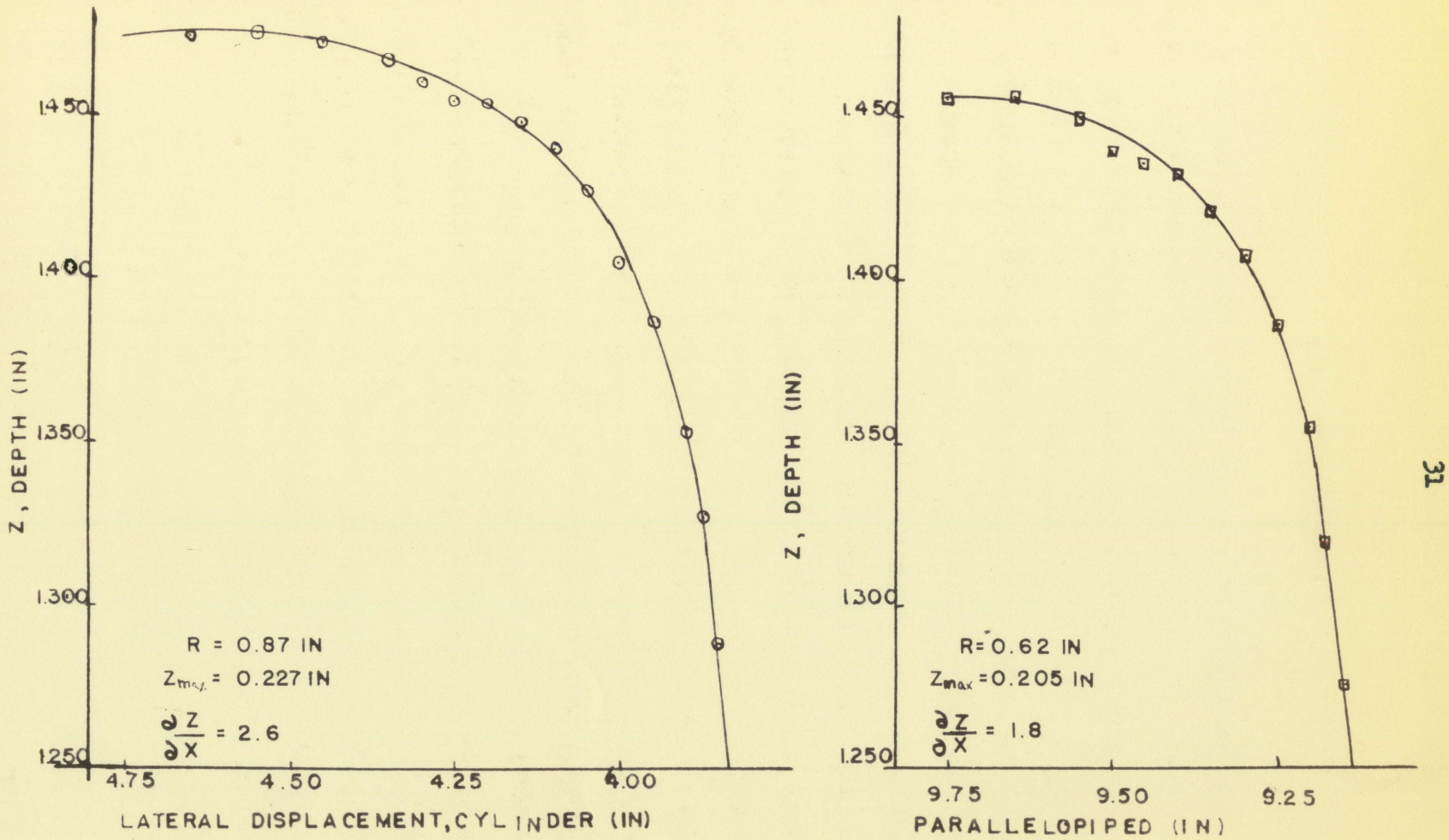

Figure 10. Membrane cross section for data determined in Experiment 2

IX. CONCLUSIONS

It has been shown that neutron flux in a bare critical core, as determined from one group theory, is analogous to the deflection of a membrane loaded with a pressure proportional to the depth at any point in the membrane. This analogy has been used to predict the size of a square critical core; however, there is one significant discrepancy shown in Figure 11. In this figure, the cosine curve that one should obtain in a square reactor is compared to the curve of membrane depth shown in Figure 9. Both curves are normalized to an amplitude of unity.

See P 34

The flat appearance of the membrane is evidenced in all of the curves shown, and can possibly be explained with the following analogy. The capillary effect in a small tube which is inserted into a fluid allows the fluid to rise inside the tube; however, as the tube diameter is increased, the forces at the surface will lift the same volume of fluid at the boundary without disturbing the level at the center.

The percentage difference in the length scales, as determined from equation 37, is due to this flattening effect. The percentage difference from equation 34 is due to inaccuracy in the measuring device near the boundary as well as this flattening effect. It would appear that these differences compensate for each other in equation 34.

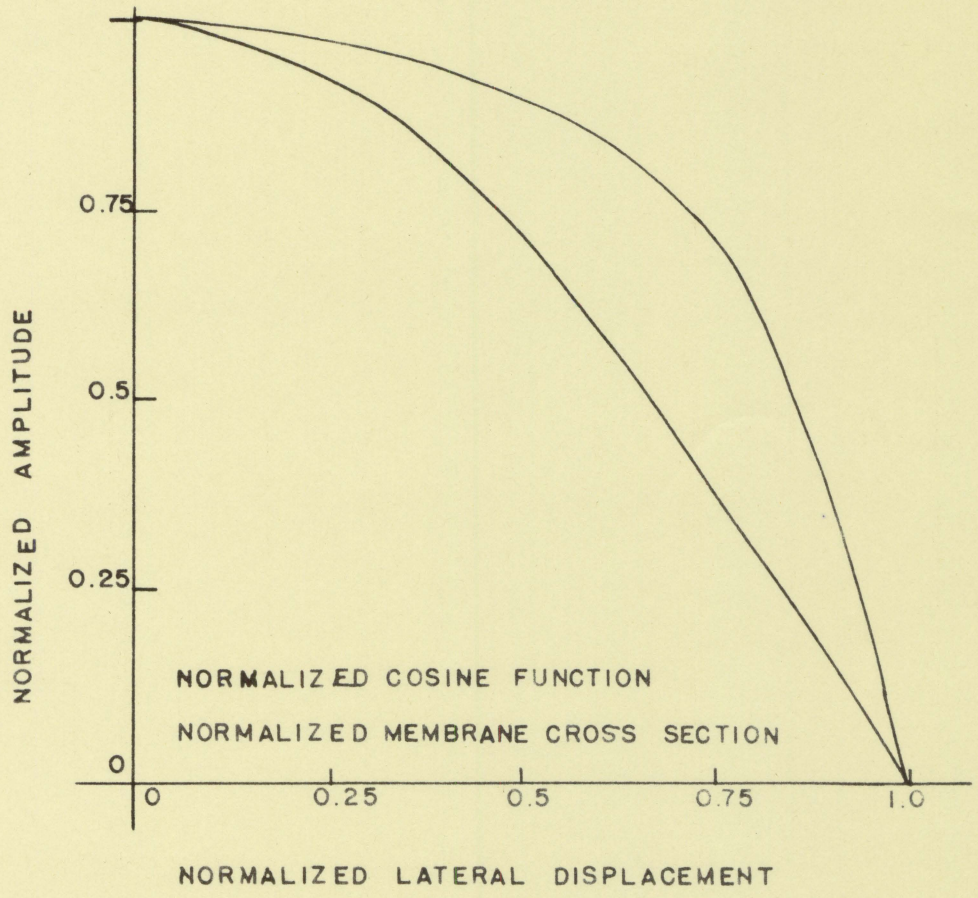

Figure 11. Comparison of flux intensity with membrane depth

X. RECOMMENDATIONS FOR FURTHER STUDY

Since the apparent flattening of the curves is a function of the forces in the fluids and the distances over which these forces act, there is then a certain critical combination of specific gravities and cross sectional areas which will enable one to distend the membrane so that the z surface will be exactly analogous to the ϕ surface. If further study is to be made in this analogy, this optimum condition should be determined.

An interesting problem would also be the determination of the depression of flux in a reactor due to a control rod or an arrangement of control rods. However, more precise means of measurement would be needed. Two adjustments would improve the accuracy. One of these would be to replace the circular depth micrometer which has a tendency to describe a circle with a vertical sliding depth micrometer. The other would be to improve the lateral positioning device with a screw type slide for more precise measurements.

XI. ACKNOWLEDGEMENTS

The author wishes to thank his major professor, Dr. Glenn Murphy, for the suggestion which began this study, for his help and encouragement, and for arranging the use of the T. & A. M. facilities for the construction and operation of the model.

XII. BIBLIOGRAPHY

1. Glasstone, S. Principles of nuclear reactor engineering. Princeton, New Jersey. D. Van Nostrand Company, Inc. 1955.

2. Glasstone, S., and Edlund, M.C. The elements of nuclear theory. Princeton, New Jersey. D. Van Nostrand Company, Inc. 1952.

3. Hetenyi, M. Handbook of experimental stress analysis. New York. John Wiley and Sons, Inc. 1950.

4. Murphy, G. Similitude in engineering. New York. The Ronald Press Co. 1950.

5. Murphy, G. Advanced mechanics of materials. New York. McGraw Hill Book Company, Inc. 1946.

6. Piccard, A., and Baes, L. Mode experimental nouveau relatif à l'application des surfaces à courbure constante à la solution du problème de la torsion des barres prismatiques. 2nd. Int. Congress for Applied Mechanics. Zurich. 1926. Proc.

7. Soodak, H., and Campbell, E. Elementary pile theory. New York. John Wiley and Sons, Inc. 1950.

8. Sunatani, C., Matuyama, T., and Hatamura, M. The solution of torsion by means of a liquid surface. Tokyo. Tohoku University Technical Report 12. 1938.

9. Timoshenko, S. Theory of elasticity. New York. McGraw Hill Book Company, Inc. 1934.

10. Weinberg, A., and Wigner, E. The physical theory of neutron chain reactors. Chicago. The University of Chicago Press. 1958.