A case study of middle school students' development and application of three-dimensional geometry language in hands-on and computer learning environments

by

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CHAPTER I. INTRODUCTION

Geometry is one of the mathematical disciplines taught during the middle school years. Several instructional environments are used to develop student understanding of geometry concepts. Two such environments are computers and hands-on physical materials (Coxford, 1991). In both environments, the development and use of mathematical language can assist students in learning. This study examines the development and application of three-dimensional geometry language among middle school-aged students in hands-on and computer learning environments. This chapter is divided into six sections: 1) background information from the literature, 2) statement of the problem, 3) purpose of the study, 4) research question, 5) definition of terms, and 6) summary of the chapter.

Background

Mathematics education is constantly changing to fit the needs and demands of society. Mathematics is viewed as a important component in all aspects of life: school, home, and workplace (National Research Council, 1989). Because of the technological thrust of American society, strong mathematical skills are essential for embracing opportunities, obtaining employment, and making informed decisions as consumers; therefore as a result, an increased awareness of the necessity to provide quality and effective mathematics instruction has occurred.

Professional mathematics organizations such as the National Council of Teachers of Mathematics (NCTM) and National Council of Supervisors of Mathematics (NCSM) have conducted an analysis of mathematics education. These organizations identified a need for changing the current mathematics curriculum, methods of instruction and methods of evaluation (Reys, Suydam, & Lindquist, 1992). Reports and recommendations from NCTM, NCSM, and other professional mathematics organization provided direction for the needed reform in mathematics education. Below is a description of four major issues in mathematics education that have been identified by various professional mathematics organizations as areas of needed reform.

Mathematical Language

The development of mathematical language is an area of growing emphasis in mathematics education (Durkin, 1991). Learning in mathematics depends upon the knowledge and use of its language. Durkin (1991) described mathematics education as an area that focuses on language from the beginning to the end of its learning. As defined by Pimm (1987), mathematical language includes two components: written and spoken language. Both components interlock to provide the basis for classroom communication between two essential groups: teachers and students.

The need for the development of mathematical language has been well documented (Mumme & Shephard, 1990; Tracy, 1994; Pimm, 1987). From the literature on mathematical language, three reasons have been identified for the development of mathematical language. Mathematical language needs to be developed to aid in classroom communication and empower student construction of knowledge.

To aid in the classroom communication process is one reason for developing mathematical language (Mumme & Shepherd, 1990). Classroom communication occurs in two sets of interactions: students to students and teacher to students. For mutual understanding to occur, common language needs to be used among teachers and students. In these interactions, students need to have the opportunity to communicate what they are thinking and listen to what others are thinking (Tracy, 1994; Mumme & Shepherd, 1990). Thus, classroom communication is enhanced through the use of a common language.

The second reason for developing mathematical language relates directly to student learning. Having students develop mathematical language will allow the students to have the necessary mechanisms to communicate their mathematical thinking and understanding (Mumme & Shepherd, 1990). With this developed mathematical language, students can express their ideas and develop their own methods for solving problems which provides an opportunity for students to be responsible for their own learning. Students can then develop confidence in themselves and their mathematical thinking ability (Mumme & Shepherd, 1990).

Mathematical language is often regarded as more complex and specialized than language in other subject areas (Durkin, 1991). Moreover, mathematical terms often have multiple meanings that are different from the students' use of the term in everyday language. For example, in mathematical language, the term square root refers to a factor of a number that when multiplied by itself gives the original number. In everyday language, square root may infer that a root, the foundation of a plant or tree, is in the shape of a square, a four-sided figure. Because of the multiple meanings of mathematical terms, students often struggle with the mathematical definitions because they are unfamiliar and perceived as difficult to learn.

Students typically memorize the definitions of mathematical terms (Dickson, Brown, & Gibson, 1994). Memorizing terms often results in two problems: students having poor attitudes about mathematics and students forgetting the definitions when solving related problems (Dickson, Brown, & Gibson, 1994). Thus, solely memorizing mathematical terms can limit a student's ability to understand and internalize mathematical concepts. Students need to move beyond merely memorizing terms; they need to understand the terms and be able to use the terms to assist in the development of their conceptual understanding.

Geometry

Geometry became an essential component of mathematics education in the 1860s when colleges began requiring students to enroll in a geometry course (Peterson, 1973). Currently, the majority of precollege geometry instruction occurs at the middle school level with introductory activities conducted throughout the K-8 grade levels. In most elementary mathematics textbooks, geometry is included in one chapter of instruction and that chapter is typically located near the end of the textbook. Geometry skills traditionally taught at the K-8 level include volume, surface area, measurement, lines, and angles of primarily two dimensional objects. Usiskin (1987) reported that this was an outdated curriculum. An expansion of geometry instruction in mathematics education is needed; that is, the amount of geometry instruction needs to be increased at all grade levels, and the topics covered at each grade level need to change.

Usiskin (1987) argued that the curricular focus of geometry needs to be reassessed based upon the contemporary definitions of geometry and skills needed for today's society. A detailed geometry curriculum based on the reassessment of geometry instruction should be specified for all grade levels (Usiskin, 1987). Three-dimensional geometry is one area that should be added to the revised geometry curriculum (Reys, Suydam & Lindquist, 1992). This topic is one of several that should be added to the curriculum to enhance geometry learning.

Learning Environments

In mathematics, classroom teachers often use a variety of methods and instructional environments to help students learn mathematical concepts. The teacher and the curriculum are the primary factors that determine how mathematics is taught (Reys, Suydam & Lindquist, 1992). Effective mathematics instruction emphasizes the use of meaningful experiences to develop understanding. Moreover, such instruction challenges the students to investigate further the concepts they are learning. (Capps & Pickreign, 1993). Two instructional environments that can be used to create meaningful mathematical language experiences are hands-on and computer.

A hands-on learning environment includes the use of physical manipulatives such as pattern blocks or an abacus. Using hands-on materials for instruction provides the foundation for future learning in mathematics (Reys, Suydam & Lindquist, 1992). Concepts can be introduced and developed through the use of manipulatives. Furthermore, the use of manipulatives increases the possibility of students constructing and retaining mathematical ideas (Reys, Suydam & Lindquist, 1992).

More than just a change from the traditional methods of teaching, the computers provide an interdisciplinary tool for developing and reinforcing concepts (Reys, Suydam & Lindquist, 1992). Computers provide abstract environments where students can manipulate variables, test hypothesis, identify relationships between concepts, and model and visualize numerical data. Software programs are available to assist students in learning mathematics. These software programs are designed in a variety of forms from games to simulations to microworlds (Kaput, 1992).

Application of Concepts

In learning mathematics, students need to have the opportunity to engage in meaningful experiences. Through these experiences, students will discover definitions, terms, and rules in mathematics (Reys, Suydam, & Lindquist, 1992). In the research literature, it is explained that if students are involved in meaningful experiences, they have a higher chance for transfer and retention of concepts (Driscoll, 1980). If provided with effective and meaningful learning environments, students have the ability to internalize concepts and apply them to other learning situations. Students move through different levels of abstraction in their developing years. Therefore, the teacher needs to aid students through these levels by providing experiences and developing language to ensure the abstraction of concepts (Driscoll, 1980).

Hands-on learning experiences with physical materials are one method to aid students in internalizing concepts in mathematics (Driscoll, 1980). Hands-on experiences have been used for many years to help develop meaning and understanding (Reys, Suydam, & Lindquist, 1992). For example, play money has been used to develop concepts such as making change or spending money, and base-ten blocks have been used to develop counting skills. Moreover, real-life objects such as coffee cans and boxes have been used to represent cylinders and cubes. From these experiences, students develop formal definitions of concepts (Curcio, 1985). The definitions developed through hands-on experiences are more likely to be internalized by the students and applied to other situations than definitions that have only been memorized. In shifting from a hands-on to another learning environment, students sometimes are unable to apply the mathematical concepts they have learned (Sowell, 1989). For example, students who have learned to subtract two-digit numerals that involve borrowing are unable to apply the concept of borrowing when presented with problems of three-digit numeral subtraction. To successfully apply knowledge between environments, students must be able to apply the mathematical concepts they have learned to a new situation that requires the use of the concepts to solve the new problem (Sowell, 1989).

Statement of the Problem

The results of several studies indicate the need to emphasize the use of mathematical language to develop understanding (Tracy, 1994; Durkin, 1991; Mumme & Shepherd, 1990). These investigations have demonstrated that developing mathematical language facilitates higher achievement levels, increased in communication, and the emergence of mathematical concepts (Davis, 1994; Souviney, 1993; Hiebert & Wearne, 1993; Reineke, 1993). Thus, student learning in mathematics can be enhanced by the development of mathematical language.

Because geometry instruction has expanded in content and materials, various instructional environments are being used to expand understanding. Two of these environments are the use of hands-on materials and computers. Hands-on materials are used to introduce visualization and manipulation skills (Reys, Suydam & Lindquist, 1992). The computer can reinforce and further develop visualization and manipulation skills in a more abstract environment. Moreover, computer software programs can provide opportunities to develop more complete understandings of geometry concepts.

Educators need to examine geometry instruction in both hands-on and computer learning environments. Common in both environments is the mathematical language students use to develop understanding. Little research exists on the development and use of mathematical language to facilitate geometry learning in hands-on and computer learning environments. Moreover, an in-depth investigation into students' development of mathematical language in a hands-on environment and their application of that language in a computer learning environment may provide illumination into the role of language in students' conceptual development of mathematics.

Purpose of the Study

Driscoll (1981) believed that language is the key to the application of knowledge among learning environments. The purpose of this study was to investigate the development and application of mathematical language in geometry instruction. Within this purpose, there were three specific research goals.

1. To identify and implement experiences that aid the participants in developing mathematical language in geometry.
2. To examine the use of mathematical language as a method to develop participants' understanding of geometry concepts.
3. To describe the participants' use of mathematical language as they apply their geometry knowledge in a hands-on and a computer learning environment.

Research Question

When students are taught three-dimensional geometry concepts through the use of mathematical language in a hands-on learning environment, do they apply the developed mathematical language to assist in understanding three-dimensional geometry concepts in a computer learning environment?

Definition of Terms

Three-dimensional Objects

The six shapes used to illustrate three-dimensionality. These shapes are torus, cylinder, cone, square, sphere, and super spheroid.

Object Manipulation

The maneuvering of three-dimensional objects in one of three methods: position, rotation, or scale.

Real-life Objects

Hands-on materials used to represent three-dimensional objects. The following is a list of these objects:

sphere - ball	square - paper
torus - bagel	cone - party hat
cylinder - coffee cans	super spheroid - shoe box, dice

Three-dimensional Structures

The combining of three-dimensional objects to create one structure. This refers to construction with hands-on materials or computer-based objects.

Mathematical Language

The specific mathematical terms and phrases students use to communicate their knowledge of three-dimensional geometry. Examples of possible three-dimensional geometry terms and phrases are included in Appendix I.

Application

The ability to use mathematical knowledge to solve new problems. The use of mathematical concepts in an environment or learning situation different than the one in which the concept initially was learned.

Hands-on Environment

The use of physical manipulatives to learn the concepts of three-dimensionality. This environment allows students to develop three-dimensional mathematical language and use the language to solve problems.

Computer Environment

The use of a computer software program to apply the three-dimensional geometry concepts. Wireman is the computer software program used as an environment for participants to apply their knowledge of three-dimensionality.

After-school Mathematics Program

The ten sessions conducted with the participants for an hour after-school each day for two weeks.

Summary

In recent years, mathematics instruction has undergone many changes. Professional mathematics organizations have been at the forefront of these changes. Both what is taught and what depth concepts are taught have been the focus of several discussions. Two areas of change include the emphasis on the use of mathematical language and the amount and methods of geometry instruction.

Because students view mathematical terms as complex and specialized, many terms in mathematics are often memorized. The development of mathematical language will enable students to move beyond memorization to expand their understanding of mathematical concepts. As a result of mathematics education reform, geometry instruction is to be taught at all grade levels and include instruction on three- as well as two-dimensionality.

This study used recent reforms in mathematics education as a framework for the development and use of geometry mathematical language. Viewing mathematical language as a tool for learning concepts, this study used a hands-on learning environment to develop three dimensional geometry language and describe the application of geometry language in a computer learning environment to further develop geometry knowledge.

CHAPTER II. LITERATURE REVIEW

Geometry is an academic subject in which the development of mathematical language may assist student learning. By focusing on the nature and definition of language, a definition of mathematical language can be developed through applying the definition of language to the subject of mathematics. Furthermore, the development of mathematical language is similar to the development of language in other subject areas. This chapter is divided into two major sections. The first section summarizes contemporary perspectives and the research literature on mathematical language and mathematical language development. The second section is a discussion about the use of language in hands-on manipulative and computer learning environments. To set the stage for these sections, an overview of geometry instruction is presented first.

Geometry

According to the Mathematics Dictionary (James & James, 1992), geometry is "the science that treats the shape and size of things." Many terms, such as parallel, symmetrical, and equilateral, are used to describe these "shapes and sizes." Researchers divide school geometry into dimensions to assess the topics geometry instruction should include. Reys, Suydam, and Lindquist (1992) identified two dimensions of geometry: solid and plane geometry. Solid geometry focuses on three-dimensional objects; plane geometry focuses on two-dimensional objects. Both types of objects are studied to describe and classify their properties. Examples of properties in two- and three-dimensional geometry include the number of sides, length of sides, and shape of sides. Concepts in three-dimensional geometry are related to concepts in two-dimensional geometry to aid in student understanding.

Usiskin (1987) classified geometry into four dimensions: 1) measurement-visualization, 2) physical-real world, 3) representation, and 4) mathematical-underpinnings. Measurement-visualization refers to the visualization of objects and shapes. Physical-real world includes studying the properties of real world objects. Representation focuses on geometry's relationship to other mathematical concepts, and mathematical-underpinnings centers on geometry as a section of the mathematical world. Topics within geometry most often do not fit in one dimension only; they are multi-dimensional. Thus, geometry instruction should include experiences in all four dimensions.

Two professional organizations, the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics, have defined what geometry skills need to be taught at the various pre-college grade levels. These organizations based their findings on research, classroom practice, and societal needs. In the late 1980s, the National Council of Teachers of Mathematics (NCTM) published K-12 curriculum and professional standards for mathematics education reform. In both the K-4 and 5-8 standards, geometry is included as one of the strands of instruction (Reys, Suydam, & Lindquist, 1992). Geometry is identified as an important aspect of students' knowledge and everyday lives. According to the NCTM standards, understanding and applying geometry is an important skill that students need to be able to use effectively. Some geometry skills that students should develop include modeling, classifying, and defining shapes, developing relationships among various geometric figures, and applying geometry to everyday life.

The National Council of Supervisors of Mathematics (NCSM) identified twelve components of essential mathematics including problem solving, mathematical reasoning, estimation, and measurement (Reys, Suydam, & Lindquist, 1992). Geometry is one of the twelve components of essential mathematics. Three specific instructional objectives were described within the geometry component. Students need to: 1) gain understanding of geometric concepts that are necessary to function in the three-dimensional world, 2) verbalize and visualize various objects and describe their motion, and 3) explore geometric concepts in problem solving situations (Reys, Suydam, & Lindquist, 1992). The NCSM considers these topics to be essential and necessary to function in our current technological society.

Because of traditional approaches in mathematics education, a well-defined geometry curriculum does not exist at the elementary level, and beyond a one-year course for secondary students, geometry is not reinforced in the high school and college years (Usiskin, 1987). The teaching of geometry throughout K-12 schooling may facilitate student learning in all areas of mathematics. Geddes (1992) stated that studying geometry helps develop other mathematics concepts and divergent and critical thinking skills. Because geometry skills are interrelated to other mathematical ideas, geometry helps increase mathematical understanding (Geddes, 1992). For example, geometry concepts can be associated with fractions, decimals, number concepts, ratio, and probability (Dolan, 1991).

Geometry can help develop divergent and critical thinking skills. In using geometry, students may not take the same direction to complete a problem. This helps students learn that there can be more than one way to solve a problem. As students solve problems, they learn to reason their way through the problems and explain their investigations to others. Also, depending on the methods of instruction used to teach geometry, critical thinking skills can be enhanced if students are encouraged to question answers.

Meaningful learning experiences for students are important in developing geometry skills. According to Geddes (1992), students should be given experiences in which they can discover different properties and relationships. These experiences may include modeling, mapping, and engaging in geometric activities whether it be in the physical world or the abstract world. Within these experiences, students need to be to able predict, test, and redefine their conceptions. To do this, students need to plan, create, sort, and classify to develop an understanding of geometry principles (Geddes, 1992). Careful selection of activities allows students to take responsibility for their learning of geometry and to develop the knowledge they need to apply these concepts to the real world.

Mathematical Language and Mathematical Language Development

Research about language provides the basis for understanding mathematical language and its roles in learning mathematics. Studying language development can also provide insight into methods for developing mathematical language. In this section, a definition and discussion of language and its use in the classroom is provided; it is followed by a discussion of mathematical language. Finally, approaches to language development and their implications for the development of mathematical language are discussed.

Language

Lemke (1990) described language as "the system of resources for making meaning". Sainsbury (1992) defined language as "elements in a whole system of interpersonal communication within a shared context." Both definitions imply that language is an essential element in the communication process. Language is used in many situations, at many different times, and among all people. Therefore, understanding and being able to use language is an important skill that everyone needs to learn. Developing language skills begins at an early age and is a continual process. Formal education is one environment for language to be used and developed. Language use in education can take on several forms and play many roles.

Language in the classroom is prevalent in four forms: speaking, listening, reading, and writing. All of these forms interact with each other and are essential in the communication process (Anders & Pritchard, 1993). In the classroom, speaking and listening focus on the oral part of instruction, and reading and writing focus on the written aspect of instruction. All four forms of language have their role in the school curriculum. Speaking provides students with a chance to talk through their ideas with others; listening allows students to process what others are saying; reading involves understanding what is written in books and other resources; and writing allows students to clarify their ideas by inscribing them in document form.

By analyzing the research literature on language's role in the classroom, three primary roles have been identified: communication, clarification of thinking, and conceptual development (Wilkinson & Calculator, 1982; Curcio, 1990; Lesh, Landau & Hamilton, 1983). Using language in the communication process is its first role in the classroom (Wilkinson & Calculator, 1982). Using language effectively to communicate is essential in the classroom. Both teachers and students use language to express their ideas with each other. Teachers facilitate instruction and learning through language, and students provide interaction during learning through language. The classroom is an environment for this constant interaction and communication among students and teachers (Hills, 1986). Therefore, effective use of language is essential to meet the defined objectives of classroom instruction (Wilkinson & Calculator, 1982). An example of the importance of language in communication is evident when new concepts need to be taught to students. In this situation, the teacher must assess whether or not students have the needed experiential background and language to understand the new concepts (Morine-Dershimer, 1985). If they lack this background and language, misunderstandings tend to occur.

The second role of language in the classroom is to assist in the clarification of one's thinking (Curcio, 1990). Questions are asked in classrooms to have students' clarify their thoughts and ideas (Martino & Maher, 1994). Students use language to explain their knowledge of the concept. They can write their answers, listen to others' answers, discuss their explanations, or read others' explanations depending on the learning environment. Also, students develop questions to be asked of other students and teachers (Martino & Maher, 1994). Similarities and differences between students' answers to the questions can be discussed. After this questioning, the students have the opportunity to re-evaluate their thinking. Questioning of students and teachers can occur in any of the four forms of language: speaking, listening, writing, and reading.

The third role of language in the classroom is to foster the development of conceptual models. Conceptual models are defined as networks of meanings and relationships used to make decisions when learning a concept (Lesh, Landau, & Hamilton, 1983). Conceptual models involve three interacting components: 1) students' understanding of the concept, 2) the written and/or spoken language within the concept, and 3) the clarification of understanding when the concept is related to the real world (Lesh, Landau, and Hamilton, 1983). Language is prevalent in all of these components. Students use written or spoken words to apply their understandings to models and real world events. Therefore, knowledge of the language of a concept is important in developing relationships among meanings and terms within the concept. For example, to understand the concept of subtraction, students need to identify language such as "take away," "borrow," and "minus". Knowledge of these terms is essential to understanding the concept of subtraction. The terms identify the problem as one which will incorporate subtraction. Students may not develop a complete understanding of the concept without knowing and using the terminology of the concept.

Language in the classroom plays a vital role in teaching and learning. Because language aids in communication, clarification of thinking, and conceptual development, its use in different academic subject areas is essential for student learning. Mathematics is a subject in which student learning could benefit from the development of its language.

Mathematical Language

In recent years, there has been a call for reform in mathematics education. In the late 1980s, the National Council of Teachers of Mathematics published professional and curriculum standards to help guide this needed reform. These standards were published for grades K-12. Within each of these sets of standards, there were five common learning goals identified. These five goals were for students to: 1) learn to value mathematics, 2) reason mathematically, 3) communicate mathematically, 4) become confident in their mathematical abilities, and 5) become mathematical problem solvers (NCTM, 1989). The third goal, communicating mathematically, is the focus of this research. Language, in all its forms, is an essential element of communication; therefore, mathematical language needs to be fostered to achieve this goal. The following description provides a definition of mathematical language and its role in mathematics.

Mathematics is one of many subject areas that relies on the correct use and understanding of its terminology and language. Mathematics language is defined as "ways of representing, thinking, talking, agreeing, and disagreeing" when using different mathematical tools to enhance knowledge (Reineke, 1993). Mathematical language is a key component in mathematical communication; its principal function is to transmit meaning and understanding in a mathematical world (Durkin, 1991). Many mathematical terms have multiple meanings; often having one meaning in a mathematics environment and a different meaning in everyday life. Mathematical terms used in everyday language can be understood through context while the same terms used in a mathematics context have different meanings (Cuevas, 1990). Mathematical language can be used in several ways to enhance one's mathematical knowledge. In mathematical processes, students use language to aid in clarifying, identifying, refining, and making connections within different mathematical concepts (Mumme & Shepherd, 1990). Without this mathematical language, students may have problems expressing and developing their understanding of a concept.

In summary, mathematics language is defined as the use of its terminology to communicate understanding. Students use mathematical language in the classroom to explain and clarify mathematical ideas. Therefore, the development of the mathematical language is a significant process that needs to be investigated in order to better prepare students to communicate mathematically.

Language Development

Curcio (1990) identified a method for language development which is referred to as a language-based approach to teaching and learning. The goal of this method is to integrate the language students experience into a meaningful context. For example, Curcio (1990) applied this method to the teaching of fractions. If students were learning about fractions, they would use words such as one-half, common denominator, and parts of a whole while completing an activity on dividing pizzas into equal parts. This example illustrates that mathematical language can be learned in a meaningful environment. The objective of the language-based approach method of language development is to have students develop discipline-specific language to use as part of their everyday language when needed (Curcio, 1990).

In the language-based approach, students need to participate in language development activities. Anders and Pritchard (1993) reported that language development instruction needs to occur in all forms of language: speaking, writing, listening, and reading. Activities that involve opportunities in these forms help students to develop their own meanings and understandings (Anders & Pritchard, 1993). These activities allow students to actively participate in their own learning by discussing their thoughts and ideas. As a result, the language the students develop through these activities becomes a means to communicate with other students and teachers in all subject areas.

Mathematical Language Development

Mathematics is one of many subjects in which language development aids to facilitate understanding (Tracy, 1994). In all concepts and components of mathematics, students need to have the opportunity to read, write, listen, and speak mathematics (Capps & Pickreign, 1993). Pimm (1987) contended that part of learning mathematics is to speak like a mathematician in order to develop a complete understanding of a concept. In speaking like a mathematician, students use mathematical language as they are learning. Three topics need to be addressed to provide insight into mathematical language development: 1) methods of mathematical language development, 2) difficulties that occur during mathematical language development, and 3) research results on mathematical language development.

Methods of Mathematical Language Development To develop mathematical language, students should be given opportunities to communicate mathematically. In applying the language-based approach of language development to mathematics, students should develop and use mathematical terms and language in a meaningful context in mathematics class (Curcio, 1990). For example if adding money is the mathematical topic being studied, then the teacher may have the students simulate buying items at the grocery store. The grocery store would be the meaningful context in which the student could count change.

A variety of instructional activities can be used to provide students with opportunities to use all four forms of language to develop mathematical language. Examples of such opportunities to develop mathematical language include having students: work in groups and communicate methods they have used to derive answers; write about mathematics in journals; or create their own mathematics glossaries (Capps & Pickreign, 1993; Wilde, 1991). Easley, Taylor, and Taylor (1990) suggested several methods for incorporating dialogue and communication in the mathematics classroom. These methods included encouraging students to express themselves, allowing students to work through their difficulties without the teacher providing the answer, encouraging constructive discussions on alternative answers, and using written work of students to evaluate learning. Mathematical learning environments should incorporate these experiences to facilitate the development and application of mathematical language.

Difficulties in Mathematical Language Development Difficulties in acquiring mathematical language can occur. Problems in development must be addressed so students are able to overcome them. Four common difficulties may arise when students are developing mathematical language (Dickson, Brown, and Gibson, 1984). The first two difficulties relate to reading and comprehension. Nearly one-third of errors in mathematics are caused by reading comprehension problems (Garaway, 1994). Students cannot be expected to solve a problem if they are unable to read the entire problem or understand the mathematical language used to convey the problem. Inability to read the problem increases the chances of students' not being able to comprehend to the problem.

The third and fourth difficulty in developing mathematical language relate to transforming word problems into mathematical operations and attaching meaning to symbols (Dickson, Brown, and Gibson, 1984). Students must be able to identify the operation implied in the mathematical terms and language of a word problem in order to translate the words into the appropriate mathematical operations. Inability to do this significantly decreases a student's chance of solving the problem correctly. In addition, students often experience difficulties in connecting a concept to a mathematical symbol. Therefore, they may miscalculate an answer, describe a shape incorrectly, or incorrectly transfer symbols.

Research on Mathematical Language Development Several researchers have examined language development in mathematics. Many studies have focused on creating activities and experiences that allow students to develop mathematical language. The activities were then assessed to analyze the role of mathematical language. The assessment instruments used in these studies include classroom observations, interviews, taped conversations, and samples of student work (Hiebert & Wearne, 1993; Reineke, 1993; Souviney, 1993; Davis, 1994). The following is a summary of several studies that have examined the development of mathematical language and its effect on student learning.

Hiebert and Wearne (1993) conducted a study on instructional tasks, classroom discourse, and student learning. The goal of the study was to provide descriptions of what occurred in student learning after the students completed the instructional tasks. Two second-grade classrooms were used: one classroom was taught traditionally using their regular mathematics textbook and the other classroom was taught using a method in which students used language to develop relationships between what they were learning and what they knew. The mathematics concept taught was place value. Through written assessments and classroom observations, Hiebert and Wearne (1993) found that the students taught with an emphasis on mathematical language gave longer responses, showed higher levels of performance, and spent more time on each of the problems than students taught through traditional methods. Therefore, the enhanced learning of the experimental class was found to be a function of the language-rich mathematical environment created. When students used mathematical language to develop and articulate relationships, significant gains in learning occurred.

Reineke (1993) examined the use of a discussion-based approach with fourth graders to facilitate students' thinking about simple mathematical functions. Using classroom observations and student interviews, Reineke (1993) conducted the study over a period of ten weeks. Removing the traditional methods of teaching mathematics, the teacher emphasized the development of mathematical understandings and connections. Upon completion of the unit, students in the class successfully went beyond knowing just the mathematical terms to engaging in mathematical discussions during the lessons. The development of mathematical language allowed students to converse with their peers about mathematical concepts; this discussion-based approach effectively facilitated students' understanding of mathematical functions.

Davis (1994) examined the change of the role of the mathematics teacher from teller to listener in seventh graders studying fractions. For the study, two types of classrooms were identified: a listening and a telling. In the listening mathematics classroom, students engaged in mathematical conversations with other students and the teacher. The teacher was there to facilitate the conversations in order to aid learning. Similar to traditional classroom instruction, the telling mathematics classroom relied on the teacher as the primary resource for learning new information. The teacher disseminated the knowledge for the students to absorb. Davis (1994) found that the listening classroom provided an atmosphere that was more conducive than the telling classroom for the development of mathematical relationships and the emergence of mathematical concepts. In the listening classroom, students consumed the knowledge and then used this knowledge to construct their conceptual understandings of fractions. Encouraging students to develop and use mathematical language produced significantly positive results in their learning of mathematics.

Souviney (1993) conducted a case study on the relationship between mathematics achievement, cognitive development, and language. This case study involved students in grades two, four, and six at five different schools. The researchers developed trial instructional lessons and trained the teachers to use the lessons. Classroom observations, cognitive development tests, and achievement tests were used for assessment. The researchers found that language used in the classroom and in textbooks affects mathematics learning. Mathematics achievement and cognitive development increase when mathematical language appropriate for the topic is used.

The results of these studies indicate that development of mathematical language can have a positive impact on students' conceptual understanding of mathematics. Through the creation of language-rich educational environments, students developed and used mathematical language to enhance their understanding of mathematical concepts.

Mathematical Language Development and Use with Manipulatives and Computers

Mathematical language can be used in a variety of learning environments that differ in levels of abstraction. Two of these environments are a hands-on manipulative environment and a computer environment. This section consists of a discussion of the development and use of mathematical language in each of these learning environments.

Manipulatives

Manipulatives are materials used to aid instruction and to link the concrete to the more abstract representations of a concept (Reys, Suydam, & Lindquist, 1992). Examples of manipulatives in mathematics include base-ten blocks and abacuses. Manipulatives are chosen based on what is perceived to be effective in the learning process. Capps and Pickreign (1993) contended that "manipulatives can aid in the development of mathematics language." Beginning the learning process with concrete representations of concepts is essential for successfully moving learners to symbolic and abstract representations of concepts. Language becomes a key component in an environment that includes the use of manipulatives (Capps & Pickreign, 1993). Mathematical language to be used in future discussions of mathematical concepts is developed while using manipulatives (Holden, 1987). In the research literature, three outcomes for language use with manipulative environments were identified: 1) developing meaning, 2) building connections, and 3) assessing understanding.

To communicate mathematically, students need to develop an understanding of terminology. Language use with manipulatives aids students in developing meanings for complex and specialized terms in mathematics (Capps & Pickreign, 1993). That is, as students work with manipulatives and discuss what they observe, they develop meaningful definitions of the terms.

Using language with manipulatives also aids students in building connections among different representations (Capps & Pickreign, 1993). Mathematics involves work in many environments including physical, pictorial, graphic, verbal, and mental. Language provides the foundation that is needed at all levels to understand the mathematical concepts (Capps & Pickreign, 1993). Because mathematical language often is not reinforced outside the mathematics classroom environment, mathematical language needs to be used within all of the environments of mathematics instruction (Pimm, 1987).

Having students use language with manipulatives also provides a means for assessing students' understanding (Capps & Pickreign, 1993). Instruction with manipulative materials usually occurs at the beginning of the learning experience. As students are discussing their discoveries with their peers, the teacher can diagnose errors in their understanding as conveyed by their language (Capps & Pickreign, 1993). Because student learning is still in the developmental stages, steps can be taken to help students restructure their understanding to ensure future success.

Using manipulatives as a framework for mathematical conversations aids students in developing meaning, building connections, and providing a means for teacher assessment of student understanding. Success in using manipulatives to develop mathematical language may assist students in developing more abstract and complex conceptions of mathematical phenomena.

Computers

The presence and use of computers in the classroom has increased dramatically in the past fifteen years (Becker, 1991). Because of this increase, researchers have investigated several issues about the use of computers in the classroom. One such issue is how language is used in a computer learning environment. Two types of language interactions occur with computers: multiple students with computer, and computer with computer.

The first type of language interaction with the computer involves more than one student working with a single computer. Students use their language to communicate with each other about the tasks to be completed on the computer. Hoyles, Sutherland, and Healy (1991) studied the role of discussion while using the computer in a mathematics environment. They discovered that what students discussed affected their interaction with the computer. The language students used was critical in overcoming problems that occurred while using the computer. Several factors influenced the amount and type of discussion that occurred during group use of computers. Mercer (1994) identified the location of the equipment, the software used, and the nature of the task as three of these factors.

The computer as a medium for communication is the second type of language interaction that occurs with computers. Kelly and Wiebe (1994) reported their use of Internet in the mathematics classroom as a means for student use of electronic mail to solve mathematics problems collectively with students from around the world. Written language was used to communicate via the computer with other students. Communicating via electronic mail required students to use and understand mathematical language in order to discuss solutions to problems. Bulletin boards and databases are other Internet resources where students must use their mathematical language to interpret data and collaborate with others.

Computers provide environments for students to engage in discussions with students in the same classroom or students in classrooms around the world. In both interactions, students used the computer as a tool to stimulate discussion and solve mathematical problems and as a means of mathematical communication.

Summary

The use of mathematical language can occur in different topics of mathematics; geometry is one of these topics. Geometry instruction should include activities to classify, define, and clarify geometric figures, to apply geometry to everyday life, and to visualize and describe geometric motion. Geometry knowledge assists in developing other mathematical concepts as well as critical and divergent thinking skills. The development of mathematical language in geometry has the potential to aid in student learning.

In recent years, emphasis on the development of mathematical language has greatly increased. Research results indicated that the development and use of mathematical language aids students in identifying relationships among concepts, communicating effectively with peers, constructing individual knowledge, and improving mathematics achievement. Thus further study on the development and use of mathematical language may help educators better understand the role of and incorporate mathematical language in instruction.

Both hands-on manipulative and computer learning environments were identified as two instructional settings in which the development and use of mathematical language can occur. The hands-on learning environment can provide effective introductory learning experiences with concepts. The use of language to discuss occurrences can help students conceptual meaning. A computer learning environment allows students to communicate with each other about a task they need to accomplish on the computer or with others via computer electronic mail to collectively solve problems.

CHAPTER III. METHODOLOGY

The purpose of this chapter is to describe the methodology used during the after-school mathematics program to describe the participants' development and application of mathematical language in the hands-on and computer learning environments. This chapter includes five sections: 1) participants, 2) research design, 3) instruments, 4) research procedure, and 5) data analysis. The method of selection of the participants and an explanation of the research design and are provided first. The instruments and research procedure for the after-school mathematics program are then described followed by an overview of the data analysis of procedures used to answer the research question.

Participants

The participants for this study were seven sixth grade students from a middle school in central Iowa. The participants consisted of three females and four males. Four days prior to the expected start date of the after-school mathematics program, the instructor/researcher gave the classroom teacher ten permission slips and requested that she select six students to participant in the study. In class, the teacher announced to her students that a two week after-school mathematics program was going to be conducted by a college student from Iowa State University. She told them that six participants from the class were needed. To identify a pool of candidates to participate in the study, the teacher asked the students three questions: who could participate in a program after school for two weeks? who could get a ride home after the program? and whose parents would allow them to participate? Ten students who met these requirements took the permission slips home to obtain parental approval to participate. Approval for the participants to cooperate in the program needed to be granted by the parents/legal guardians. Nine students indicated their interest in participating in the after-school mathematics program by returning the signed permission slips. The selection of the final six participants was determined by the teacher who selected the students based on overall interest and attitude towards school and mathematics in particular. A seventh participant was added when one student and his family expressed a great deal of interest in being involved in the after-school mathematics program.

Research Design

The research design chosen for the after-school mathematics program was a descriptive case study. A single definition of a case study was not identified in the literature, rather a variety of definitions of this research approach are reported in the research literature (Lichtman, 1993; Cashman & McCraw, 1993). Stake (1994) identified three types of case studies: intrinsic, instrumental, and collective. An intrinsic case study focuses on a given case. The case is studied in-depth to provide deeper understanding into the specific case. In an instrumental case study, the case is often secondary. The case is chosen because it relates to something larger; the case is studied to provide understanding into a larger phenomenon. The collective case study is the study of multiple cases on a specific phenomenon. More than one case is studied to provide greater insight into the phenomenon.

For the after-school mathematics program, an instrumental case study approach was used. The case, middle school students, was chosen to provide insight into a larger phenomenon, the development and application of mathematical language to foster geometry learning. The case for this research study was the seven middle school students participating in the after-school mathematics program. The seven participants were from the same central Iowa middle school, nearly the same age, and enrolled in the same sixth grade classroom.

As previously stated, the case of an instrumental case study has a secondary role; that is, the study of a particular question and just the case itself is the primary focus (Stake, 1994). The primary interest in this case study was the development of three-dimensional mathematical language. The research question for this case study was: will students apply three-dimensional geometry language developed in a hands-on learning environment to a computer learning environment?

The goal of case study research is to provide a detailed representation of an environment so the reader can gain a clear understanding of what occurred (Cashman & McCraw, 1993). In this descriptive case study, the goal was to describe the three-dimensional mathematical language the participants developed in the hands-on learning environment and the mathematical language they applied in the computer learning environment.

Instruments

Many studies investigating the development and application of mathematical language used a case study approach. In these studies, the researchers primarily used classroom observations, interviews, and samples of student work as methods of data collection (Hiebert & Wearne, 1993; Reineke, 1993; Souviney, 1993; Davis, 1994). For the case study reported in this thesis, the following data gathering methods were used: background questionnaires, participant interviews, discussions, group observations and participant journals.

To aid in the clarification of meaning and decrease misinterpretation of data, the use of triangulation is often used in case study research. Triangulation is defined as the use of multiple sources of data to understand the same phenomenon (Glesne & Peshkin, 1992; Ary, Jacobs, & Razavieh, 1990; Stake, 1994). Triangulation of data in this study was established through the use of participants' journals, whole and small group discussions, group observations, and interviews. Both individual and group data was collected of the participants during the after-school mathematics program.

Background Questionnaire

During the first lesson of the after-school mathematics program, a background questionnaire was given to the participants to complete (Appendix A). Items in the questionnaire asked their age, their interests both inside and outside of school, and their plans after graduating from high school. Each participant individually completed the twelve questions by writing their responses. Information collected from this document provided the instructor/researcher with background data on the participants. Responses to the questions were used as a tool for the instructor/researcher to become familiar with the participants and to describe each participant and the research setting.

Participant Interviews

Each participant was interviewed twice during the after-school mathematics program. The purpose of the interviews was to provide an opportunity for the instructor/researcher to have one-on-one contact with each participant to gather data with regards to their experiences in the after-school mathematics program. The interviews were used to begin and end the after-school mathematics program. The first interview occurred in the first lesson and was used as a method to get to know the participants' and their perceptions toward mathematics. Based on Spangler (1992), five open-ended questions about mathematics were identified. The instructor/researcher asked the participants the five questions in fifteen minute individual interviews. The first interview questions are reported in Appendix B.

The instructor/researcher conducted the second set of interviews during the tenth lesson; each interview was approximately ten minutes. Each participant's responses to the interview questions were used to describe the student's knowledge and application of mathematical language. Four questions were used to encourage the students to communicate about their experiences. The second interview questions are identified in Appendix D. The questions were asked of each individual and were used because they related to the purposes of the study.

Both interviews were audiotaped and then transcribed. Notes were also taken by the instructor/researcher during the interviews. All of the interviews were conducted by the instructor/researcher.

Discussions

Two types of class discussions were used in the instructional unit: whole and small group. Whole group discussions involved all of the participants and the instructor/researcher in one group. Small group discussions involved two or three participants per group. The instructional activity dictated the type of discussion the instructor/researcher facilitated, and multiple discussion types were used with some activities. Questions formulated for the lessons and asked by the instructor/researcher served as the catalyst for the discussions. Examples of these questions are included in the daily lesson plans in Appendix E. Each discussion during the after-school mathematics program was audiotaped and then transcribed. Notes were also taken by the instructor/researcher as the participants interacted.

Observations

During the after-school mathematics program, observations of the participants were made by the instructor/researcher. Each lesson was videotaped and reviewed by the instructor/researcher to confirm the phenomena observed. Through observations, the instructor/researcher specifically noted participants' interactions with each other, participant interaction with the instructor/researcher, and participants' overall disposition during the after-school mathematics program. Information from this data gathering method was used to describe the participants (individually and collectively), their experiences, and the overall setting during the case study. The observation data, the responses from the first interviews, and the background questionnaire were used to set the stage for the data analysis process.

Mathematics Journals

Mathematics journals were used to collect written data from the participants. Based on the instructional objective of each lesson, journal questions were formulated by the instructor/researcher for the lessons in the hands-on environment and the lessons in the computer learning environment. The journal questions are reported in Appendix C. As defined by Wilde (1991), open-ended and process questions were used. Open-ended questions gave students an opportunity to write as much as they wanted and encouraged the participants to think through their responses. Process questions asked students to describe in detail how they would complete a specific task. Each participant responded to each journal question by writing in their mathematics folder; each response was dated. Participants' journal responses included illustrations, examples, definitions, and other written information.

Research Procedures

Gaining Access

Gaining access in a case study involves the process of acquiring permission to work with the participants and collect the data (Glesne & Peshkin, 1992).

Gaining access for the after-school mathematics involved acquiring permission from five sources: Iowa State University, the administration of the cooperating school, the sixth grade classroom teacher, the parents/legal guardians of the seven participants and the participants themselves. The Human Subjects Committee at Iowa State University reviewed and approved the research study's procedures and instruments. Permission from the cooperating school principal and the school district superintendent was granted. The sixth grade classroom teacher agreed to identify the students to participate, and parental permission was granted by each participant's parents or legal guardians. The participants gave their consent by attending the after-school mathematics program every day for two weeks and engaging in the program's activities.

The objective of the first lesson was for the instructor/researcher to get to know the participants and to introduce the participants to the mathematical environment of the after-school mathematics program. In addition to the background questionnaire, the instructor/researcher conducted individual interviews of each participant. Along with gathering information from the participants, the interviews allowed the instructor/researcher to gain each participant's trust and cooperation for the after-school mathematics program. This was essential given the nature of the study.

In a case study, the goal is to observe and, in some cases, become engaged with the participants in a natural setting (Ary, Jacobs, & Razavieh, 1990). In the after-school mathematics program, the instructor was the researcher of the study. By having the researcher serve as the instructor, the learning environment created was more natural and comfortable for the participants to discuss and engage in the activities.

In addition to the background questionnaire and the first set of interviews, the participants were to complete an activity on mathematics in everyday lives during the first lesson. Each participant was given a newspaper with which they were to identify ten articles where mathematics was discussed. This activity assisted the participants in placing mathematics in a meaningful, real-world context. The goal of the activity was to assist the participants in recognizing the presence of mathematics in their world and prepare them to realize the utility of geometry in everyday life.

Instructional Setting

Seven sixth grade students from a middle school in central Iowa volunteered to participate in the after-school mathematics program. The program was conducted over a two week period. The seven participants attended one hour sessions each day after school. The sessions were conducted in the regular classroom of the participants. All supplies and equipment for the after-school mathematics program were provided by the instructor/researcher. Supplies and equipment included hands-on materials, portable computers, video camera, mathematics journals, and audio cassette recorders. The instructional setting consisted of two learning environments: hands-on and computer. Following an overview of the instructional setting, a detailed description of each environment is provided.

The topic of the instructional unit for the after-school mathematics program was three-dimensional geometry and the development of mathematical language. The lesson plans for the unit are included in Appendix E. The goal of the lessons was to introduce and reinforce the concepts of position, scale, and rotation in three dimensionality using objects common to the participants. Throughout the after-school mathematics program, the intent was to provide the participants with a language-rich atmosphere to develop three-dimensional mathematical language and understanding of geometry concepts.

The instructional unit for the after-school mathematics program was divided into ten lessons; one lesson was implemented per day. Each lesson was approximately one hour in length. The following is an outline of lessons:

Lesson 1	• Introductory interviews and background questionnaire • Mathematics in everyday world activity
Lesson 2	• Three-dimensional objects and three-dimensionality
Lesson 3	• Scale of three-dimensional objects in three-dimensional space
Lesson 4	• Position of three-dimensional objects in three-dimensional space
Lesson 5	• Rotation of three-dimensional objects in three-dimensional space
Lesson 6	• Activity to incorporate concepts from Lessons 2-5
Lesson 7	• Computer basics • Wireman introduction
Lesson 8	• Computer activity to develop skill in using Wireman
Lesson 9	• Computer activity for assessment of the application of mathematics language and concepts
Lesson 10	• Final interviews

A constructivist approach to teaching and learning was used by the instructor/researcher to implement the lessons. Two main beliefs of constructivism were used to design the unit on three-dimensional geometry for the after-school mathematics program. The first belief of constructivism used to design the unit was "student responsibility for learning." In the unit, participants were responsible for constructing their own knowledge and meaning of three-dimensionality (Reys, Suydam, & Lindquist, 1992; Duffy & Jonassen, 1992). The instructor/researcher provided experiences for the participants to construct perceptions and meanings. These experiences were usually conducted at the beginning of the lesson to provide a framework for later discussions. Multiple meanings and perceptions were encouraged; there was not one meaning or perception that was viewed as correct. To further construct meaning and understanding, the participants discussed their thoughts and ideas with each other.

"Instructor as facilitator" was the other constructivist principle that directed the unit (Reys, Suydam, & Lindquist, 1992). Throughout the unit, the participants engaged in several discussions of three-dimensional concepts; these discussions helped the participants develop and use mathematical language. The role of the instructor/researcher was to facilitate these discussions. To aid the participants in their thinking, the instructor/researcher often asked questions. The role of the participants then was to create their own definitions and conceptual understandings from their experiences and the whole and small group discussions.

Hands-on and computer environments were provided to develop and apply the three-dimensional concepts. The hands-on learning environment was designed as a setting in which the participants could learn the three-dimensional concepts and develop their mathematical language. In the hands-on learning environment, physical manipulatives were used to assist the students in learning the concepts of three-dimensionality. The participants were to use the manipulatives to formulate their initial conceptual understandings. Then, their basic understanding of three-dimensional concepts was to provide the framework for discussion and communication. Through the discussions, the participants were to use their developed mathematical language to describe and further develop their understanding.

The computer learning environment was incorporated as a setting in which the participants could apply their mathematical language to complete tasks and further their understanding of three-dimensional geometry. In the computer learning environment, Wireman software was used to provide the participants with an environment to apply the three-dimensional mathematical language they developed in the hands-on environment. To complete the computer-based activities, the Wireman software program required the participants to have knowledge of the same concepts used in the hands-on environment. The participants were to complete specific tasks on the computer and record in their journals a description of the procedures they used to complete the tasks. These descriptions served as a basis for whole and small group discussions.

Hands-on Learning Environment Five lessons of the ten lesson unit focused on the use of hands-on materials. Four lessons were designed to develop conceptual understandings and meanings of three-dimensional geometry, and the fifth lesson was designed to have the participants incorporate their understandings of the preceding four hands-on lessons to solve a task. The format for each lesson included a review of the previous day's lesson, an introductory discussion, activities with manipulatives, follow-up discussions, and journal writing.

Lessons two through six focused on engaging the participants in hands-on activities to develop their three-dimensional mathematical language. Lesson two was used to introduce the participants to the concept of three-dimensional geometry through experiences with three-dimensional objects. Real-life objects such as basketballs, boxes, coffee cans, and toy rings represented three-dimensional objects. The participants were to analyze the objects to determine the number of faces, vertices, and edges. After this activity, a discussion of three-dimensionality occurred.

Lesson three was designed to develop the concept of scale in three-dimensional space. Different sizes of the same object were compared for their similarities and differences. Then, the participants were asked to identify ways to change the scale by discussing how a small object could be made into a large object. The goal was to have the participants realize different methods of changing the scale of an object.

Position of objects in three-dimensional space was the focus of the fourth lesson. Using string taped to the walls and ceiling, a simulation of three-dimensions was constructed in the classroom. Three different colors of string were hung in the room; each color represented a different axis. A ball was placed in a specific location in the string simulation, and the participants were to give the instructor/researcher directions to the location of the ball. The objective of the activity was for the participants to use mathematical language to describe movement in three-dimensional space.

The fifth lesson of the after-school mathematics program was about rotation of three-dimensional objects. With a participant on each side of a four-sided table, each participant was to drawn an object placed in the middle of the table from the four different views. After the first view was drawn, the instructor/researcher rotated the object, and the participants continued drawing. The participants were then asked about different ways to rotate an object. The goal of the activity was to have the participants identify ways to rotate objects similar to the ways of rotating an object around the x, y, and z axes.

The sixth lesson of the after-school mathematics program was to have the participants incorporate the concepts they had learned in the previous four lessons to complete a task. In this lesson, the task given to the participants was to create a structure using three of the three-dimensional objects about which they had previously learned, draw a picture of the structure, and write directions on how to create it in their journal. Then, the participants were to describe their structure to a partner so the partner could create the structure using play dough.

In summary, the goal of the hands-on activities was to have the participants develop their understanding of three-dimensional geometry through the development and use of three-dimensional geometry language. Hands-on materials provided common experiences and a foundation from which all of the participants could communicate mathematically with each other. Three methods were used to encourage the participants' use of mathematical language during the hands-on activities: 1) mathematics journals, 2) discussions and 3) hands-on materials. The mathematics journals provided the participants with an opportunity to write and read using mathematical language. Whole and small group discussions were to used to encourage students to speak and listen using mathematical language.

Computer Learning Environment Three of the lessons in the after-school mathematics program consisted of activities in the computer learning environment. The software program used was Wireman (Figure 1). Wireman was developed at Lawrence Livermore National Laboratories for the National Education Supercomputer Program. Wireman allows the user to manipulate three-dimensional wire framed shapes. Several options within the software program give the user the opportunity to change their view of the object and the object's position, scale, and rotation.

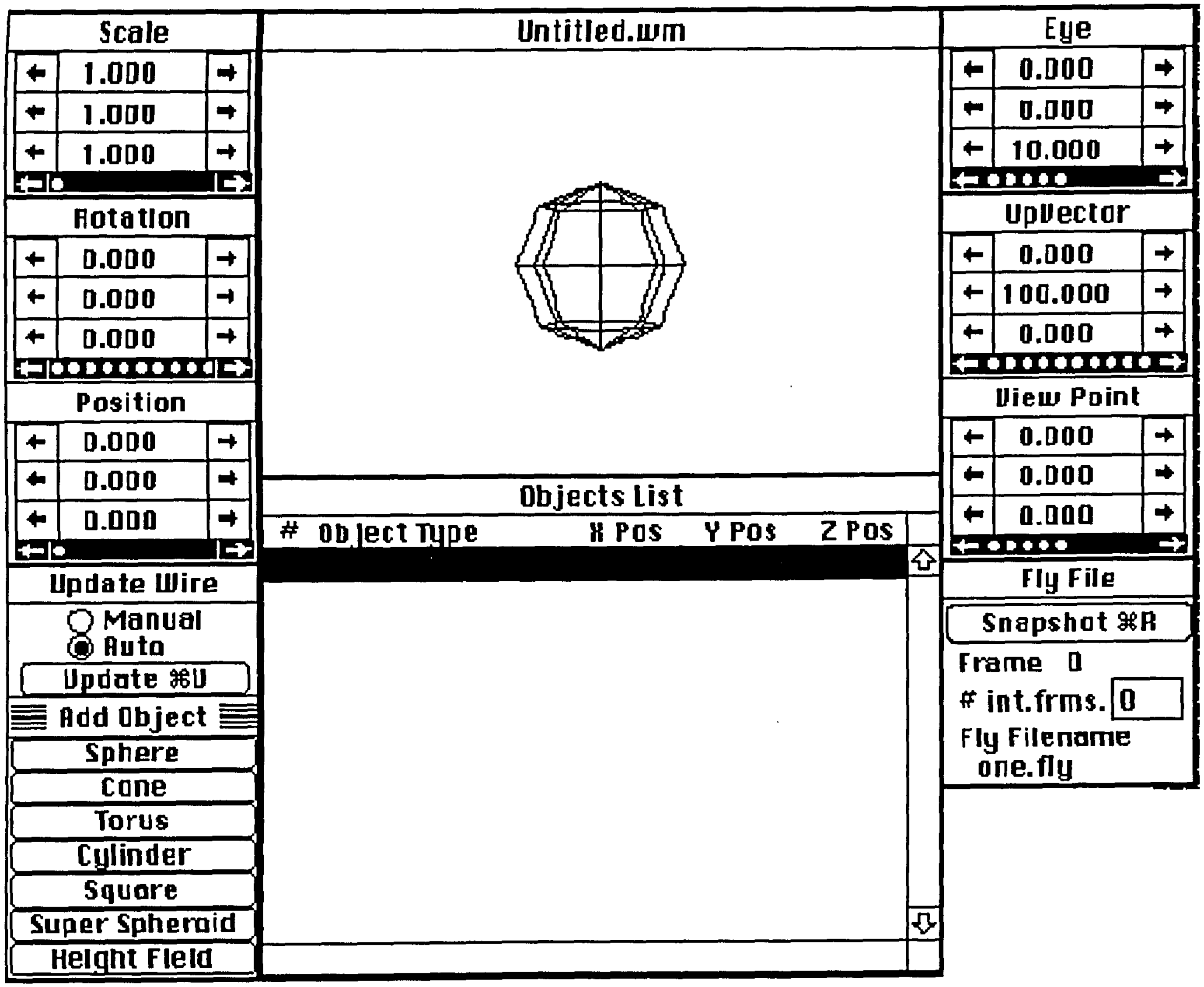

Figure 1. Wireman Computer Screen

Wireman activities were designed to encourage the participants to apply their knowledge of three-dimensional geometry and mathematical language in an environment different than the hands-on environment. Because Wireman provided abstract representations of three-dimensional geometry concepts, the instructor/researcher considered the computer to be an appropriate environment for the participants to apply the mathematical language they developed with the hands-on materials.

Lessons seven through nine were the computer activities. Lesson seven was an introduction of computer basics and the Wireman program. The participants were given an introduction to the computer; topics included in the introduction were turning on the computer, using the mouse, and opening files. After the introduction to the computer, the participants were given the opportunity to experiment with Wireman and to become familiar with its functions. To facilitate their experimentation, the instructor/researcher directed the participants to make the letter "T" using the various objects available in Wireman.

Lesson eight provided the participants with an opportunity to develop their skill in using Wireman. Two tasks were given to the participants to complete. The first task was to create an ice cream cone on the computer using the objects, position, scale and rotation tools in Wireman. Then, the participants were to change the ice cream cone into a person wearing a dunce hat (Appendix H). The second task the participants were given was to create a structure of a Lifesaver lollipop using the capabilities of Wireman (Appendix H). Both tasks provided the participants with opportunities of use scale, position, and rotation in completing the tasks.

The final computer lesson of the after-school mathematics program focused on gathering data concerning the participants' application of three-dimensional mathematical language in the computer learning environment. Much like they had done for the hands-on incorporating activity, the participants were to create a three-object structure using Wireman. After building the structure with the computer, they were to sketch it and write in their journal directions on how to create the structure. Using the written directions as guidelines, each participant orally instructed the instructor/researcher on how to design their structure in the computer environment. To avoid the participants' use of unclear phrases and directions, the instructor/researcher was the manipulator of the computer. When unclear directions were given, the instructor/researcher questioned the participants in a manner that would encourage them to clarify their directions.

To conclude the after-school mathematics program, lesson ten consisted of a second interview with each participant. Similar to the first interview, four open-ended questions were asked to provide an opportunity for the participants to orally discuss their participation in the after-school mathematics program (Appendix D).

Data Analysis

In this descriptive case study, data analysis occurred in five sections, a description of the: research setting, participants, mathematical language development during the hands-on activities, application of mathematical language during the computer activities, and participants' final perceptions and reactions to the after-school mathematics program. Consistent with the research question, the primary focus of the data analysis was the mathematical language. For reference purposes, a list of three-dimensional geometry language, terms, and phrases the instructor/researcher thought the participants may develop through the after-school mathematics program was compiled prior to the after-school mathematics program (Appendix I).

An essential element of case study research is to analyze and describe the setting in which the case exists. An analysis and detailed description of the instructional setting was conducted. The description of the setting was constructed using information from the classroom teacher and the classroom observations of the instructor/researcher. The description of the setting provided a basic understanding of the environment in which the research took place.

The description of the participants compiled from the data in the background questionnaire, first interview, classroom observations, and initial journal entry was included in the second section of the data analysis. This section consisted of three components; descriptions of: each individual participant, the groups' prior knowledge of geometry and use of computers, and the groups' perceptions of mathematics.

Data from the background questionnaire, individual interviews, and classroom observations of the participants' interests and behavior were used to formulate each participant's description. A portrayal of the group dynamics was also provided. To assess participants' prior knowledge of geometry and computers, the instructor/researcher used data from the background questionnaire and the first journal question. The third component of the participants' descriptions was an analysis of each participant's perceptions of mathematics as reported in the first interview. Themes in their responses were identified and used to report the findings.

The third section of the data analysis was a description of the development of mathematical language during the hands-on activities. Each of the hands-on lessons was reviewed for the use of mathematical language by the participants. Discussions of group activities, classroom observations, and journal entries were analyzed for the use of three-dimensional geometry mathematical language by the participants. The mathematical terms and phrases each participant used to convey mathematical ideas was reported. An overall summary of the three-dimensional geometry language used during the hands-on activities was given to provide a basis for understanding the utility of the instructional approach for the participants and to provide a framework for the analysis of mathematical language in the computer activities.

A description of the application of mathematical language during the computer activities was the fourth section of the data analysis. Each computer activity was described in terms of the use and application of mathematical language in the discussions, classroom observations, and the journal entries. The computer lessons were analyzed for students' application of the mathematical language developed in the hands-on activities. The language the participants used was compared to the language they used in the hands-on environment. A summary of the results of the application of the mathematical language in the computer learning environment was provided for each computer lesson.

The final section of the data analysis was a summary of each participant. Using the data from the second interviews as well as other previously gathered data, the instructor/researcher prepared a summary description of the three-dimensional geometry language development and application experience of each participant.

Summary

The purpose of this chapter was to explain the methodology used in assessing the research question about the development and application of three-dimensional geometry language. The research design for this study was a descriptive case study. According to Stake (1994), there are three types of case studies: intrinsic, instrumental, and collect. This case study used an instrumental case study because the case, middle school students, was used to obtain greater understanding of a larger context, mathematical language. Seven participants, selected by their classroom teacher, volunteered to participate in this descriptive case study.

During the after-school mathematics program, the participants were involved in a unit of instruction on three-dimensional geometry; the focus of the unit was to provide a language-rich environment for students to learn geometry. Hands-on activities were used to develop the mathematical language associated with three-dimensional geometry. Computer activities were incorporated as an environment where the participants could use and apply their mathematical language to complete specific tasks and further their knowledge of three-dimensional geometry.

In this case study, both group and individual data were collected. Group discussions, journal questions, observations, and interviews were the primary methods of data collection. These methods combined the use of group and individual data. The data analysis techniques common to case study research, such as triangulation, were applied to describe the participants' development and application of three-dimensional mathematical language.

CHAPTER IV. RESULTS

After ten days of implementing the after-school mathematics program lessons with the seven participants, the data collection was complete. The group and individual data collected were then analyzed in relation to the research question, and the results are presented in this chapter. A description of the setting and the participants is given to provide the framework for the other results. As was mentioned in chapter three, the after-school mathematics program was comprised of two components: a series of hands-on activities and a series of computer activities. Examples of participant written and oral responses are used to support the findings of the study. This chapter includes five sections: a description of the: 1) research setting, 2) participants, 3) hands-on activities, 4) computer activities, and 5) summary of participant experiences.

Research Setting

A rural middle school in central Iowa was the research setting for the after-school mathematics program. The middle school included four hundred thirteen students in grades five through eight. Two elementary and one high school were also part of the consolidated school district. The school day went from 8:05 until 3:05 except on Wednesday when classes were dismissed one hour early. The school district consisted of mostly white students from two income levels: lower and middle class.

At the time the after-school mathematics program was implemented, the middle school was being remodeled to fit the growing needs of the district. Four sixth grade classrooms existed in the school, each consisting of approximately twenty-five students. The sixth grade classroom of the participants switch with another sixth grade class. One teacher was responsible for science and mathematics instruction for both classes, and the other teacher instructed language arts and reading. Each teacher was responsible for their own social studies instruction.

The after-school mathematics program was conducted in the participants' regular classroom. Each session occurred after the regular school day and was approximately one hour in length. The classroom was organized in a traditional manner: rows of desk facing the chalkboard in the front of the room. Five IBM computers typically were located along the side wall of the classroom; although during the remodeling of the school, these computers were not located in the classroom. Throughout the program, the instructor/researcher often rearranged the desks to accommodate the after-school mathematics program activities; several extra desks from the classroom were not used for the after-school mathematics program and were moved, if needed.

Other equipment in the classroom included the camcorder for video taping the lessons, audio recorders for taping discussions, and the container of hands-on manipulatives. The camcorder was placed in the back of the classroom; the audio recorders were placed next to each participant; and the hands-on materials were in the front of the classroom. Seven portable Macintosh computers were brought for participant use during the after-school mathematics program. A computer was placed on each participant's desk during the computer activities.

The atmosphere for the program was less formal that of a regular school day. No penalty was given for those students who misbehaved. Different than traditional methods of instruction, the instructor/researcher typically sat in a chair near the participants who were seated in their desks. The instructor/researcher had full responsibility for the participants without much interruption from the regular classroom teacher. With the exception of one day, the regular classroom teacher was not in the classroom during the ten sessions; on the day she remained in the classroom, she was attending to a student in detention. Otherwise, all of the other students from the participants' class were either involved in after-school sports or had gone for the day.

Participants

The seven participants volunteered to be involved in the after-school mathematics program. No school credit was given for participation in the program. The reasons given for volunteering to participate included using computers, getting ahead of other students, love of mathematics, and the stipend the instructor/researcher provided for participation. The following is a description of each of the seven participants. In the first lesson, information about the participants was collected from individual interviews, a background questionnaire, and classroom observations. The interviews focused on the participants' perceptions of mathematics. The background questionnaire provided a framework for describing each participant's interests, likes and dislikes of school, and plans after high school graduation (It should be noted that completing high school was a personal goal and assumption held by each participant). The observations provided descriptive information about how the participants presented themselves, interacted with others, and their overall disposition towards the after-school mathematics program.

Sue

Sue was an eleven year old female participant. Her favorite subject was mathematics and study hall, and her least favorite subject was science. Sue enjoyed riding her bike, spending time with friends, and playing with her nephew. After graduation, she wants to pursue becoming a doctor. From observations, Sue seemed like an ordinary child; she dressed and behaved like any other sixth grade student. She appeared to be a quiet, well-behaved and a curious individual. While the other participants began working on the computer, Sue asked approximately four questions before she began. The types of questions she asked were usually about how to complete the activity. For example, Sue asked "Now, what are we supposed to do? What do you mean by that? Am I doing this right?" Asking these types of questions indicted to the instructor/researcher that Sue was an individual who was careful and conscientious in completing assigned tasks.

Linda

Linda was an eleven year old female participant. Mathematics, as well as band and science, were her favorite subjects. Her least favorite subject was social studies. In her spare time, she enjoyed playing sports and her clarinet. After graduating from high school, she hopes to become a veterinarian. Linda always dressed nicely. When her mother picked her up every day, she also was dressed nicely because she just came from work. In working with the after-school mathematics program activities, Linda became frustrated easily when she could not solve a problem. For example, during the introductory activity on the computer, she began to cry when everyone else completed the task before her. Linda said, "This will take us forever. It takes me awhile." From these statements, Linda appeared to possibly be self-conscious about herself and her ability to complete tasks. However, if given the time and the encouragement to finish a task, Linda was able to complete the task.

Mary

Mary was a ten year old female participant. While science was her favorite subject, reading was her least favorite subject. Her hobbies included playing softball and drawing. She hoped to one day be a veterinarian. Mary participated significantly during some lessons, but in other lessons, she did not say very much. For example, during her first interview with the instructor/researcher Mary provided detailed answers and gave five or six responses to the questions. Then, at her final interview, she gave one or two abrupt answers while mainly looking around the room at others. The thoughts and actions of others mattered to her. Mary dressed and acted like many other sixth grade student by wearing contemporary clothes and participating in school activities. Often, Mary was laughing at the boys who were misbehaving. She helped encourage their actions. In doing this, Mary seemed to want to be accepted and like by others.

Steve

Steve was a twelve year old male participant. Steve's favorite subject was mathematics, and his least favorite was reading. In his spare time, Steve enjoyed collecting baseball cards. He hoped to attend college after high school and major in an area that included mathematics and science. Steve often dressed in tee-shirts and sweatshirts which displayed different athletic teams. Steve was very attentive to what was being taught. He always had his hand raised when a question was asked, but if someone distracted him, he began to misbehave. He wanted to do activities beyond what the group was doing. When the instructor/researcher was describing the introductory computer activity, Steve was designing a person using different objects in the Wireman software program.

David

David was a twelve year old male participant. While English and language arts were his least favorite subjects, science was his favorite. Outside of school, David enjoyed watching and participating in sports. After high school graduation, he wants to become a Navy Seal. David was the tallest participant in the after-school mathematics program. He was very interested in sports both playing and watching. David often disrupted others and was talkative. Many times the instructor/researcher had to reprimand him for not paying attention. He seemed disinterested at times. For example, during the hands-on activity lesson to develop the concept of position, he kept moving his desk back to try to break the string simulation of a three-dimensional model. Originally, only six participants were included in the after-school mathematics program, but David really wanted to participate in the program. Therefore, after a discussion with school staff, he was allowed to participate.

Tim

Tim was a twelve year old male participant. Similar to Steve, Tim identified mathematics as his favorite subject and reading as his least favorite subject. He identified playing and watching sports as his hobby. After high school graduation, Tim wants to attend college. Tim's appearance was like any other sixth grade students; he wore the same clothes, his hair was styled the same, and he acted like others. Tim was a quick learner and often kept to himself. Because he was quiet and hard-working, the instructor/researcher often did not know that Tim was in the room. He was quiet during discussions, and he worked on the project that was given. When he raised his hand to respond to a question, he often had a detailed answer.

Peter

Peter was an eleven year old male participant. His favorite subject was mathematics, and his least favorite subject was social studies. In his spare time outside of school, Peter enjoys reading. Upon graduating from high school, he does not yet know what he wants to do. During the activities, Peter finished problems quickly; he often asked what he could do next. He was also teased often by some of the other participants because he differed from them in appearance and attitude. Peter was a short and heavy-set boy who often wore tee-shirts and blue jeans without a belt. He often smelled suggesting that maybe he did not do much personal grooming. While others were teasing him, Peter often laughed along with them because he received attention.

The group, as a whole, functioned effectively together. Different participants provided different insights into the discussions. The female members of the group were often quiet, cooperative, willing to learn and actively involved in the activities. Two of the male members of the group, Tim and Peter, were willing to provide answers and often remained on task. Two participants, David and Steve, were very good friends. David often misbehaved during the lessons creating distractions for others. He often attempted to make Steve follow his lead. For example, while Steve was working on an activity, David started to tease Peter. David tapped on Steve's shoulder to get him to listen and laugh. Many of the participants exerted indirect pressure via body language on those who misbehaved because they wanted to have fun and learn.

Overall, the seven participants were very talkative throughout the after-school mathematics program. Both males and females were involved equally during the discussions. Their talk included both information on what they were learning and irrelevant information on topics outside of the unit such as the homecoming football game, incidents in school, and teasing of one of the participants. Placing the participants in a language-rich environment and having the program after-school encouraged the participants to talk. The language-rich atmosphere allowed them to talk more than many of their regular classrooms. Also, because the program was conducted after-school, a more informal and less threatening environment was provided.

Prior to instruction, three factors that could affect the planned implementation of the after-school mathematics program were identified. They were: the participants' computer experience, geometry knowledge and perceptions of mathematics. The first factor perceived by the instructor/researcher to potentially affect the after-school mathematics program was the amount of computer experience that the participants had. The participants' answers from the background questionnaire provided the instructor/researcher with information on how much time would be needed for the computer basics lesson. All of the participants had used computers in the past either at school or at home (Table 1); thus, the instructor/researcher implemented the computer lessons as planned.

Their knowledge of geometry was a second factor that could potentially affect the participants' responses during the unit activities. The first journal question "what is geometry?" identified their knowledge on the topic of instruction. Approximately half of the participants stated that geometry had to do with shapes. Many were unfamiliar with the specific term, 'geometry' (Table 2). Because the unit of instruction was at the introductory level, the level of geometry knowledge of the participants was appropriate for the after-school mathematics program.

The participants' perceptions of mathematics was the third significant factor that could affect the after-school mathematics program. In the first interview, the participants were asked a series of open-ended questions to gather information on their overall disposition and attitudes toward mathematics. Interview items included: what is mathematics?; how do you use mathematics in your daily life?; and describe someone who is mathematically talented. It was anticipated that the level of experience and strong attitudes about the unit of instruction by the participants could influence their level of involvement in the activities of the after-school mathematics program. Responses to the interview questions varied among the participants, but the responses did not indicate a need to modify the after-school mathematics program.

A summary of the participants response to one item on the background questionnaire, the interview questions, and the first journal entry are reported in Tables 1-8. Frequency counts were completed to describe the group of participants. All but one participant had used a computer at school prior to the after-school mathematics program, and all but one participant had access outside of school to a computer. Four of the seven participants were not sure what geometry was. Most of those four thought geometry had to do with shapes. Five of seven participants identified the operations of division, multiplication, addition, and subtraction as words they thought of when they heard the word 'mathematics.' The most unique response for using mathematics in daily life was counting the nutrition value of food. As the instructor/researcher expected, most of the participants identified science as the subject most like mathematics. In choosing a subject least like mathematics, there was not a clear consensus among the participants.

Table 1. Frequency Count of Location of Participants' Computer Experience

Location of experience	Frequency of responses
School and Home	3
School and Friend's home	1
School and Brother's home	1
School only	1
School (Total)	6
Home only (Total)	1

Table 2. Frequency Count of Participants' Definition of Geometry

Definition	Frequency of responses
Has to do with shapes	3
Nothing	1
Not exactly sure (Total)	4
Kind of math (Total)	2
Finds sums to large mathematics problems (Total)	1

Table 3. Frequency Count of Words Participants' Thought of When Say Mathematics

Words	Frequency of responses
Estimating, rounding	6/7
Multiplication, addition, division, subtraction	5/7
Fractions	2/7
Geometry	1/7

Table 4. Frequency Count of Participants' Use of Mathematics in Daily Lives

Use	Frequency of Responses
School subject	5/7
Shopping	4/7
Nutrition value of food	1/7
Collecting baseball cards	1/7

Table 5. Frequency Count on the Participants' Perceptions of the Subject Mathematics Is Most Like

Subject	Frequency of responses
Science	5
Social Studies	1
Physical Education	1

Table 6. Frequency Count on the Participants' Perceptions of the Subject Mathematics Is Least Like

Subject	Frequency of responses
Reading	3
Language Arts	2
Social Studies	2

Table 7. Frequency Count of Participants' Description of a Mathematically Talented Person

Description	Frequency of responses
Good grades (A's and B's)	6/7
Works hard	3/7
Gets work done on time	2/7

Table 8. Frequency Count of Participants' Description of a Mathematician

Description	Number of responses
Works with numbers	3/7
Extra Work	2/7
Banks, stocks, loans	2/7
School teacher	1/7
Keeps to themselves	1/7

Hands-on Activity Lessons

The five hands-on activity lessons were included to develop the participants' knowledge of three-dimensional geometry concepts and language. Most of the lesson time was spent having the participants communicating mathematically through whole and small group discussions and writing in their journals. As stated previously in chapter three, the instructor/researcher only asked the participants questions to encourage them to clarify and explain their thoughts and ideas. The participants' role was to listen and respond to others' ideas. No direct answers were given by the instructor/researcher; the participants were responsible for identifying answers during the group discussions. The following is a description of the participants' development of and use of three-dimensional geometry language during the five hands-on activity lessons. The description is based on the analysis of the triangulated data from the discussions, the classroom observations, and the journal entries.

Lesson 1 - Introduction

The introductory lesson was an opportunity to get to know and develop rapport with the participants. As previously stated, three tasks were given to the participants to complete: a background questionnaire, an individual interview with the instructor/researcher, and a mathematics in everyday life activity. For the latter, the participants were asked to look through a newspaper and record in their journal ten items that related to mathematics.

For the remainder of the hands-on activity lessons and the computer activity lessons, the data are presented in two subsections. A summary of the lesson's activities and the general disposition of the participants are presented in the overview subsection. The three-dimensional geometry language used by the participants during the implementation of the lessons is reported in the lesson implementation subsection.

Lesson 2 - Three-dimensional Objects

Overview The goal of the lesson on three-dimensional objects was to introduce the concept of three-dimensionality to the participants. An object characteristic worksheet was completed by the participants. Working in pairs, the participants were to analyze six objects and describe them by identifying the number of faces, edges, and vertices of each object. During the worksheet activity, the participants used real-life objects such as party hats and basketballs to simulate three-dimensional objects. After everyone completed their worksheet, a discussion about their results was conducted. Then, the discussion moved to talking about three-dimensionality and how it compared to two-dimensionality. The instructor/researcher observed that the participants were more active and talkative than they had been in the introductory lesson.

Lesson Implementation To begin the lesson, the instructor/researcher asked the participants, "Does anyone know what three-dimensional means?" Only two participants responded. One participant explained that three-dimensional has "something that sticks out and is thick." Another stated that three-dimensional is "when you stare at something with those red glasses." Stimulated by the discussion, Linda then asked "what is three-dimensional anyway?" Careful not to give the participants the answer, the instructor/researcher replied "you will find out."

The next part of the lesson was for the participants to complete the object characteristic worksheet and become familiar with the objects. In pairs, the participants worked on their answers using real-life objects. To prepare the participants for the worksheet, the instructor/researcher asked the participants what the definitions of faces, vertices and edges were and which real-life object (from the hands-on materials) represented which three-dimensional object. For example, the instructor/researcher showed a basketball and asked "what object is this?" The participants responded with "a sphere." For the most part, the participants used the three-dimensional names of the objects when completing the worksheet. In addition to their three-dimensional names, two objects were also called by different names; a torus was referred to as a "donut," and a super spheroid was called a "three-dimensional box, three-dimensional square, and rectangle." While completing the worksheet, the participants used three terms to identify the characteristics of the objects: face, edge, and vertices. These terms were given on the worksheet. Face was the only term that the participants called by a different term. One pair referred to a face as a "flat side."

The participants were asked to state their answers in the group discussion following the worksheet activity. Groups disagreed on their answers to some of the objects. Both the names of the objects and characteristics were used to work through differences in responses. For example, when Steve's group stated that three was the number of edges on a cylinder, Sue's group questioned their response by stating there were zero edges. The instructor/researcher asked Steve's group to explain where the edges were. Steve's group responded that the edges were on the top, bottom, and middle of the cylinder and pointed to them. Laura's group stated that those were not edges because they did not stop. Agreement then came that there were zero edges on a cylinder because the flat sides did not meet.

The journal question asked the participants to define three-dimensional geometry. In their responses, all of the participants used the language and terms developed in the earlier activity. Four participants identified the term "shapes" as part of the definition of three-dimensional geometry. Three participants gave and drew examples of the three-dimensional objects; those objects were a cone, sphere, and square. Two responses to the journal question included the terms faces, vertices, and cones. Other language used by the participants to describe three-dimensionality was "sticks out and not flat."

Lesson 3 - Scale

Overview The goal of the lesson on scale of three-dimensional objects was to help students learn what occurs when the scale of an object is changed. In the lesson, different sizes of real objects were used for scale comparisons. For example, a small coffee can was compared to a large coffee can; both were different scales of a cylinder. The participants used play dough to create three scales of the same object. For example, they were to construct three sizes of spheres. A discussion occurred to compare the objects and identify ways to change the scale of an object. Most of the participants, including both males and females, had trouble staying on task throughout the lesson.

Lesson Implementation In the introductory discussion of the lesson, the instructor/researcher asked the participants, "What is meant by changing the scale of something?" One participant identified scale as "changing the looks of something." The instructor asked the participants "how do you changed the look of something?" Responses given were "mold it," "break it," "smash it with a hammer," "write on it," and "build onto it." After these response, Steve stated that scale means to make bigger or smaller.

The next activity was to have each participant construct three sizes of a sphere using play dough. In comparing the three sizes, the participants described them in three different ways: big, medium, small; little, medium, large; and large, extra large, and double extra large. David identified the objects as being "all round"; this was his attempt at clarifying the idea that all of the objects were still the same even though they were different sizes. The conversation switched to focus on how to change one of the spheres to another. For example, the instructor/researcher asked the participants about how to change the smaller sphere to the larger one. The responses given were "add more," "add to it," "mix it up," and "combine the little and the big." Then, the participants were asked how to make the larger sphere like the smaller one. They answered "take some off; compact it; rip it apart." From the conversations in both situations, the participants explained that changing the scale meant changing the amount of material in a object but not changing its shape.

The participants were questioned on what words they would use to describe change in scale. Steve responded with "add or subtract." Laura stated making the object "smaller or bigger." Other replies included make medium, make extra large, and more or less. The instructor/researcher asked the participants what was meant by "making extra large." The participant responded by stating "making it bigger than large." To culminate the discussion, the instructor/researcher asked the participants if changing the scale meant changing the shape of an object. Laura responded by stating that "yes, because it is like a chalkboard that cannot be made bigger." From this comment, Laura may have believed that by making a chalkboard bigger it was not a chalkboard anymore. Therefore, the shape was different. After further questioning on the relationship between changing the scale and the shape of an object, all of the participants responded that all of the objects before were spheres; the objects were only different sizes.

In the journal question, the participants were shown two different scales of a cone and were asked how they would tell someone to change between the sizes. The following replies were given: "make one cone bigger," "add more," "subtract," and "divide." Peter was the only participant that stated that you need to change the shape of it. Other responses that varied from those given during the activity discussions were "poking it with a pencil and changing the color."

Most of the participants were very restless today; they often were not on task. David often was trying to show off and get others to laugh. Therefore, an answer that did not make sense may have been given primarily to get a reaction from other students.

Lesson 4 - Position

Overview The position of three-dimensional objects in three-dimensional space was the topic of the fourth lesson of the after-school mathematics program. The goal of the lesson was to have the participants learn to move in three-dimensional space. A simulation of three-dimensional axes was constructed in the room. Three different colors of string were used to represent the different axes. A discussion on what the simulation represented was conducted. Then, the instructor/researcher placed a ball in a specific location within the string simulation. Participants gave directions on how to move to the ball. The participants then chose a location in the classroom and wrote directions in their journals on how to get to the location. Later that evening was the homecoming game for their school, and the participants' minds were on other things at times during the lessons.

Lesson Implementation At the beginning of the lesson, the instructor/researcher asked the participants, "Does anyone know what position means?" Three responses were given to the question. Steve stated that "position was where something is put like if there is something in the back and there is something up here." Sue suggested that position was "where numbers were: ones, tens, and hundreds." Linda explained position as "the world like longitude and latitude."

The instructor/researcher then asked the participants "what do the three colors of string represent?"; their responses indicated they were not sure. Steve was the only one to give an answer; he compared the string to "length, width, and height." By asking the participants "how many colors of strings are there?" and "what are we studying?", the instructor/researcher attempted to have the participants associate the three colors of string with three dimensionality. Peter realized the association and said "three-dimensional."

The next part of the lesson involved changing the position of a ball within the string simulation of the three axes. The instructor/researcher held the ball in one location and asked the participants to give directions to the instructor/researcher to move the ball to the predetermined location. In doing this activity, the participants identified several words to describe positions. These words were "forward, back, left, right, up, down, and straight." One description of position that differed from the three-dimensional language the instructor/researcher anticipated; the terms, "north, south, east, and west" were used to describe position. The participants were successful in having the ball finish in the correct location.

The next activity of the lesson was to have the participants choose an object in the room and describe in their journal how to get to that location using the three axes. Two participants, David and Tim, did not complete this task because they were misbehaving. After the participants finished, the instructor/researcher read each set of directions while one of the participants moved around the room following directions. The language used in the participants' written directions included "forward," "back," "right," "east," and "straight." If the directions were inaccurate, corrections were made by the author of the directions. A major problem occurred because the participants did not note in their journals the direction one needed to face at the onset. The instructor/researcher had given this information to the participants prior to the activity, but many were talking to each other and not paying attention.

For the final activity of the position lesson, the instructor/researcher placed the ball in a specific location in the classroom. The participants were to write in their journal directions on how to get to where the ball was located. The participants used the same terms as they did when describing the location of the ball; the terms were "down, up, left, right, back, east, and west." Different participants had different orders of operations to get to the location of the ball, but all of the participants ended up in the same location.

Lesson 5 - Rotation

Overview The goal of the lesson on rotation of three-dimensional objects was for the participants to learn about different ways to rotate three-dimensional objects. The primary activity for the lesson involved the participants in a drawing exercise where they were charged with the task of sketching an object from multiple perspectives. An object, such as a cylinder, was placed in the middle of the table, and the participants were to draw the object from different views. The instructor/researcher rotated the object when the participants were ready. This activity was repeated twice using a super spheroid and a cylinder. A discussion was conducted on the various ways to rotate an object. The participants were very attentive and on task during this lesson.

Lesson Implementation As an introductory activity, the instructor/researcher asked the participants "what can you tell me about rotation?" A majority of the participants responded. Linda identified rotation as "the earth spinning." Related to that definition was Tim's description of rotation as "something circling around an axis." Peter described rotation as "a little spinning ball." All these definitions were most likely derived from the participants' study of the rotation of the Earth in a science course.

The next activity in the lesson was a group activity. Divided into two groups, each group was seated around a four-sided table. An object was placed in the center of the table, and the participants were to sketch the object from their perspective at the table. The instructor rotated the object on the table until everyone had the opportunity to sketch the object from four different views. After completing the sketching tasks, the participants compared their drawings with the sketches of those in their groups. Each group came to the conclusion that two of the four views were the same depending on the object. For example, with sketches of a cylinder, View A and C were alike and View B and D were the same (Figure 2).

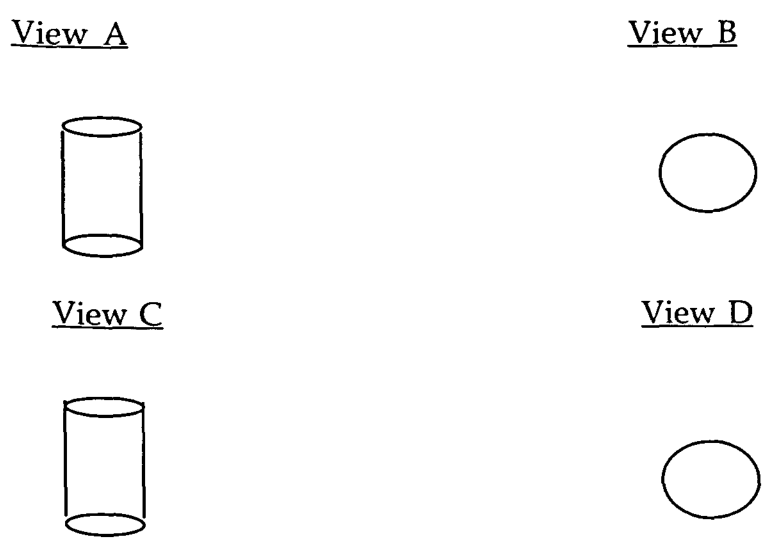

Figure 2. Rotation Views of a Cylinder

In discussing the results of the group activity, the instructor/researcher asked the participants to "identify words to describe rotation." Several participants responded with the following language: spin, turn, and flip. Related to her previous understanding and use of the term, Linda described rotation as "the earth spinning". She also used the word "revolve" to describe rotation. When the instructor/researcher inquired about the ways of rotating objects, the participants chose an object and illustrated the process of rotation. They used hand motions to simulate the directions an object can be rotated.

In their journals, the participants wrote questions they would ask someone if they were told to rotate an object. Linda responded with "rotate the object by flipping it, turning it, or spinning it?" Mary asked "diagonal? flip? turn?" David responded with a different answer than other participants. He asked "up, down, side to side, or diagonal?"

Lesson 6 - Incorporating Hands-on Activity Lesson

Overview The goal of the incorporating hands-on activity lesson was to have the participants reinforce and link all the concepts they had learned during the preceding hands-on activities. In the activity for this lesson, the participants were to incorporate all of the concepts from the previous lessons to create a three-dimensional structure. Each participant was to choose three objects and create a structure. Then, the participants were to write directions explaining how to create the structure in their journals. Working in pairs, each participant was to describe to their partners how to make the structure while their partner created it using play dough. The participants appeared to have trouble focusing on the task they were to complete; often they were doing something different than what was asked of them.

Lesson Implementation The language the participants used during this activity is described below. The three-dimensional geometry language used came from their description to a partner and the written journal directions. The following is an explanation of the language used based on the concepts taught during the hands-on activities.

The identification and classification of three-dimensional objects was the first concept taught in the hands-on activity lessons. During the incorporating activity, the majority of participants were successful in using the geometry names of the objects (i.e. sphere, cone, torus, super spheroid, square, and cylinder). All of the participants, with the exception of Sue, chose three different objects to create their structure. One participant, Peter, called the objects by different names. He identified a sphere as a "circle" and a torus as a "donut." His partner questioned these names, and Peter changed the names to sphere and torus.

The scale of three-dimensional objects was the second concept explored during the hands-on activity lessons. All of the participants used language developed in the lesson on scale when creating their three object structure. One particular term, "big," was used by all participants. Other language used included "smaller, littler, medium, add, and size." Three terms that were not mentioned in the lesson on scale were used to describe scale: huge, taller, and stretch it. When most of the participants described to their partners how to create the three-dimensional structure, they usually used words to compare the three objects. For example, Linda told her partner that "this was made with a sphere, a square, and a cone. The circle was the biggest, the cone was the smallest, and the square is medium."

The third concept introduced in the hands-on activity lessons was position of three-dimensional objects in three-dimensional space. Only three participants used the three-dimensional geometry language that they had used in the lesson on position; the language was "down," "up," "straight," and "right." Other language to describe the position of objects was "on top of," "put on," "bottom," "higher," and "underneath." From this language, the partners were able to position the objects accordingly. For example, Sue told Laura to place the "two spheres; one on top of the other." Laura followed the directions and successfully arranged the spheres in the structure.

Rotation was the final concept explored in the hands-on activity lessons. In the incorporating activity, none of the participants used the task of rotation to create their structure. Therefore, no language on rotation was spoken. All of the objects were used as if the participants were looking at them from a side view. For example, Mary used a cone, cylinder, and torus. The cylinder was drawn to see the side view such as where the lettering is on a coffee can. The cone was located directly on top of the cylinder with the rounded side of the cone on the round top of the cylinder. The tip of the cone was placed through the hole of the torus; therefore, the side of the torus was the only portion visible.

Summary of Hands-on Activity Lessons

The goal of the hands-on activity lessons was to provide an atmosphere for the participants to develop three-dimensional geometry language. This mathematical language was used to understand, interpret, and complete tasks that were to help the participants learn about three-dimensionality. A constructivist approach was used to explore the concepts. The instructor/researcher questioned the participants, and the participants created their own understandings of the concepts. If the participants strayed from the assigned task, the instructor/researcher tried to guide the actions of the participants back to the topic of the activity. Mathematical language was used in both whole and small group discussions. Often in small group discussions, the participants questioned the ideas of others. However, in whole group discussions, the instructor/researcher, through questioning of their comments, was the person who most often challenged the participants' ideas. Through the use of mathematical language, the participants were able to work through the problems given in the activities.

In the language-rich, hands-on learning environment, the participants identified terms and language associated with three-dimensional geometry. Because the instructor/researcher only questioned the participants, they were responsible for their own description and definitions of the concepts. Nearly fifty percent of the mathematical language used during the hands-on activity lessons was identified by the instructor/researcher prior to the after-school mathematics program as terms the participants would use to understand three-dimensional geometry. The other fifty percent of the language consisted of terms and phrases the participants created to describe, define, and develop their understanding of the three-dimensional concepts.

Computer Activity Lessons

The three computer activity lessons were used to support the participants' application of the three-dimensional mathematical language developed in the hands-on activities. The participants were to use their knowledge and language of the three-dimensional concepts to complete tasks on the computer. Similar to the hands-on environment, the instructor/researcher was the discussion facilitator in the computer learning environment. The participants were responsible for solving the problem of the activity. Then, in discussing their solutions, the participants were to interact and respond to each other's thoughts and ideas. The following is a description of the three computer activities based on the data collected during the discussions, the classroom observations, and the participants' journals. After this description, a summary of the application of the three-dimensional geometry language is provided.

Lesson 7 - Computer Basics and Wireman Introduction

Overview The goal of this lesson was for the participants become familiar with the Macintosh portable computers and the Wireman software program. During the first part of lesson, the instructor/researcher talked with the participants about the basics of a Macintosh computer. The second part of the lesson was spent having the participants experiment using Wireman. Prior to using the computers, the instructor/researcher asked the participants what rules they should follow when using the computers. All of the participants were excited to use the computers; therefore, the participants were very quiet and cooperative during this lesson.

Lesson Implementation Most of the participants had used only IBM compatible computers prior to the after-school mathematics program. Therefore, the first ten minutes of the lesson consisted of an explanation and demonstration of the hard drive, opening of the hard drive, opening folders, and double-clicking to open files. The participants also needed to become comfortable with the use of the track pad as their mouse.

After the participants opened the Wireman program, a discussion was conducted about the familiar components of the program in relation to what has been discussed in the hands-on activity lessons. Responses given included "the shapes: torus, sphere, cone, cylinder, square, and super spheroid" and the words: "scale, position, and rotation." The instructor/researcher then demonstrated how to remove the windows and commands the participants would not need. For the computer activities, the participants were limited to the concepts they were introduced to during the hands-on activity lessons.

During the remainder of the lesson, the participants experimented with the Wireman program. Specifically, the goal of this activity was for the participants to observe how the objects were placed on the screen and explore the purpose of the three sets of numbers below position, scale, and rotation commands (Figure 1 in Chapter 3). During this part of the activity, the participants asked questions when what they intended to do and what occurred on the computer screen were not the same.

Lesson 8 - Develop Skill in Using Wireman

Overview The goal of this lesson was to provide the participants with activities that would allow them to become more familiar with Wireman. The participants were to develop their skills in creating structures with the wire frames. Each structure they were to create for this activity included making a change in scale, position, and rotation of the object. The participants were asked to complete two tasks during this lesson. The first task was to create an ice cream cone using the tools of Wireman. Then, the participants were to change the ice cream cone to a person wearing a dunce hat. The second task was to create a Lifesaver lollipop using Wireman. The participants chose the objects they wanted to use to create the structures. The participants were cooperative during this lesson; the computers appeared to draw their attention to staying on task.

Lesson Implementation In creating the ice cream cone, the participants unanimously chose the cone and sphere as the objects. The participants were able to place the objects on the screen with few questions asked of the instructor/researcher. Examples of questions asked were "which one of the three numbers do I push under position?" or "why is my sphere doing that?" The instructor/researcher helped the participants with their questions but did not provide the answers. All of the participants were successful at creating the ice cream cone using Wireman.

To complete the task, the participants were asked to change the ice cream structure to a person with a dunce hat. A picture of this structure was drawn on the chalkboard to clarify what was meant by the term 'dunce hat.' Steve stated that it was a "face with a birthday hat." While they were changing the structure, the participants were to write directions explaining their actions in their journals. The descriptions in their journals were very brief; often they did not communicate the complete actions. For example, David wrote "rotate the cone 170 degrees up and move the sphere down one." Many of the descriptions included "pushing this bottom three times" indicating that this was how participants thought directions should be given. At this time, the participants did not use any of the conceptual terms in their directions. All of the participants knew that there were two steps to changing the ice cream cone to the dunce hat. These steps were changing the position of the two objects and rotating the cone.

The second task of the computer activity lesson was to create a Lifesaver lollipop using Wireman; the instructor/researcher drew a picture of this structure on the chalkboard. The objects chosen varied among participants. Sue, Linda, and Mary chose a torus and square. Steve and Peter chose two tori, and

Tim chose a torus and a cylinder. With the torus, the participants described rotating it until "it is standing up, it looks like a lifesaver, or you see a hole in the middle." With the other objects, the participants described different methods based on the object they chose. Mary and Sue selected a square and stated that the object needed to "rotate until it is standing up or flipped." Peter used a torus and "rotated it to a stick." Tim chose a cylinder and moved its position "down." Steve picked a torus and said to "extend and make long with scale and turn so it's going up and down." Many methods were used to create the structures. In examining the three-dimensional geometry language in the participants' journal entries, the three-dimensional concepts, the names of the objects, position, and rotation, were used to complete the activities.

Lesson 9 - Computer Activity Lesson for Assessment of Application

Overview The goal of this lesson was to have the participants demonstrate their application of three-dimensional geometry language to their learning of geometry. The activity given was similar to the incorporating activity of the hands-on activity lessons. The participants were to select three different objects and create a structure. The participants were given time to draw a picture of their structure in their journal, create it once on their computer, and write directions on how to create the structure in their journal. After completing this, each participant was to explain to the instructor/researcher how to create the structure while the instructor/researcher completed the action on the computer. These explanations were audio taped for further analysis. One of the participants, David, misbehaved quite frequently during this activity and seemed disinterested. His structure was not complex, and his explanation was brief. The other participants were on-task most of the time.

Lesson Implementation In the activity, the participants used mathematical language to describe to the instructor/researcher how to create the structure. The following is a description of the three-dimensional geometry language used. Most of the three-dimensional geometry language the participants used had been used during the hands-on activities.

Identification and classification of three-dimensional objects was the first concept taught during the hands-on activity lessons. In completing the computer activity lesson on creating the three-object structure, the participants identified the objects by name. They used objects such as cone, sphere, cylinder, super spheroid, and torus. No other terms were used to describe the objects.

The second concept taught during the hands-on activity lessons was scale of three-dimensional objects. Often, the participants used the term "scale" to describe what they needed to do to the structure. Scale was also called "size" by some participants. For example, Mary said "you need to make the cone twice its size." To explain further, the participants used the words, "bigger, taller, smaller, and littler" to describe the specific change in the object. Two participants used phrases that referred to scale but were never mentioned in the hands-on activities. Mary described changing the scale by "making an object bigger so it is flatter." The word "flatter" had not been used before. Steve used the phrases "extend it" and "longer"; both terms had not been mentioned previously.

Position of the objects was the third concept taught in the hands-on activities. All of the participants, with the exception of David, described a change in the position. The terms they used were "up, down, and under." Two participants, Sue and Mary, pointed on the computer screen where the object should be. They instructed the instructor/researcher to move the object "toward this way and over here." These phrases were not used in discussions or journals in the hands-on or computer activities. From their responses and drawings, most of the participants only placed objects on top of each other not side by side. Therefore, language such as "right, left, back, and forward" were not used.

The last concept taught during the hands-on activities was rotation of three-dimensional objects. All of the participants, with the exception of David, used rotation in their structure. When describing their structure, most of the participants used the concept term "rotation" to explain the action to be taken. For example, Peter said to "rotate the cone on the middle one up to 40." The only other word that had been used in the hands-on activity discussion was "turn." Linda stated that "the super spheroid needed to be turned back." Mary identified another word to describe rotation that was not used in the hands-on computer activities; the word was "slant."

Summary of Computer Activity Lessons

The computer activity lessons provided an opportunity for the participants to apply the language they developed and used during the hands-on activity lessons. During the computer activity lessons, the participants described their structures both by writing directions in their journals and verbally communicating the directions to the instructor/researcher. Often, the participants' spoken directions were in greater detail than their written directions because the instructor/researcher encouraged them to clarify their verbal directions. During the participants' application of mathematical language in the computer learning environment, the instructor/researcher observed three significant occurrences: the consistent use of the conceptual terms, a naive conception regarding scale, and the use of the three sets of numbers associated with the position, scale, and rotation commands in the computer program. These occurrences are presented below.

Through analysis of participants' verbal and written directions for the construction of the three object structure, the use of the conceptual terms was prevalent; the conceptual terms the participants used were scale, position, rotation, and the names of the three-dimensional objects. The participants used the terms at two different times during their descriptions: initially, as the first instruction or in response to a question from the instructor/researcher. After placing the first object on the screen, most of the participants gave the three-dimensional name of the object during their initial instruction. For example, Tim told the instructor/researcher to "use a torus." He then said "rotate it using the second set of numbers." Steve chose a cone to be placed on the screen first. Then, he told instructor/researcher to "extend it with scale." If the participants advised the instructor/researcher to manipulate the object without mentioning a term or phrase that specified the meaning of direction, the instructor/researcher asked the participants how to perform the manipulation. For example, Steve said to "get a sphere and place it under the cone." The instructor/researcher said "how?" Steve declared "with position." Mary stated "slant the cone so its diagonal point toward this corner." The instructor/researcher said "how?" Mary replied "with rotation." For the majority of the time, the participants were able to use the conceptual terms.

Of the three conceptual terms, scale seemed to be the most difficult for the participants to use and understand. Three participants appeared to possess naive conception regarding scale. These participants believed that an object would become larger and smaller if the z-axis was changed on the position menu. Peter and Sue instructed the instructor/researcher to "move the position back so the object would become smaller." Steve said to "move the position forward to make the object bigger." This naive conception may have been overcome if the participants had learned about the concept of perspective. In changing the perspective from which they observed the objects, the participants would notice that their objects were not larger or smaller than they had perceived. Then, they may have come to the conclusion that their manipulations changed the position and not the scale as they intended. With the exception of these three participants, all of the others appeared to understand the concept of scale and were able to use scale to change the size of an object.

Three sets of numbers were displayed on the screen for each of the commands: scale, position, and rotation. The participants were not told that the three numbers corresponded with the x, y, and z axes; however, they were able to identify the axis to which each set of numbers corresponded. For example, Tim told the instructor/researcher to "change the scale by two or three times on each one to make it bigger." He knew that changing all three numbers made the object equally larger. Peter said to "make the cone go up by selecting the middle on position." He knew that the middle set of numbers on the position command caused the object to move vertically without knowing it was called the y-axis.

Summary of Participant Experiences

To summarize the participants' experiences during the after-school mathematics program, a final interview with each participant was conducted by the instructor/researcher. Four open-ended questions aimed at identifying the participants' definitions of three-dimensional geometry, math language, and perceptions of how the hands-on learning environment impacted their learning in the computer environment were asked. The following is a synthesized description of each participant's development and application of three-dimensional geometry language as a result of their participation in the after-school mathematics program. As was previously mentioned, group and individual data were collected of participant experiences in the after-school mathematics program. Due to the nature of the research question and case setting, data were analyzed in terms of group experience. In addition, a summary of individual data were analyzed to provide insight into the group experience. Each participant's summary includes three sections: overall disposition, use of mathematical language, and knowledge of three-dimensional geometry. The summaries are based upon the triangulation of data from the following sources: instructor/researcher observations of participants, transcriptions of whole and small group discussion tapes, participant journal entries, and the concluding interviews.

Sue

Overall Disposition For the most part, Sue was continually on-task during the after-school mathematics program. Most of the time, she was excited to be there and was ready to learn; she always entered the classroom with a smile on her face. Sue was quiet and reserved throughout the entire program especially during the whole class discussions; she rarely raised her hand to contribute answers. In small group discussions, Sue was more out-going and willing to contribute. For example, while completing the object characteristic worksheet, the participants worked in small groups; Sue was in a group of three. She communicated her ideas more often than one of the other members of the group, Mary. During the second interview, Sue identified the use of play dough as an activity that helped her learn three-dimensional geometry language. Most activities using play dough occurred in small groups.

Use of Mathematical Language Sue often was more detailed and thorough than the other participants in the use of mathematical language in her journal entries both during the hands-on and computer activity lessons. When giving detailed descriptions of the structures, Sue used complete sentences and ideas that incorporated mathematical language to aid her descriptions. For example, when describing the location of the ball during the hands-on activity lesson on position, Sue began by telling the reader where to start from: "if you are in the middle of the room, facing the front..." Sue identified the journal as an activity that helped her learn the mathematical language associated with three-dimensional geometry.

Throughout the two learning environments, Sue used a combination of the concept terms and other terms and phrases to describe her understanding of the concept. For example, during the computer activity for assessment, Sue told the instructor/researcher to "go to position, the second one, to make it go up." "Position" was a concept term, and "up" was another term used as part of her understanding of position. Both terms helped Sue successfully describe to the instructor/researcher what action was to be taken.

Knowledge of Three-dimensional Geometry In the first interview, Sue stated, "I did not know much about geometry, but from the little I know, it seems pretty interesting." This suggested that the information on geometry that she learned during the after-school mathematics program was new knowledge for her.

In the final interview, Sue defined three-dimensional geometry as "looking into glasses and seeing something you do not normally." This definition was a complete change from the one she recorded in her journal during the lesson on three-dimensional objects. Her journal definition of geometry was "three-dimensional is something that is fat and sticks up." Sue also expanded her final definition of three-dimensional geometry to include "position, rotation, and directions to get to a spot." Sue used this part of the definition to describe her complete understanding of three-dimensionality.

Linda

Overall Disposition Linda often spent more time completing activities than the other participants in the after-school mathematics program. In the first interview, Linda identified mathematics as "thinking hard"; she worked and thought hard during the different activities. In conversations with the instructor/researcher, Linda often verbally expressed her entire thought process. Linda often corrected herself when she believed her ideas might be incorrect. Linda was often frustrated with the other participants for misbehaving because their actions would disrupt her concentration and learning.

Linda was not afraid to take chances in completing the activities. For example, in response to the journal question "what is geometry?", Linda gave the following answer: "I really don't know what the definition is, but I will try." If given sufficient time to complete the activities, Linda successfully completed them. For example, for her final computer activity, she spent nearly three times the amount of time describing her structure as did the other participants. The extra time she took may have been caused by a lack of confidence in her abilities or because she was very conscientious and wanted to do the activities completely and correctly. Linda became frustrated when she gave an incorrect answer; she then took her time in giving another answer.

Use of Mathematical Language During the hands-on experiences, Linda used both the concept terms and other terms and phrases to express her thoughts and ideas. However, during her experiences using the computer, Linda often focused on the numbers for each command rather than the action for each command. For example in her written directions of the three-object structure, she described the structure in terms of numbers: "go to position; make 1.000 and 1.000." When giving the same directions verbally to the instructor/researcher, Linda would become confused when the directions would not work. She had to re-focus her thoughts on what she really wanted to accomplish. The instructor/researcher attempted through questioning to have Linda focus on her conceptual understanding of position, scale, and rotation to decide her actions.

Knowledge of Three-dimensional Geometry At the conclusion of the after-school mathematics program, Linda expanded her initial definition of three-dimensional geometry to include some of the concepts learned in the hands-on activity lessons. Her initial definition was "things that have many sides and shapes." Linda then added a description of three-dimensional as "something you can shape, position, and scale." Her knowledge of geometry had broadened to include some of the concepts learned in the hands-on activity lessons.

In the final interview, Linda identified the play dough activity as "helping her fit the pieces together" to complete the computer tasks. The incorporating activity of the hands-on activity lessons required the use of play dough. The goal of the incorporating activity was to link the concepts learned in the hands-on activity lessons. Based on the above response Linda gave during the final interview, the play dough activity accomplished its intended goal.

Mary

Overall Disposition Mary successfully completed most of the activities. Her responses to questions in whole group discussions and her descriptions in her journal entries were often brief and concise. For example, in her final interview, she was asked, "what activities helped her learn mathematical language?" She looked around the room and hesitated before giving a response; the response was "turned the box and play dough." She did not elaborate on what each of these responses meant and how each specifically helped her in learning.

Throughout most of the hands-on and computer activity lessons, Mary's attention did not always appear to be on the activity. She often glanced around the room and looked at other participants. She often laughed with David when he was misbehaving. Mary spent the most time and gave the most attention to the final computer activity lesson on creating a three-object structure using the Wireman program. Her structure and verbal directions were more detailed that they had been in the previous lessons. The reason for the greater attention when using the computer may relate to her reason for participating in the program: to spend more time on the computer.

Use of Mathematical Language Mary was able to use both the names of the concepts and everyday language to describe the three-dimensional geometry concepts. During the hands-on and computer activity lessons, Mary most often used common language such as "right, left, up, and down" as her mathematical language, but she was able to associate that language with the concepts. For example, during one of the computer activities, Laura was describing to the instructor/researcher what to do with the cone on the screen.

She said, "move it down so it is close to the corner." The instructor/researcher asked, "how do I move it down?" Sue said, "you use position." This situation also occurred with the concept of scale. Rotation was the one concept she often used the name of the concept in her descriptions.

Knowledge of Three-dimensional Geometry When asked to describe three-dimensionality in the final interview, Mary gave the exact same definition as she did in the journal response of the first lesson: "something that sticks out." Mary identified "position, scale, and rotation" as three-dimensional language she learned, but she did not include them in her definition of three-dimensional geometry. This may suggest that Mary did not perceive these concepts as being a part of three-dimensionality.

Steve

Overall Disposition Steve was always willing to provide a response to a question. Nearly every time a question was asked, he raised his hand and attempted to answer the question. Steve was willing to take chances during his learning process. For example, in the first journal entry on what is geometry, Steve answered "a math study that finds large numbers or sums to large math problems quickly." He obviously did not know the definition of geometry, but he attempted to make an educated guess.

Steve enjoyed the computer activity lessons the most. He often finished the assigned task quickly and then experimented on his own. For example, during the first computer activity lesson, while others completed the tasks, Steve created a picture of a man using several wire frame objects in Wireman. The computer was the first response Steve gave when asked what activities helped him learn three-dimensional geometry language. This response, as well as the interest he exhibited and advanced activities he undertook, led the instructor/researcher to believe the computer was the most effective and beneficial experience for him.

Use of Mathematical Language Steve was the dominant member of the participants. He attempted to be the leader of the small group discussions. In a small group situation, Steve was the one participant who used his mathematical language to guide the other participants in discovering answers. For example, Steve and Peter were partners in completing the object characteristic worksheet of the lesson on three-dimensional objects. When Peter identified a square as having eight edges, Steve disagreed and pointed to and verbally counted the edges on the real life object representation of a square; Peter then agreed. Steve's actions and his use of mathematical language during small group discussions indicated to the instructor/researcher that this type of instructional environment helped Steve learn; moreover, he identified two hands-on activities that were conducted in small groups as the most beneficial to him. These activities were the object characteristic worksheet and the drawing of the rotation.

During the final computer activity lessons, the participants were asked to write in their journal a description of the three-object structure they created and then describe it to the instructor/researcher. Steve's written and verbal descriptions were nearly identical. For example, in the written description, Steve stated "get a cone and extend it with scale." In the verbal description, Steve used the same phrase. From this phrase, the instructor/researcher was able to complete the action. Other participants often changed their descriptions when verbally giving them to the instructor/researcher; they expanded their verbal descriptions because they included insufficient mathematical language in their written descriptions to describe the action to be taken. Steve was the participant whose written description was changed the least when given verbally. This suggested to the instructor/researcher that Steve was capable of using his mathematical language in two forms, written and verbally, to clearly express himself.

Knowledge of Three-dimensional Geometry As stated previously, Steve initially gave an incorrect definition of geometry when he defined geometry as "a math study that finds large numbers or sums to large math problems quickly." From this response, the instructor/researcher assumed that Steve did understand what geometry was; thus, what Steve learned during the after-school mathematics program would be new knowledge to him.

In his journal entry following the lesson on three-dimensional objects, Steve described three-dimensional objects as "having faces, edges, and vertices." In his concluding remarks during the second interview, Steve used the phrase, "has front, back, sides, bottom, and top," to describe what three-dimensional meant. Similar to the first description, this phrase describes some three-dimensional objects such as the super spheroid or cone. In his definition of three-dimensional geometry during the hands-on activity lessons and in his final interview, he also described three-dimensional geometry as "something that sticks out." This response may indicate that he may not have connected the other three-dimensional concepts of position, scale, and rotation with three-dimensional geometry.

David

Overall Disposition David was the participants with the most behavior problems. As stated before, he often misbehaved and was not on-task during the after-school mathematics program. After being allowed to participate in the after-school mathematics program because of the intense interest shown by him and his family, David consistently disrupted the activities forcing the instructor/researcher to stop the lesson several times in attempt to get him on-task.

David began the program with a positive attitude and a willingness to learn. He stated in the background questionnaire that he was participating because he "thought he would learn something." Throughout the first three hands-on activity lessons, David gave responses in whole group discussions and wrote detailed journal entries.

David's attitude seemed to change after the first three hands-on activity lessons. When asked to complete a task, he acted as if the instructor/researcher was inconveniencing him. Before starting the assigned task, he often did other tasks, such as play with the string during the position lesson and create structures other than those requested in Wireman. Also, David's explanations in the computer activity lessons' whole and small group discussions, journal entries, and final interview were brief; he seemed disinterested. During the last day of the program, David made a comment during a conversation with the instructor/researcher that he "did not want to be here anyway." This comment was contrary to the attitude he and his family exhibited prior to and early in the program.

Use of Mathematical Language David's declining interest in the program affected his use of mathematical language. The after-school mathematics program emphasized student responsibility for their own learning. David's actions did not reflect this principle; David appeared to not take responsibility for his own learning. As the after-school mathematics program progressed, it became difficult to identify David's mathematical language use to explain his understanding. The instructor/researcher assessed David's journal questions and responses in group discussions to identify his use of mathematical language to understand the concepts. David appeared to able to use the mathematical language, but he had difficulty expressing his thoughts and ideas in a meaningful way. For example, during the activity on scale, the participants were asked to describe in their journal how to change a small object to a large object. David's response was "you can stack up until you get from one place to another." From this response, the instructor/researcher believed that David probably knew that he needed to add more material to the small object, but he had difficulty expressing this idea using meaningful mathematical language.

Knowledge of Three-dimensional Geometry Identifying David's development and application of mathematical language was difficult. In the first journal question, David used terms and phrases he learned in the three-dimensional object lesson to define three-dimensionality. His definition was "three-dimensionality is a shape that has faces, edges, and vertices." At the end of the after-school mathematics program, David described three-dimensionality as "different sizes and shapes." From this description, it was evident to the instructor/researcher that David lost interest after the first few hands-on activity lessons.

In the first interview, David identified mathematics as "not his favorite subject." As a result of his lack of enjoyment of mathematics, his disinterest in the after-school mathematics activities may have caused his misbehavior and his inability to grasp the concepts.

Tim

Overall Disposition Tim was a hard-working participant. When tasks were given to him, he immediately began to solve the task. Both the hands-on and the computer activity lessons seemed to interest him. In the background questionnaire, Tim identified the stipend he was to receive for participating as the reason he participated in the after-school mathematics program. Tim's involvement did not reflect this comment. He appeared to be the participant who was most interested and involved in learning.

Tim was also the only male participant to not instigate misbehavior during the after-school mathematics program. The instructor/researcher may have overlooked him because he was so well-behaved. This behavior was contrary to what the instructor/researcher thought at the beginning of the after-school mathematics program. When the instructor/researcher first met the classroom teacher, Tim was staying after-school for detention because of behavior problems. When Tim decided to participate in the program, the instructor/researcher's preconceived thoughts suggested that he may be a discipline problem. Tim's actions during the program were in contrast to the instructor/researcher's assumptions.

Use of Mathematical Language In most activities, Tim used mathematical language to participate effectively in the whole and small group discussions. He also used mathematical language to write complete and thorough explanations in his journal entries. For example, when describing how to change a ice cream cone to a dunce hat using Wireman, he wrote the following description: "Change the rotation of the cone; then change the position so it is in the upper middle; then change the position of the sphere so it is in the middle." Through his use of mathematical language in this description, Tim appeared to understand the three-dimensional geometry concepts of position and rotation. Tim's understanding then led him to successfully complete the task of changing the ice cream cone to a dunce hat.

Knowledge of Three-dimensional Geometry From his participation throughout the program and his responses during the final interview, Tim seemed to be the participant who benefited most from the instruction. When asked about the three-dimensional geometry language he learned, Tim identified "position," "scale," "rotation," and two names of three-dimensional objects. The instructor/researcher then asked Tim, "what activities helped him learn the mathematical language?" Tim responded " the computers (Wireman), the play dough, the strings, and the objects (such as the party hat and ball). All of these activities, he stated, were program activities meant to develop his understanding of the three-dimensional concepts.

Tim appeared to be aware of how the hands-on activity lessons provided the framework for using the computer. The instructor/researcher asked Tim, "how did what you learned in the hands-on activities help you complete the computer activities?" Tim responded with "using scale, position, and rotation and learning the different ways to change each." He was the only participant to specifically identify that there were several ways of changing an object's position, scale, and rotation.

Peter

Overall Disposition During the after-school mathematics program, Peter often was teased by the other participants because of his body odor, a funny comments, or inappropriate actions. Even though most of the participants teased him, he was able to solve the problems and effectively participate in the activities. Peter was effective in finishing the tasks he had to complete, often inquiring about what he could do next. Especially during the computer activities, Peter created the structures quickly and then asked "can I create something else now?"

Peter often searched for the quickest method to complete the tasks. For example during the final computer activity lesson, the participants were asked to write directions in their journal about how to create a three-object structure using Wireman. Peter wrote the following: "go to the wire mold." Instead of writing detailed directions, he wanted the instructor/researcher to use a structure already created. This incident, as well as others that occurred throughout the after-school mathematics program, lead the instructor/researcher to the notion that Peter often identified creative and clever methods to solve problems. He appeared to want to be different than the other participants to possibly gain attention.

Use of Mathematical Language Peter gave very few answers during the whole group discussions possibly because he was afraid the other participants would laugh at him. His level of participation in the small group discussions and his journal entries was greater, but he often did use mathematical language to express his understanding. For example, when describing his three-object structure during the hands-on incorporating activity, Peter stated "stick the donut; put it around the cone and put a ball on top of the cone." From this description, Peter appeared to describe the structure, but he did not use many of the terms and phrases he was to have developed during the hands-on activity lessons.

In his journal entries during the computer activities, Peter often used only the exact number and the command to describe where the object was located. For example, he described the cone as having "a rotation of 40.000 in the middle and 180.000 on the bottom." He did not use any mathematical language to describe what actually occurs when "the middle rotation is changed to 40.000." This response suggested that Peter wanted to complete tasks in the quickest method possible. Without focusing on learning three-dimensional geometry, the response also indicated that his understanding of three-dimensional concepts was not complete.

Knowledge of Three-dimensional Geometry In the first journal question on "what is geometry," Peter gave no response. This indicated he did not know what geometry was. After the lesson on three-dimensional objects, Peter identified three-dimensional objects as "spheres and squares." His knowledge of geometry had expanded although it was not yet complete. Throughout the after-school mathematics program, the instructor/researcher had a difficult time identifying what Peter was learning because he often did not express his ideas and views and often completed the task quickly but not thoroughly.

However, in his final interview responses, Peter did give an indication to the instructor/researcher of what three-dimensional geometry concepts he knew. Peter was able to incorporate the knowledge he learned to summarize his experiences. In response to three of the four final interview questions, Peter identified the concepts of position, scale, and rotation. For example, the instructor/researcher asked Peter "how did the hands-on activities helped you complete the computer activities?" Peter stated that "I wouldn't have known what scale was, ... position was, and ... rotation was; so I would not have been able to complete the computer activities." From these responses, Peter appeared to have expanded in his knowledge of three-dimensional geometry because of his experiences during the after-school mathematics program.

Summary

As indicated by their responses in their initial interviews and the journal entries, none of the participants had received formal geometry instruction prior to the after-school mathematics program. From their written responses, overall disposition and oral comments, the participants developed various levels of mathematical language through their participation in the after-school mathematics program. However, the level and extent of that development and use was contingent upon the level of responsibility for learning taken by each student and comfort level during the instructional activities.

Those participants who assumed responsibility for their learning tended to develop and use mathematical language more that those who did not. For example, Steve was the participant who was always raising his hand to provide answers and was not afraid to take chances. Because of these actions, Steve appeared to be responsible for his own learning. He used mathematical language to describe his thoughts and ideas. Tim was another participant who exhibited actions that suggested he knew he was responsible for his own learning. When given tasks to complete, Tim started immediately and worked completely through the tasks.

The participants' comfort level during the various instructional activities also affected the extent that they developed and used mathematical language. For example, as stated previously, Sue and Peter contributed more during small group discussions. Therefore, Sue and Peter's language development and usage during these instructional activities was greater than in other instructional activities. Sue also identified the journal activities as helping her learn three-dimensional geometry. Through these activities, Sue appeared to be more comfortable using the mathematical language to express her ideas that she was in large group discussions.

Summary

The purpose of this chapter was to describe the results of the case study. A description of the research setting and each participant was given to provide a background for the other results of the after-school mathematics program. Seven participants volunteered to be part of the program which occurred in the participants' sixth grade classroom. An informal learning environment was created by the instructor/researcher.

A description of each participant was given to provide an understanding of the after-school mathematics program participants. Then, the participants were described as a group to demonstrate how they functioned together. Throughout the program, the participants were often talkative and restless. Because the program was immediately after-school, the participants attention span and level of concentration for academics were lower than during the regular school day.

During the hands-on activity lessons, the participants used different terms and phrases to describe the concepts of three-dimensional geometry. Through activities in writing, listening, and speaking using three-dimensional geometry language, the participants were able to develop and use to use their language to describe their understandings of three-dimensional concepts. Some of the mathematical language they used was identified prior to the program; other phrases were constructed by the participants during the learning process.

During the computer activity lessons, the participants used their mathematical language to reason through the tasks with varying levels of success. Because the instructor/researcher was the recipient of the directions for creating the final computer structure, the participants' application of language was identified. Unclear phrases by the participants were questioned by the instructor/researcher to clarify their meaning. The instructor/researcher observed three occurrences during the last computer activity: the consistent use of the conceptual terms, a naive conception regarding scale, and an understanding of the three sets of numbers in each command of the Wireman program.

Even though data analysis in group settings, individual assessments were conducted throughout the after-school mathematics program to identify the participants' progress. In a summation of each participant's experiences, the participants differed in three ways: overall disposition, use of mathematical language, and knowledge of three-dimensional geometry. The extent to which the participants developed and used mathematical language to develop understanding varied based upon each participant's level of responsibility for their own learning and comfort during the instructional activities.

CHAPTER V. SUMMARY, DISCUSSION, AND RECOMMENDATIONS

The purpose of this chapter is to provide a summation of the case study research conducted through the after-school mathematics program. The chapter is divided into four sections. The first section in the chapter is a brief summary of the study's background and methodology. Next, a discussion of the major findings of the study is presented. The third section of the chapter identifies four areas for future research; the last section provides concluding remarks.

Summary

In recent years, professional organizations such as the National Council of Teachers of Mathematics (NCTM) and the National Council of Supervisors of Mathematics (NCSM) have identified the need for reform in mathematics education (Reys, Suydam, & Lindquist, 1992). Recommendations for changing the mathematics curriculum have been provided by these two organizations. As identified by these professional organizations and the research literature, several components of mathematics instruction are included in the reform. Four of these components provided a framework for this case study: mathematical language, geometry, application of concepts, and learning environments. All four of these components play an important role in aiding students' understanding of mathematical concepts.

The geometry curriculum has recently been expanded both in the number of concepts taught and the grade levels in which instruction occurs. According to the NCTM, geometry instruction should provide learners with opportunities to classify, define, and develop relationships (Reys, Suydam, & Lindquist, 1992). According to the NCSM, geometry instruction should include activities in developing visualization skills and understanding three-dimensional concepts (Reys, Suydam, & Lindquist, 1992).

The development of mathematical communication among students is one of the five common learning goals identified by the NCTM (1989). Such communication involves the use of mathematical language; this language aids students in clarifying and understanding mathematical concepts. Mathematical language can occur in four forms: speaking, writing, listening, and reading (Anders & Pritchard, 1993). To develop mathematical language, students need an opportunity to engage in mathematical communication in a meaningful context. Having students work in groups, write in journals, and explain their methods of solving a problem are techniques that encourage students to use mathematical language (Capps & Pickreign, 1993; Wilde, 1991; Easley, Taylor, & Taylor, 1990).

The use of hands-on manipulatives and the use of computers can enable the development and use of mathematical language. Language use with manipulatives helps students develop meaning and build connections (Capps & Pickreign, 1993). Language also can be a method for assessing understanding. The computer encouraged direct interaction among students in a single classroom and enables interactions among students in different parts of the world (Hoyles, Sutherland, & Healy, 1991; Kelly & Wiebe, 1994). Both interactions allow students to use mathematical language to communicate.

The purpose of this study was to create a setting for the development and application of three-dimensional language in a hands-on and a computer learning environment and to describe the participants' experiences in this setting. Driscoll (1981) identified language as a key component in the application of concepts and knowledge among environments. Using this as a framework, this study focused on three-dimensional geometry and the development of its mathematical language.

The participants for the after-school mathematics program were seven sixth grade students from the same middle school classroom. Because the study was a descriptive case study, the goal was to describe the seven participants' experiences in developing and applying three-dimensional geometry language. A description of the instructional setting and each participant was used to establish the context for the description of the participants' experiences with three-dimensional geometry.

Five types of instruments were used to gather data regarding participants' experiences in the after-school mathematics program. They were: background questionnaire, interview questions, classroom observations, discussions, and mathematics journals. A background questionnaire and the participant interview before instruction were used to help the instructor/researcher get to know each participant. The classroom observations by the instructor/researcher were used to describe the instructional setting, participants' interactions with each other, and mathematical language development and use. The discussions and the mathematics journals were used to describe the development and use of mathematical language by the participants. Finally, responses to the second interview helped summarize the participants' development and use of three-dimensional geometry language.

A ten-lesson unit on three-dimensional geometry was taught in the after-school mathematics program. Instruction occurred in both hands-on and computer learning environments. The unit lasted for two weeks with ten one-hour sessions. To create a classroom setting that would most likely put the students at ease and would resemble a typical classroom, the researcher was the instructor for the after-school mathematics program. The unit was developed based upon the constructivist principles that the student is responsible for his/her own learning and the instructor is to facilitate student learning.

A language-rich classroom atmosphere was provided in both the hands-on and the computer learning environments. The hands-on learning environment was used to develop three-dimensional language and conceptual understandings. Hands-on manipulatives were used to provide a basis for discussing three-dimensional concepts. In these discussion, the participants developed used three-dimensional language to convey their understanding. The computer learning environment was used to apply the three-dimensional language and concepts. Wireman was a tool that appeared to have the potential for illustration of three-dimensionality and to give students experiences with advanced levels of abstraction.

Discussion

The goal of this case study was to describe the participants' development of three-dimensional geometry language in a hands-on learning environment and their application of that language in a computer learning environment. Through analysis of the data gathered, a detailed description of the development and use of three-dimensional geometry language was completed. The following is a discussion of the results of the case study. The discussion consists of four sections: research setting, hands-on learning environment, computer learning environment, and participant experiences.

Research Setting

The after-school mathematics program was conducted in the classroom of the seven participants. The program was after-school for one hour and lasted for two consecutive weeks. The classroom was arranged traditionally with the desks in rows facing the chalkboard in the front of the room. Often, during the program, the instructor/researcher changed the position of the desks to accommodate the activities. A camcorder, audiotape recorders, and portable computers were brought to the classroom by the instructor/researcher for use during the after-school mathematics program.

The atmosphere for the program was more informal than a regular classroom. The instructor/researcher often sat along side the participants during discussions, and the classroom teacher often was not in the room leaving the instructor/researcher in charge. Communication by all of the participants was encouraged by the instructor/researcher-facilitated discussions.

Hands-on Learning Environment

The five hands-on manipulative activity lessons provided the participants with a language-rich atmosphere to develop and use mathematical language. Opportunities to write, speak, listen, and read using mathematical language occurred throughout the hands-on lessons. The role of the instructor/researcher during these lessons was to facilitate discussions by questioning the participants. In the discussions, no one right answer was accepted; the participants were encouraged to express their thoughts and ideas and formulate and note multiple answers.

During the hands-on manipulative activities, the participants were successful at identifying terms and language associated with the concepts. In the introductory lesson for each concept, the participants used mathematical language to reason through their understanding. For example, in the lesson on the scale of three-dimensional objects, the participants used phrases such as "take some off," compact it," "add," and "subtract" to describe change in the scale of an object. These phrases appeared to help the participants understand the meaning of scale. Similar results occurred in the other hands-on activity lessons.

Following the lessons on the basic concepts of three-dimensionality, the participants were involved in an incorporating activity to link the individual concepts they had learned. The activity was to describe to a partner how to create a three object structure. The participants used conceptual terms (i.e. scale, position, and rotation) and other phrases (i.e. smaller, bigger, size, down, and right) to describe the structure.

In the introductory lesson of each concept and in the lesson to incorporate the concepts, the participants used terms not previously identified by the instructor/researcher as well as terms that the instructor/researcher had identified as possible three-dimensional geometry language. Both sets of language allowed the participants to develop basic conceptual understandings of the three-dimensional concepts.

Computer Learning Environment

The three computer lessons provided a language-rich atmosphere for the participants to apply their mathematical language. While using the computer, the participants were able to speak, listen, read, and write using three-dimensional geometry language. Participants were asked to create structures in using Wireman software, write in their journal descriptions of how to create the structure, and discuss with other participants and the instructor/researcher what they had done.

Two computer lessons focused on developing skill in and becoming acclimated to the Wireman program. Tasks to be completed were given to the participants by the instructor/researcher. The tasks chosen gave opportunities for the participants to use their knowledge of all of the concepts they had learned in the hands-on activity lessons such as three-dimensional objects, scale, position, and rotation. Analysis of the three-dimensional language the participants used in these two lessons showed that the participants applied similar language they developed in the hands-on activity lessons to the computer. The participants also used language that did not aid the instructor/researcher in identifying their understanding. Such language included "push the left button on the second row of rotation." In using this language, the participants were identifying what to do, but they were not communicating their understanding of what was to occur.

In the final computer lesson, the participants were to use three-dimensional geometry language to describe a structure they had created to the instructor/researcher. For a majority of the time, the participants applied the same three-dimensional language they used in the hands-on activity lessons to the computer activity lessons. Both conceptual terms and other descriptive phrases were used to explain their structures. For three-dimensional objects, the participants used the names of the objects. The participants used terms such as "bigger," "smaller," and "size" to describe scale. "Up," "down," and "under" were used to describe position. To describe rotation, the participants most often used the term "rotation."

From these results, there is an indication that the application of three-dimensional language developed in the hands-on learning environment occurred in the computer learning environment. The language used included the conceptual terms as well as other terms and phrases used to describe the conceptual terms. The exact role of mathematical language in learning three-dimensional geometry was not clear, but the participants were able to work through the problems given in both the hands-on and computer activity lessons and used mathematical language to do so. Providing a language-rich hands-on environment for the development of three-dimensional language helped the participants apply that language to the computer learning environment.

Participant Experiences

Each of the seven participants began with little knowledge of geometry. Most of them knew what that geometry had to do with shapes, but they were not able to define geometry any further. Through the hands-on and computer activity lessons, the participants' knowledge of three-dimensional geometry expanded. The participants were engaged in mathematical discussions in which they used three-dimensional geometry language to portray their understanding.

Throughout the after-school mathematics program, the participant's experiences often varied in three ways: overall disposition, use of mathematical language, and knowledge of three-dimensional geometry. Each participant was unique in their overall disposition. An individual's disposition often affected their learning process. The participants' use of mathematical language also varied depending on their level of responsibility for their own learning and comfort during the instructional activities. Most of the participants were able to use mathematical language to express their understanding in small and whole group discussions, interviews, and journals. Although many of the participants preferred one method over others, both conceptual terms such as "position, scale, and rotation" and common language such as "bigger, smaller, and straight" were used as mathematical language by the participants. The participants' knowledge of three-dimensional geometry also varied among the participants. Some were able to expand their initial definition of three-dimensional geometry (given in their mathematics journal after the lesson on three-dimensional objects). Some participants gave the exact definition at the beginning and the end of the after-school mathematics program.

Recommendations

This case study focused on describing seven sixth grade participants' development and application of three-dimensional geometry language. In examining the format and results of this study, four recommendations for future research are suggested.

More case studies focused on mathematical language development need to be conducted. This case study focused on three-dimensional geometry language; case studies on other areas of mathematics such as areas problem solving, estimation, and computation skills may provide insight into how students use language to develop and convey their understanding. These case studies could involve participants at various grade levels. Studying the development of mathematical language at various grade levels may provide insight into language's developmental role in facilitating student understanding.

Making the after-school mathematics program a regular part of classroom practice is the second recommendation for future research. For this study, the program was conducted after-school which may not have been the most ideal time for participant learning. The participants often appeared to be restless and lack discipline after being in school all day. During the after-school mathematics program, the participants often were easily distracted. If the study were conducted during the regular school day, then the program would be part of the participants' regular daily schedule. As a result, teacher and parent expectations would be different, and thus participant behavior and learning may differ also. This could possibly change some of the results of the study.

The third recommendation for future research is to conduct the study for a longer period of time. With only two weeks for the after-school mathematics program, the participants did not have an extended period of time to full develop their mathematical language. Conducting the activities for three or four weeks would give the participants a chance to fully develop and use their mathematical language; the application of which, it is hypothesized, would be greater and more natural. This type of study could provide a deeper understanding of how the development of mathematical language occurs over a period of time.

Finally, incorporating more three-dimensional geometry concepts into the participants' learning experience would provide them with a more complete understanding of geometry and help them see relationships among concepts. In the after-school mathematics program, the three-dimensional geometry concepts taught were objects, position, rotation, and scale. The next step may be the concepts of perspective of viewpoint. Wireman has the capability of allowing the user to view objects from various viewpoints; incorporating this concept along with those previously mentioned may help students better understand all of the concepts.

Conclusion

The development of mathematical language can have a significant impact on mathematics understanding. If given the opportunity to read, write, speak, and listen mathematics, students can develop and use mathematical language. The instructor's role is to facilitate mathematical discussions in which the students use the mathematical language to communicate and develop their understanding of geometry. This study described the participants' experiences in developing and applying three-dimensional geometry language to aid in their understanding of three-dimensional geometry. The after-school mathematics program participants developed three-dimensional geometry language and applied that language to understanding three-dimensional concepts in a more abstract environment, Wireman.

When given a single means of communicating their understanding of a concept in typical classroom instruction, students' understanding of the concept may be limited because of individual learning styles and personal preferences. However, providing multiple methods for students to communicate understanding of a concept may help them develop more complete knowledge of the concept. In this study, a language-rich, constructivist environment was created for the participants to develop and apply mathematical language. Focusing on three-dimensional geometry, this environment used activities such as journals and small group projects that encouraged the participants to use the four forms of language. The level of communication varied; it was apparent that each participant had a preferred method of participation with which they were most comfortable. As a result, their mathematical language increased when they were given the opportunity to communicate in their preferred manner. Therefore, providing multiple ways for students to convey their understanding of a concept may better accommodate groups of students and complement individual learning styles.

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APPENDIX A. BACKGROUND QUESTIONNAIRE

BACKGROUND QUESTIONNAIRE

1. How old are you?

2. What grade are you in?

3. Gender: (circle one)

MALE FEMALE

4. In what field are your parents/guardians employed?

5. How many children are there in your family (including yourself)?

6. What is your favorite subject?

7. What is your least favorite subject?

8. What computer experience do you have?

9. What are your hobbies?

10. Why did you choose to participate in this program?

11. What are you currently studying in mathematics?

12. What do you plan on doing after you graduate from high school?

APPENDIX B. FIRST INTERVIEW QUESTIONS

FIRST INTERVIEW QUESTIONS

1. What words do you think of when I say "mathematics"?

2. How do you use mathematics in your everyday life?

3. What subject or subjects is mathematics most like? Least like? Why?

4. Describe a person in your class or school who you think is mathematically talented? (no names, please)

5. Imagine a mathematician
   - describe what he/she would be like, where work, what do, what work with, or other information
   - draw a picture

APPENDIX C. JOURNAL QUESTIONS

JOURNAL QUESTIONS

Lesson 1:

- What is geometry?

Lesson 2:

- What does it mean to be three-dimensional? - draw pictures, give examples

Lesson 3:

- If I had a torus that was this big (show example), describe to someone how you would scale the torus to this other torus (show example)?

Lesson 4:

- If I placed the ball over here, give me three-dimensional directions on how to get to this location?

Lesson 5:

- If someone asked you to rotate something, what questions would you ask them to accomplish this?

Lesson 6:

none

Lesson 7:

- How does Wireman relate to mathematics?

Lesson 8:

- How would you give directions to me on how to change this picture?
- Write directions on how to create a Lifesaver lollipop.

Lesson 9:

- Write directions on creating your structure.

Lesson 10:

none

APPENDIX D. SECOND INTERVIEW QUESTIONS

SECOND INTERVIEW QUESTIONS

1. What three-dimensional mathematical language did you learn?

2. What activities helped you to learn three-dimensional mathematical language?

3. If you were to describe to fellow classmates what three-dimensional means, what would tell them?

4. How did what you learned in the hands-on activities help you to complete the computer activities?

APPENDIX E. DAILY LESSON PLANS

DAILY LESSON PLANS

Lesson 1:

Objectives:

- To get to know one another
- To interview students to assess knowledge about mathematics

Concepts to learn:

- The presence of mathematics in our lives

Time: 1 hour

Materials: first interview questions, questionnaire, 6 newspapers, paper, pencil, journals

Steps of Instruction:

1. Students and teachers informally introduce themselves to each other.
2. Give students each a journal.
3. Describe the purpose and audience of mathematics journals - your class at school who knows nothing about these concepts you are learning.
4. Interview each student individually. Ask them the five questions from first interview questions.
5. Have students fill out the background questionnaire.
6. Give each child a section of a newspaper.
7. Have them find 10 places in the paper that involve mathematics.
8. Have students mark the ones that have to do with geometry.
9. Discuss the results of their findings.
10. Introduce what will be happening in the next couple of weeks.

11. Have students answer the following question in their journals.

    "What is geometry?"

Evaluation:

- answers to questionnaire
- interview questions
- journals

Questions:

1. What was the result of your newspaper search?

Lesson 2:

Objectives:

- To introduce students to 3-dimensional objects

Concepts to learn:

- Solid 3-D geometry objects

Time: 1 hour

Materials: real-life objects: balls, dice, boxes, party hats, bagels, toy rings, coffee cans, and paper, worksheet for reporting findings, journals, pencils

Steps of Instruction:

1. Discuss with students what they know about three dimensions or X Y Z axis.
2. Put the seven students in pairs.
3. Pass out a set of "real-life" objects to each group.
4. Pass out to each pair the worksheet for classifications of objects - discuss terms in column headings.
    - faces - flat side
    - edges - faces meet to form edges
    - vertices - edges meet to form vertices
5. Ask the students what they think each of the real life objects is called in the rows of the worksheet. Discuss what to do with worksheet.
6. Have students in pairs fill out the worksheet for each object. Have them discuss what they think and why.
7. Have a class discussion on answers to sheet.
8. Discuss what it means to be three-dimensional.

9. Compare the concepts of three-dimensional versus two dimensional. Give examples.
10. Hand out mathematics journals.
11. Have them write in their journals.

    "What does it mean to be three-dimensional" - draw picture, give examples

Evaluation:

- Class discussion
- Mathematics journals

Questions:

1. What does "three-dimensional" mean to you?
2. What are the names of the shapes of these real-objects?
3. What answers did you get on each of the boxes on the classification worksheet?
4. How did you get that number?
5. Does anyone differ in their answer?
6. How do these objects compare to two dimensional objects?

Lesson 3:

Objectives:

- To have students learn what happens when the scale is changed
- To realize that there are three ways (X, Y, Z axis) to change the scale of an object

Concepts to learn:

- Scale

Time: 1 hour

Materials: play dough, real-life objects, journals, pencils

Steps of Instruction:

1. Review what was discussed in the previous lesson on 3-D objects. Share journals.
2. Ask students what they think it means when you change the "scale" of an object.
3. Have available a set of different sizes of the real-life objects.
4. Give each pair a set of the real life objects.
5. As a group, discuss each object individually and compare the sizes of the same shape. For example, pair one may have a larger cylinder such as an apple juice can and pair two will have a soup can. The class will then compare the size of these two cylinders.
6. Construct three sizes of spheres with play dough.
7. Ask students the following questions.
   - Which one of these is a sphere?
   - How do the objects compare?
   - How were the objects changed to make another object?

8. Ask students to identify different ways to change the scale.
9. Repeat step 7 and 8 with different shapes, if time.
10. Make two sizes of cones out of play dough. Have them write in their mathematics journal the answer to the following question.
    "If I had a torus that was this big (show example), describe to someone how you would scale the torus to this other cone (show example)?"

Evaluation:

- mathematics journals
- class discussions

Questions:

1. What do you think it means to change the scale of one of these real-life objects?
2. What are different ways you can change the scale of an object?
3. Which one of the following is a cylinder?
4. How do the cylinders compare?
5. How were the cylinders changed to make the other objects?
6. What does your shape look like?

Lesson 4:

Objectives:

- To learn about position of 3D objects
- To learn about different ways of moving objects in space

Concepts to learn:

- Position

Time: 1 hour

Materials: ball, three different colors of string, separate room, pencils, journals

Steps of Instruction:

1. Review what was discussed in the previous lesson on scale. Share journals.
2. Ask students what they believe changing the position of the objects mean in relation to what they learned previously.
3. Take students to a room which has been created by the instructor which has been constructed as a real model of three dimensions. Use different color string and make sure there is an origin in the middle.
4. Ask students what this room looks like. Try to get the three dimensional aspect out. Ask why there are three different colors.
5. Ask students what terms they would use to describe the movement.
6. The instructor will take a ball and stand in part of the room.
7. Have students take turns giving you directions as to how to get the ball there.
8. Move students around in the room.
9. Have them describe in words in their journals on how to give someone directions to get there.

10. Collect the journals - give the journals to another student than whose it is.
11. Have each student read those directions and move accordingly.
12. Discuss results. Did you end up where you were supposed to?
13. Repeat steps 9 - 12 except using the location of an inanimate object.
14. Have them write in their mathematics journal.

    "If I placed the ball over here, give me three-dimensional directions on how to get to this location?"

Evaluation:

- mathematics journals
- classroom discussions

Questions:

1. What did we discuss yesterday?
2. What do you think it means to change the position of something?
3. What does the room look like?
4. What do you observe as you take a look around the room?
5. What do you think the room is supposed to model?
6. Why are three different colors of string used?
7. What terms could you use to describe the ball movement?
8. Was that the correct/incorrect action based on directions given?
9. What should have been done differently?

Lesson 5:

Objectives:

- To learn about the rotation of 3D objects
- To learn about different ways to rotate an object

Concepts to learn:

- Rotation

Time: 1 hour

Materials: real-life objects, worksheet to draw views on (3 per student), pencils, table for each group, journals

Steps of Instruction:

1. Review what was discussed in the previous lesson on position. Share journals.
2. Ask students what they believe rotating the objects will do.
3. In their groups, students will put a table in the middle and will put an object in the middle.
4. Each students will sit at the four corners around the object - look at object from eye level.
5. Have students roughly sketch what the objects look like from that angle. Have students sketch it in the View A box.
6. Rotate the object one-quarter turn.
7. Have students again draw what they see. Have students sketch it in the View B box.
8. Have students repeat this until they have completely rotated the object around (4 views).

9. Have students compare their different views. Items to discuss include how their drawings compare, what does rotating do to the object, does rotating change the size of the object, are there other ways to rotate besides a quarter turn.
10. Repeat steps 4-8 using rotation with other objects.
11. Discuss as a class the results of their group discussion which relate to the issues stated above.
12. Ask students if there are other ways to rotate an object than the method we just did. Try sketching that rotation.
13. Have them write in their mathematics journals.

    "If someone asked you to rotate something, what questions would you ask them to accomplish this?"

Evaluation:

- mathematics journals
- classroom discussions

Questions:

1. What object manipulation did we discuss last time?
2. What do you think it means to rotate an object?
3. How do the drawings of the views of each group member compare?
4. Does rotating an object change the size of the shapes?
5. What is another method of rotation?
6. How did I rotate the object to get this view?

Lesson 6:

Objectives:

- To reinforce three-dimensional language learned in hands-on environment
- To have students practice what they have learned

Concepts to learn:

- Practice of three-dimensional language

Time: 1 hour

Materials: journals, pencil, paper, play dough, real objects

Steps of Instruction:

1. Discuss what we have learned thus far.
2. Ask students what is a structure they could build with these shapes.
3. Each student will think of a structure using three of the 3-D objects we learned about. He/she will sketch it out in their journals.
4. Students can use real-objects to see how things fit together.
5. Remind students to think of shapes and use the three object manipulations that we just talked about.
6. Have students write out in their journals.

   "What directions would you give your partner on how to create the structure?"
7. In their pairs, one student will describe to their partner on how to create the structure using play dough.
8. As the other student is building, the students giving instructions can observe and change the directions as necessary.
9. Show instructor final results and compare.

10. After okay from instructor, the partners can switch roles.
11. Repeat steps 7-9.
12. Discuss with the students the results of the activity.
13. Ask students what were the words they used to describe the structure.

Evaluation:

- tapes of group conversations while constructing sculptures
- mathematics journals

Questions:

1. What structures could you build with these shapes?
2. What were the difficulties in instructing your partner in building the sculptures?
3. What were the difficulties in creating what your partner instructed you to?
4. What object manipulation attributes did you use?
5. What words did you use to describe the structure?

Lesson 7:

Objectives:

- To have students become familiar with the basics of the computer
- To introduce students to the Wireman program and its function

Concepts to learn:

- Basic computer skills
- Wireman components

Time: 1 hour

Materials: seven computers, journals, pencils, real - life objects (if necessary)

Steps of Instruction:

1. Discuss with the students that we will now change the environment of learning to the computer.
2. Ask students how many have ever used a computer before.
3. Discuss basics of the Macintosh computers - mouse, saving, opening files, disks.
4. Have students open Wireman.
5. Close all windows except scale, position, rotation, and objects.
6. Discuss with the students the components of Wireman. Ask if they see familiar items.
7. Have students put on the screen one of the objects. Have students explore the three object manipulation attributes and what happens to the objects.
8. Have the students construct the letter "T" in Wireman using what shapes they wish- first on paper, then on computer.
9. As a class, discuss what they used. Display their results.

10. Repeat steps 6 and 7 using the letter "H".
11. Have students construct a "p", then "b", and then "d".
12. Allow students to continue experimenting in Wireman.

Evaluation:

- mathematics journals

Questions:

1. How many have ever used any type of computer before?
2. How do you open a file?
3. How do you save a file?
4. What components of Wireman look familiar to you?
5. What is unfamiliar?
6. What did you use to construct the letter "T"? "H"?
7. What did you change on the shapes you used in the letter "T"? "H"?

Lesson 8:

Objectives:

- To have students construct an object using the wire frame shapes and the object manipulation attributes
- To have students use the language they learned in the hands-on activities to figure out how to construct an object using Wireman

Concepts to learn:

- Develop skills in using Wireman

Time: 1 hour

Materials: seven computers, Wireman program, pencils, paper, real life objects, journals

Steps of Instruction:

1. Ask students what shapes they would use to create an ice cream cone.
2. Have students construct their ice cream cone on their own screen.
3. Discuss that now we want to change that to a dunce hat on someone's head.
4. Have them write in their journals.

   "How would you give directions to me on how to change this picture?"
5. While you are figure it out, you can experiment with it on the computer.
6. When complete, have each student give the instructor the directions he/she wrote.
7. Ask the student if this is what is to happen.

8. Ask students what they would use to create a Lifesaver lollipop. Draw a picture for them if they do not know what one is.
9. Write directions in their journal.
10. Have their partner read the written directions and create it.
11. Discuss results. Verbally discuss possible changes in directions.
12. Switch partners.

Evaluation:

- Class discussion
- Partner interactions
- Mathematics journals

Questions:

1. What wire frame shapes would you need to create a one scoop ice cream cone?
2. How can you change that to a dunce hat?
3. Is this the way you wanted to the dunce hat to look like?
4. What would you use to create a Lifesaver lollipop?

Lesson 9:

Objectives:

- To assess students use of three-dimensional language

Concepts to learn:

- none

Time: 1 hour

Materials: pencils, seven computers, journals, Wireman

Steps of Instruction:

1. Have students individually decide on a structure to create using three of the three-dimensional objects discussed in the lesson two (not the same structure as before).
2. Have students think of all the objects and object characteristics learned. Incorporate them into their structure.
3. Use mathematics journals as a place to sketch out and write directions to create the structure.
4. As an instructor, I will sit at the computer and you describe to me what I should create.
5. When the students are ready, I will take turns with each student.
6. This interaction will be separate from the rest of the students.
7. Make sure students describe and not just say "push this button three times".
8. Record these interactions.
9. If instructor is not with student creating the structure, he/she can play around with the Wireman program.

Evaluation:

- tapes of group conversations while constructing sculptures
- mathematics journals

Questions:

1. What instructions will you give me to create the structure you designed?
2. Could you be more descriptive in your explanation?

Lesson 10:

Objectives:

- To assess their language by interviewing them

Concepts to learn:

- none

Time: 1 hour

Materials: second interview questions, pencil, Wireman, seven computers

Steps of Instruction:

1. Interview students on an individual basis.
2. Four questions will be asked of each student to have them summarize their experiences. The answers to these questions will give an idea of the perceptions each student has about three-dimensional geometry and the different environments.
3. While each student is being interviewed, have the rest of the students use Wireman.

Evaluation:

- interview questions

Questions:

APPENDIX F. OBJECT CHARACTERISTIC WORKSHEET

NAME OF SHAPE	# OF FACES	# OF VERTICES	# OF EDGES	DESCRIPTION OF OBJECTS
SPHERE				
SQUARE				
CONE				
CYLINDER				
SUPER SPHEROID				
TORUS				

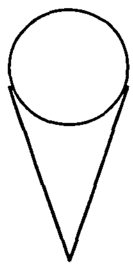
APPENDIX G. ROTATION WORKSHEET

VIEW A	VIEW B
VIEW C	VIEW D

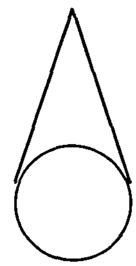
APPENDIX H. TWO COMPUTER TASKS IN WIREMAN

TASK ONE

ICE CREAM CONE

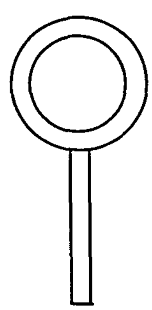


DUNCE HAT



TASK TWO

LIFESAVER LOLLIPOP



APPENDIX I. POSSIBLE THREE-DIMENSIONAL GEOMETRY LANGUAGE

POSSIBLE THREE-DIMENSIONAL GEOMETRY LANGUAGE

Three dimensional objects

- vertices
- face
- edge
- shape
- surface
- sphere
- torus
- square
- super spheroid
- cylinder
- cone

Scale

- scale
- size
- add
- subtract
- larger
- smaller
- wider
- skinnier
- fatter

Position
- forward
- back
- right
- left
- up
- down
- position
- horizontal
- vertical
- straight
- axis

Rotation
- rotate
- flip
- turn
- slide
- spin
- clockwise
- counterclockwise

APPENDIX J. HUMAN SUBJECTS APPROVAL

Last Name of Principal Investigator Cronin

Checklist for Attachments and Time Schedule

The following are attached (please check):

12. [X] Letter or written statement to subjects indicating clearly:
    a) purpose of the research
    b) the use of any identifier codes (names, #'s), how they will be used, and when they will be removed (see Item 17)
    c) an estimate of time needed for participation in the research and the place
    d) if applicable, location of the research activity
    e) how you will ensure confidentiality
    f) in a longitudinal study, note when and how you will contact subjects later
    g) participation is voluntary; nonparticipation will not affect evaluations of the subject

13. [X] Consent form (if applicable)

14. [ ] Letter of approval for research from cooperating organizations or institutions (if applicable)

15. [X] Data-gathering instruments

16. Anticipated dates for contact with subjects:

First Contact	Last Contact
August 28, 1995 *	September 30, 1995
Month / Day / Year	Month / Day / Year

* This is contact with teacher to obtain permission

17. If applicable: anticipated date that identifiers will be removed from completed survey instruments and/or audio or visual tapes will be erased:

May 30, 1996
Month / Day / Year

18. Signature of Departmental Executive Officer | Date | Department or Administrative Unit

8-21-95 Curriculum and Instruction

19. Decision of the University Human Subjects Review Committee:

✓ Project Approved ___ Project Not Approved ___ No Action Required

Patricia M. Keith 9-13-95
Name of Committee Chairperson | Date | Signature of Committee Chairperson

GC:1/90

APPENDIX K. PRINCIPAL CONSENT FORM

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

College of Education
Department of Curriculum and Instruction
N157 Lagomarcino Hall
Ames, Iowa 50011-3190
515 294-7003
FAX 515 294-6206

August 23, 1995

Deborah Hunter
Woodside Middle School
Saydel Consolidated Schools
5810 NE 14th Street
Des Moines, IA 50313

Dear Ms. Deborah Hunter:

I am a graduate student at Iowa State University working on my masters degree in Curriculum and Instructional Technology. I am interested in conducting my thesis research with middle school-aged students from Saydel Consolidated Schools. The following is an explanation of my study.

The topic of my research is an examination of the development and transfer of three dimensional geometry language from a hands-on to a computer learning environment. This study will address two problems that exist in mathematics education: the prevalence of complex and difficult mathematics terminology and the transfer of concepts from one learning environment to another. In conducting this study, I hope to provide insight into the educational benefits of the development and the use of mathematical language in the classroom. The NCTM Standards identify the need for instruction in mathematics as communication. Therefore, research is needed to aid in developing mathematical language skills as a means of communication.

For my study, a total of six middle school-aged participants will begin by being involved in a language intensive hands-on environment. Using manipulatives, students will learn about four concepts: 1) three dimensional objects, 2) scale of these objects, 3) position of these objects, and 4) rotation of these objects. Upon completion of the hands-on activities, the students will be placed in a computer learning environment to assess the transfer of the mathematical language from the hands-on setting. Attached is a copy of the daily lesson plans to be used in both the hands-on and computer learning environment.

The research for this study will last for two weeks. The six participants will attend one hour sessions each day after school for ten days. The location of the study will be in a classroom at the school in which the participants attend. All supplies and equipment will be provided by me, the researcher. After receiving approval from Saydel Consolidated Schools, the student participants will be selected by a cooperating teacher. Final approval for these students to be involved in this study will be granted in writing by their parents or guardians. Participation in this study will be on a volunteer basis. Each participant who completes all ten days of activities will receive a $25.00 stipend.

In the study, four types of data collection will be used: a background questionnaire, interviews, journals, and taped conversations. The background questionnaire will provide demographic information on the participants. Two sets of interviews will be conducted: one before instruction and one after instruction. The purpose of the first interview is to assess the participants' attitudes about mathematics. The purpose of the second interview is to assess the participants' perceptions about what they have learned. Group discussions in both the hands-on and the computer learning environment will be audio taped to identify the mathematical language used. Mathematics journals will be used to gather written data about student learning. Attached are copies of the background questionnaire, interview questions and journal questions.

I would like to obtain your approval as soon as possible so I can continue moving forward with my research. If you have any questions, please feel free to contact me at 294-6840 or Dr. Connie Hargrave (my major professor) at 294-5343. If you need additional information, we would be happy to meet with you to further discuss the research.

Thank you.

Sincerely,

Beth Cronin
ISU Graduate Student

Dr. Connie Hargrave
Assistant Professor
Department of Curriculum and Instruction

I have read the letter and understand what the study will entail. I am willing to allow six middle school-aged students from the Saydel Consolidated Schools to participate in the study described above.

Signature ____________ Principal, Woodside Position Sept 7, '95 Date

APPENDIX L. PARENTAL CONSENT FORM

IOWA STATE UNIVERSITY

OF SCIENCE AND TECHNOLOGY

College of Education
Department of Curriculum and Instruction
N157 Lagomarcino Hall
Ames, Iowa 50011-3190
515 294-7003
FAX 515 294-6206

September 7, 1995

Dear Parent/Guardian:

I am currently a graduate student at Iowa State University working on my masters degree in Curriculum and Instructional Technology. I am interested in conducting my research with six middle school-aged students at your child's school. These six students will volunteer to participate in this study for two consecutive weeks. Each session will be after school and will last for one hour per day. The study will begin on Tuesday, September 12, 1995 and will last until Tuesday, September 26, 1995. Each participant who completes all ten days of activities will receive a $25.00 stipend..

The topic of my research is the examination of the development and application of three dimensional geometry language from a hands-on to a computer learning environment. I will be collecting four types of data from the students: questionnaire, interviews, journals, and taped conversations. Prior to instruction, a background questionnaire will be completed by the participants to obtain demographic data. Two individual interviews will be conducted: one before instruction and one after instruction. Each interview will last approximately twenty minutes per participant. The first interview will assess students' perceptions toward mathematics, and the second interview will assess students' perceptions about what they have learned. Everyday, students will be asked to write responses to questions in their journals. Class discussions throughout the research process will be audio taped. Although names may be included in the data collection process, the participants will be identified by a pseudonym in the data analysis process. Next spring all tapes and transcripts from the research process will be destroyed.

Please discuss this with your child and decide if this something he/she can and wants to participate in. We need your permission in order to conduct this study. Please fill out and return the enclosed form to your child's teacher by September 12, 1995 if your child can participate. If you have any questions, please contact me, Beth Cronin at 294-6840 or Dr. Connie Hargrave (my major professor) at 294-5343. Thank you.

Sincerely,

Beth Cronin
ISU Graduate Student

Dr. Connie Hargrave
Assistant Professor
Department of Curriculum and Instruction

PARENTAL CONSENT

_____ I give my child permission to participate in this study on the following dates: Tuesday, September 12, 1995 to Tuesday, September 26, 1995.

______________________ Date

________________________ Parent/Guardian signature

Child's name: ____________________________

Child's teacher: ____________________________

APPENDIX M. PARTICIPANT LETTER

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

College of Education
Department of Curriculum and Instruction
N157 Lagomarcino Hall
Ames, Iowa 50011-3190
515 294-7003
FAX 515 294-6206

September 7, 1995

Dear Participant:

I am currently a graduate student at Iowa State University working on my master's degree in Curriculum and Instructional Technology. Your teacher has identified you and five other students as possible participants in a research study. The study will last for two weeks and will be after school for one hour per day. The study will begin on Tuesday, September 12, 1995 and will last until Tuesday, September 26, 1995. Participation in this study is voluntary. If you participate in all ten days of activities, you will receive a $25.00 stipend.

The research involves learning about three dimensional geometry. The first lessons will include hands-on activities to develop your understanding of this topic. Then, you will use the computer to apply what you have learned. I am interested in the mathematical language you will use in both of these environments. I will collect four types of information for assessment purposes: a background questionnaire, two sets of interviews, journal questions, and class discussions. In analyzing the information I receive, your name will not be used. You will be identified by another method.

A letter will be sent home to your parents or guardians to obtain their permission. Please discuss this information with them. If you have any questions, please contact me, Beth Cronin at 294-6840 or Dr. Connie Hargrave (my major professor) at 294-5343.

Thank you.

Sincerely,

Beth Cronin
ISU Graduate Student

Dr. Connie Hargrave
Assistant Professor
Department of Curriculum and Instruction