SLOWING DOWN LENGTH OF NEUTRONS IN HYDROGENOUS MEDIA

by

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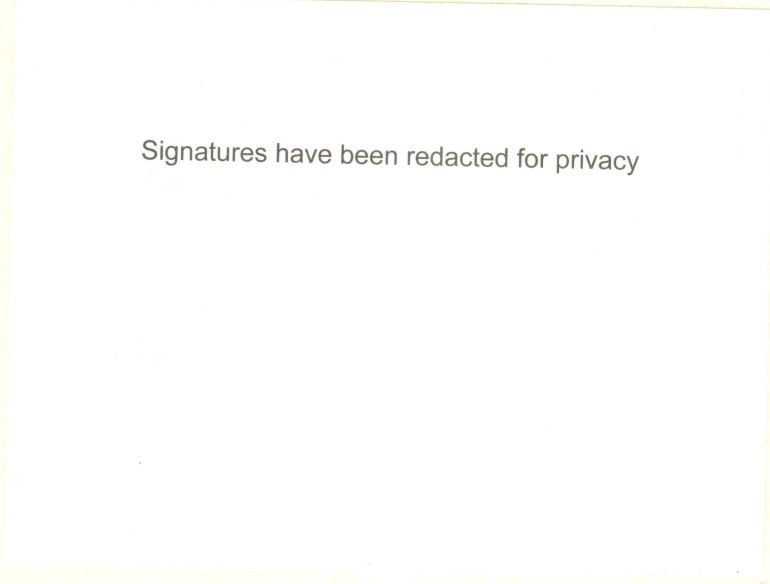

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I. INTRODUCTION

The development of thermal nuclear reactors during the past several years has given rise to a new set of problems concerning the moderator to be used in these reactors. There are many situations in which a liquid moderator would be of great value, due to its ease of removal and its inherent stability. Liquids are stable since if the power level becomes too high, the liquid will boil, thus displacing moderator with air and lowering the power level. They may also be circulated and act both as a moderator and as a cooling medium. Two liquids presently being used as moderators are water and heavy water. Heavy water is the better moderator of the two, but its cost is nearly prohibitive. Water has many advantages as a moderator. It is easily handled, does not combine readily with other materials, and is abundant and therefore relatively inexpensive. Its components, hydrogen and oxygen, are very good scattering elements, hydrogen being the best available.

Water is also an excellent solvent, and it is this property that was used in an attempt to increase its

usefulness as a moderator. Sucrose ($C_{12}H_{22}O_{11}$) was dissolved in distilled water. This led to a more dense solution of three good moderating elements: carbon, hydrogen, and oxygen.

To determine the moderating effectiveness of the solutions, the migration area was measured experimentally using a radium-beryllium neutron source and a thermal neutron detector suspended in a stainless steel tank containing the solution.

As a check on the experimental values obtained, an approximate method of calculating neutron age for hydrogenous materials (as suggested by Tittle (1)) was used in this investigation.

II. REVIEW OF LITERATURE

Little research has been done with moderators since the early days of nuclear reactor development. At that time, data were collected on all basic moderating materials and have since been distributed widely. However, almost no data are available on hydrogenous moderators other than water and heavy water, reflecting the lack of work in this area.

Many investigators have determined neutron distributions in media varying from grids of bismuth and water to quantities of furfural in an effort to find efficient neutron radiation shielding. A good moderator is an element or group of elements of low mass number and low neutron absorption, so that this work is not compatible with moderator development since good neutron shields absorb many neutrons.

At this point in the reactor program, emphasis is shifting from thermal reactors to reactors operating in the intermediate and fast neutron velocity ranges which require little or no moderator. However, it is safe to state that the demand for good moderating material will be heavy for some time to come, due both to its use in thermal power reactors and to its use as a research material.

III. THEORY

A. General Theory of Moderators

In order to compare the neutron moderating abilities of several different materials it is necessary to consider several of their fundamental characteristics quantitatively. The average logarithmic energy decrement per collision, ξ, is a partial measure of the moderating capacity of the scattering material. However, a large value of ξ, i.e. a large decrease in energy per collision, is of small value unless the probability of a scattering collision taking place is quite large. This probability is represented by the macroscopic scattering cross section Σ_s. The product of these two quantities, $\xi\Sigma_s$, is known as the "slowing down power," and is a better indication of moderating ability. Since a large value of the slowing down power would be useless if the moderator absorbed many of the neutrons, the best indication of moderating ability is the "moderating ratio," $\xi\frac{\Sigma_s}{\Sigma_a}$, where Σ_a represents the macroscopic absorption cross section.

The above quantities are easily computed for any combination of moderators and give the designer the relative merit of these moderators. If the one or two group criticality equation is to be used in a reactor computation, it is necessary to know the numerical value of a term called the migration area and represented by the symbol M^2.

To show how the migration area comes about and why it is important, it is necessary to consider the one-group critical equation for a reactor,

$$k_{eff} = \frac{k_\infty \, e^{-B^2\tau}}{1 + B^2 L_d^2} \qquad (1)$$

where

k_{eff} = effective multiplication factor

k_∞ = multiplication factor for an infinite reactor

B = material buckling for reactor

τ = Fermi age

L_d = diffusion length

$e^{-B^2\tau}$ = probability for non-leakage of fast neutrons

$\frac{1}{1 + L_d^2 B^2}$ = probability for non-leakage of thermal neutrons.

When the reactor is large, Equation (1) may be written as:

$$k_{eff} = \frac{k_\infty}{(1 + L_d^2 B^2)(1 + B^2 \tau)}$$

or

$$k_{eff} = \frac{k_\infty}{1 + B^2(L_d^2 + \tau)}.$$

The quantity $L_d^2 + \tau$ is referred to as the migration area, so that:

$$k_{eff} = \frac{k_\infty}{1 + B^2 M^2}. \qquad (2)$$

In Equation (2), $\frac{1}{1 + B^2M^2}$ represents the probability that a neutron will not leak out of the reactor.

The migration area is actually the sum of the squares of the slowing down length and the diffusion length:

$$M^2 = L_s^2 + L_d^2$$

where

L_s = slowing down length

L_d = diffusion length.

L_d may be calculated by the formula

$$L_d^2 = \frac{D}{\Sigma a},$$

where

Σa = macroscopic absorption cross section

D = diffusion coefficient

$$= \frac{1}{3\Sigma_s (1 - \frac{2}{3A})}. \qquad \text{(for weak absorbers)}$$

Calculating L_s ($L_s = \sqrt{\tau}$ in moderators with mass numbers greater than unity) is more difficult since the moderator investigated contained a large proportion of hydrogen.

The equation for Fermi age, $\tau(E) = \int_E^{E_0} \frac{D}{\xi \Sigma_s} \frac{dE}{E}$, is simply a mathematical transformation which simplifies the derivation of the equations for the spacial distribution of slowing down neutrons under the assumptions of the continuous slowing down model. This model assumes

that the discontinuous process of a neutron in losing energy as it slows down may be replaced by a continuous process. This is nearly true where the scattering nucleii are much heavier than the neutron so that it is possible for it to lose only a small part of its total energy with each collision. However, when the scattering nucleus is hydrogen, the assumption of a continuous process is completely invalid since the neutron may lose all its energy in one collision.

Although Fermi age does not apply to hydrogenous materials, there is a physical quantity present which may be likened to the result obtained under Fermi age considerations. It can be shown that Fermi age is equal to one-sixth of the mean square distance traveled by a neutron from the time of its emission to the time at which it reaches thermal energies.

B. Method of Calculation

Determining "age", or the square of the slowing down length, is not easily done analytically. Marshak (2) has developed the exact mathematical equations but their solution is a long tedious operation. An approximate

method based on an empirical correction has been devised by Tittle (1) and can be applied, after some further modifications, to the case at hand.

A number of investigators have shown that Fermi age is a good approximation of the square of the slowing down length for hydrogenous materials if the initial neutron energies are about 100 kev or less. Above this energy it is necessary to consider other methods of obtaining age data.

It is assumed that the distribution of neutrons which have just made their first collision with a hydrogen nucleus, including those which may have first struck non-hydrogen nucleii one or more times, can be represented (3) by

$$q_o = \frac{Q}{4\pi r^2} e^{-r/\lambda_o} \qquad (3)$$

where λ_o is the "effective mean free path" to the first hydrogen collision, r is the distance from the source, and q_o is the number of neutrons per square centimeter per second. The effective mean free path will include the effect of one or more collisions with non-hydrogen nucleii.

The mean square distance traveled by a neutron from the time it is born to the time it becomes thermal is

given by:

$$\bar{r}_1^2 = \frac{\int_0^\infty r^4 f(r)\, dr}{\int_0^\infty r^2 f(r)\, dr}$$

where f(r) is the functional dependence of the neutron density on the distance r from the source.

If, as stated above, the neutrons follow the distribution law given by Equation (1) at high energies and the distribution given by the point-source solution of the Fermi age equation

$$q(r,\tau) = \frac{e^{-r^2/4\tau}}{(4\pi\tau)^{3/2}}$$

at energies below 100 kev, the mean square distance is:

$$\bar{r}_1^2 = \bar{r}_f^2 + \bar{r}_s^2$$

where:

$$\bar{r}_f^2 = \frac{\int_0^\infty r^2\, e^{-r/\lambda_0} dr}{\int_0^\infty e^{-r/\lambda_0}\, dr} = 2\lambda_0^2$$

and:

$$\bar{r}_s^2 = \frac{\int_0^\infty r^4\, e^{-r^2/4\tau} dr}{\int_0^\infty r^2\, e^{-r^2/4\tau} dr}.$$

Let

$$x = \frac{r^2}{4\tau},$$

then:

$$\bar{r}_s^2 = \frac{16\tau^{5/2}\int_0^\infty x^{3/2}\, e^{-x}\, dx}{4\tau^{3/2}\int_0^\infty x^{1/2}\, e^{-x}\, dx}$$

$$= \frac{4\tau\,\Gamma(5/2)}{\Gamma(3/2)}$$

$$= 6\tau,$$

so that:

$$\bar{r}_1^2 = 2\lambda_o^2 + 6\tau. \qquad (4)$$

If it is further considered that each neutron at energy E_o is on the average degraded in energy with each hydrogen collision by an amount equal to the geometric average of the possible final neutron energies, then:

$$E_1 = \frac{E_o}{e},$$

where E_1 is the new neutron energy and e is the natural logarithmic base. Modification of Equation (2) so that the entire energy range above 100 kev is covered by the λ_o corrections results in

$$\bar{r}_1^2 = 6\tau + 2\lambda_o^2 + 2\lambda_1^2 + 2\lambda_2^2 + \cdots\cdots\cdots 2\lambda_n^2 \qquad (5)$$

where $\lambda_o, \lambda_1, \lambda_2, \cdots \lambda_n$ are the mean free paths for a first collision for neutrons of energy E_o, $E_1 = \frac{E_o}{e}$, $E_2 = \frac{E_o}{e^2}$, ... $E_n = \frac{E_o}{e^n}$, n being such that $\frac{E_o}{e^n}$ = 100 kev (approximately).

All that remains necessary is a method of determining the correct values of λ_o, λ_1, etc. This may be done empirically by using values obtained from Marshak's curve (Figure 1) to solve Equation (4) for λ_o. If λ_o is computed for several energies and the corresponding scattering mean free paths for hydrogenous (λ_H) and non-hydrogenous (λ_x) components obtained for these same

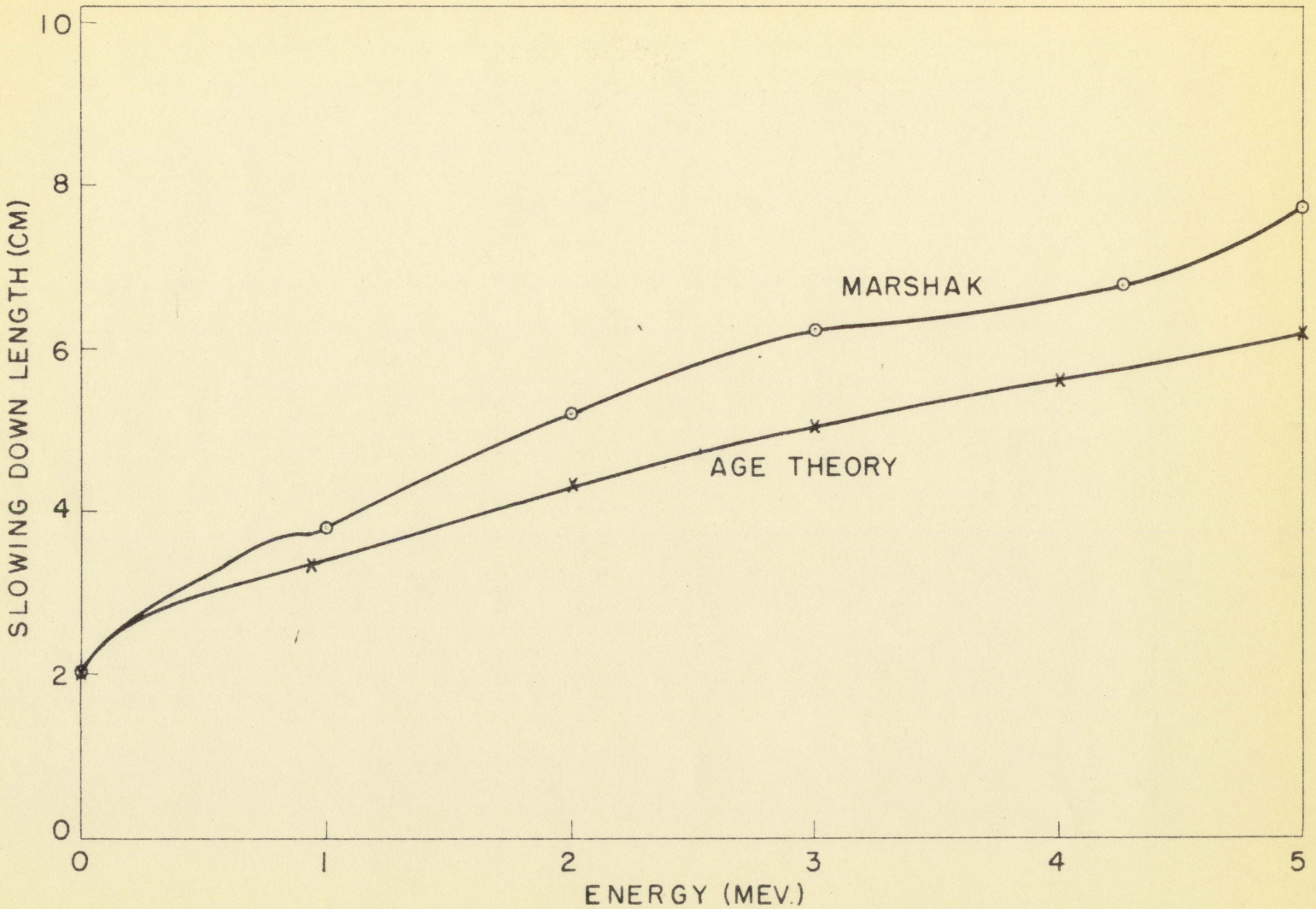


Figure 1. Comparison of slowing down lengths computed from age theory to Marshak's true slowing down lengths

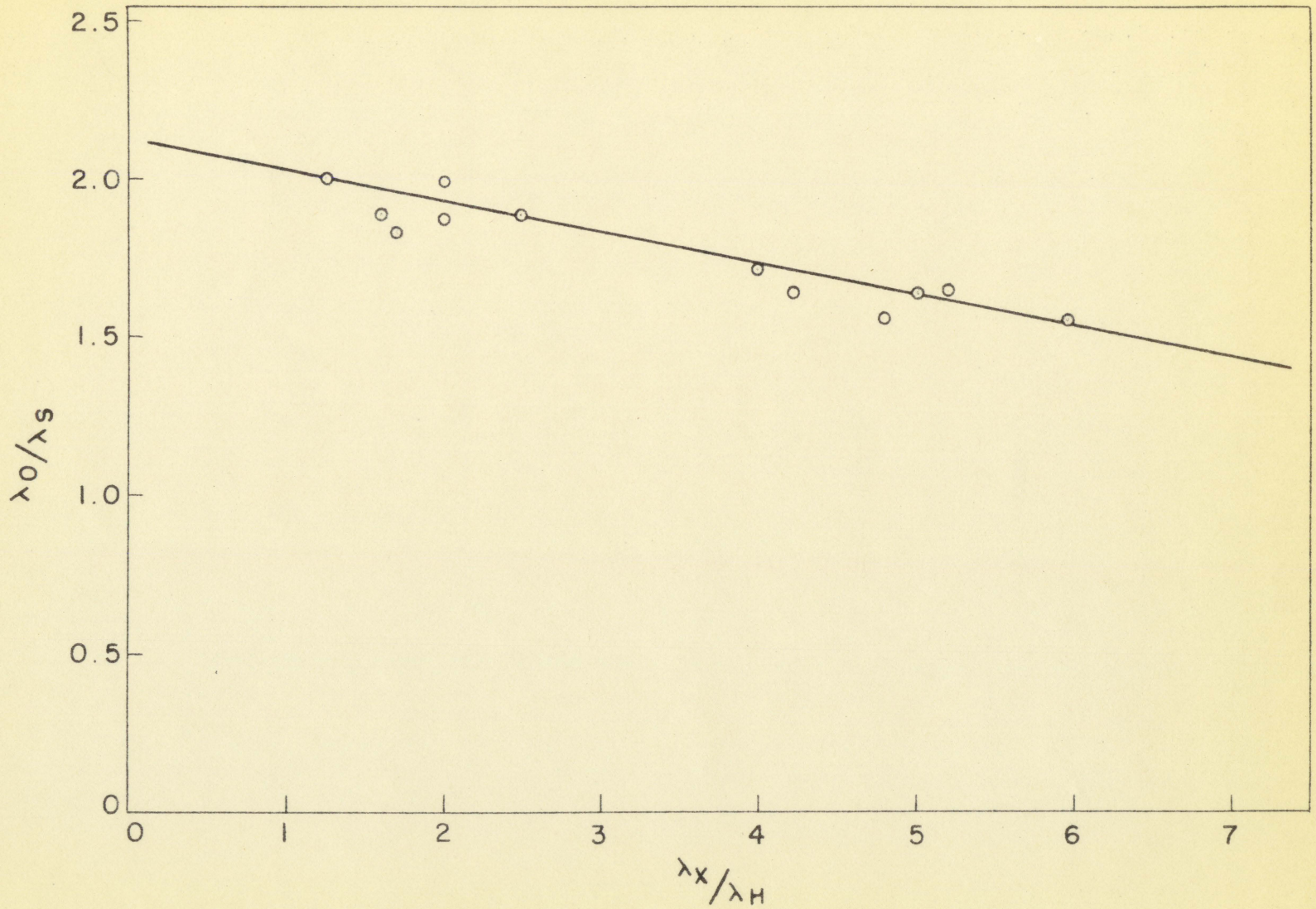


Figure 2. Effective mean free path for first hydrogen collision

energies, it develops that an approximate straight line relationship will hold between the plotted values of λ_o/λ_s and λ_x/λ_H (see Figure 2), where λ_s is equal to the total scattering mean free path. Thus if the assumption is made that the relationship of Figure 2 holds for other hydrogenous media, a general method of evaluating λ_o has been evolved.

C. Basis of Experiment

Regardless of the detailed theory which has been the topic of the foregoing pages, the practical value of the migration area for substitution into the eventual equations of pile theory is the empirical value which is obtained from the Gaussian distribution found by experiment (4). Since M^2 is equal to the slowing down length squared (L_s^2) plus the diffusion length squared (L_d^2), and $L_s^2 = 1/6\ \bar{r}_1^2$ while $L_d^2 = 1/6\ \bar{r}_2^2$ where $\bar{r}_2^2$ is equal to the mean square distance a thermal neutron travels before being absorbed, therefore:

$$M^2 = 1/6\ (\bar{r}_1^2 + \bar{r}_2^2)$$
$$= 1/6\ \bar{r}^2$$

where $\bar{r}^2$ is equal to the mean square distance a neutron travels from entrance into the solution as a fast neutron until the time it is captured as a thermal neutron.

This mean square distance ($\bar{r}^2$) may be obtained experimentally by placing a neutron source in a tank filled with moderating solution and measuring the counting rate (thermal) at various distances from the source. Then the number of neutrons in a spherical shell element dV at a distance r from the source is equal to n(r)dV or $n(r) \cdot 4\pi r^2 dr$, where n(r) equals the number of neutrons per unit volume at a distance r from the source. Therefore the mean square distance, $\bar{r}^2$, becomes

$$\bar{r}^2 = \frac{\int_0^\infty r^2\, n(r)\, 4\pi r^2 dr}{\int_0^\infty n(r)\, 4\pi r^2 dr}$$

$$= \frac{\int_0^\infty r^4\, n(r)\, dr}{\int_0^\infty r^2\, n(r)\, dr}.$$

Since the activity A(r) is equal to a constant times n(r),

$$\bar{r}^2 = \frac{\int_0^\infty K\, r^4\, A(r)\, dr}{\int_0^\infty K\, r^2\, A(r)\, dr}$$

$$= \frac{\int_0^\infty r^4\, A(r)\, dr}{\int_0^\infty r^2\, A(r)\, dr}.$$

From the data obtained, the above expression may be plotted and integrated graphically to obtain $\bar{r}^2$.

IV. EXPERIMENTAL PROGRAM

A. Material

The materials necessary for the experimental determination of the migration area of a moderator are a neutron source, a supply of moderator large enough to simulate an infinite medium, and a neutron detecting apparatus.

The neutron source was a combination of 94.7 milligrams of radium mixed with beryllium and sealed in a monel metal capsule in the form of a right circular cylinder. The cylinder was approximately 0.5 inch in diameter and 0.75 inch high. The source gives a neutron flux of roughly 1400 neutrons per square centimeter per second at 5 inches from its center. There is also a hard gamma flux associated with this source whose maximum energy of 5 mev gives a tolerance distance in air of 10 feet.

The moderating materials were various concentrations of commercial beet sugar dissolved in distilled water. The sugar was purchased locally and a typical analysis is given in the appendix. Three hundred pounds of sugar dissolved in 73.1 gallons of distilled water gave a

Figure 3. Experimental equipment.

A. Neutron counting tube

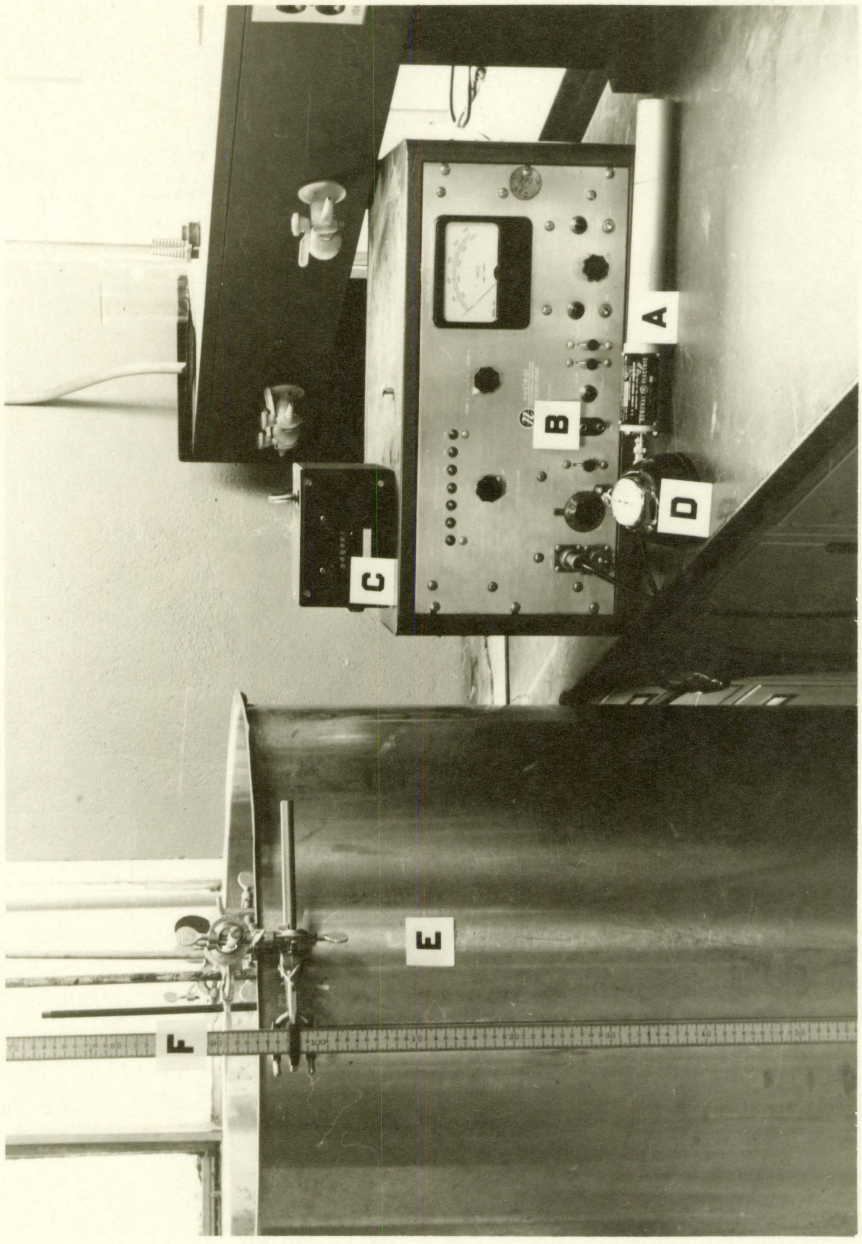
B. Scaler

C. Register

D. Stop watch

E. Tank

F. Meter stick


A
B
C
D
E
F

maximum sugar concentration of 32.2% by weight.

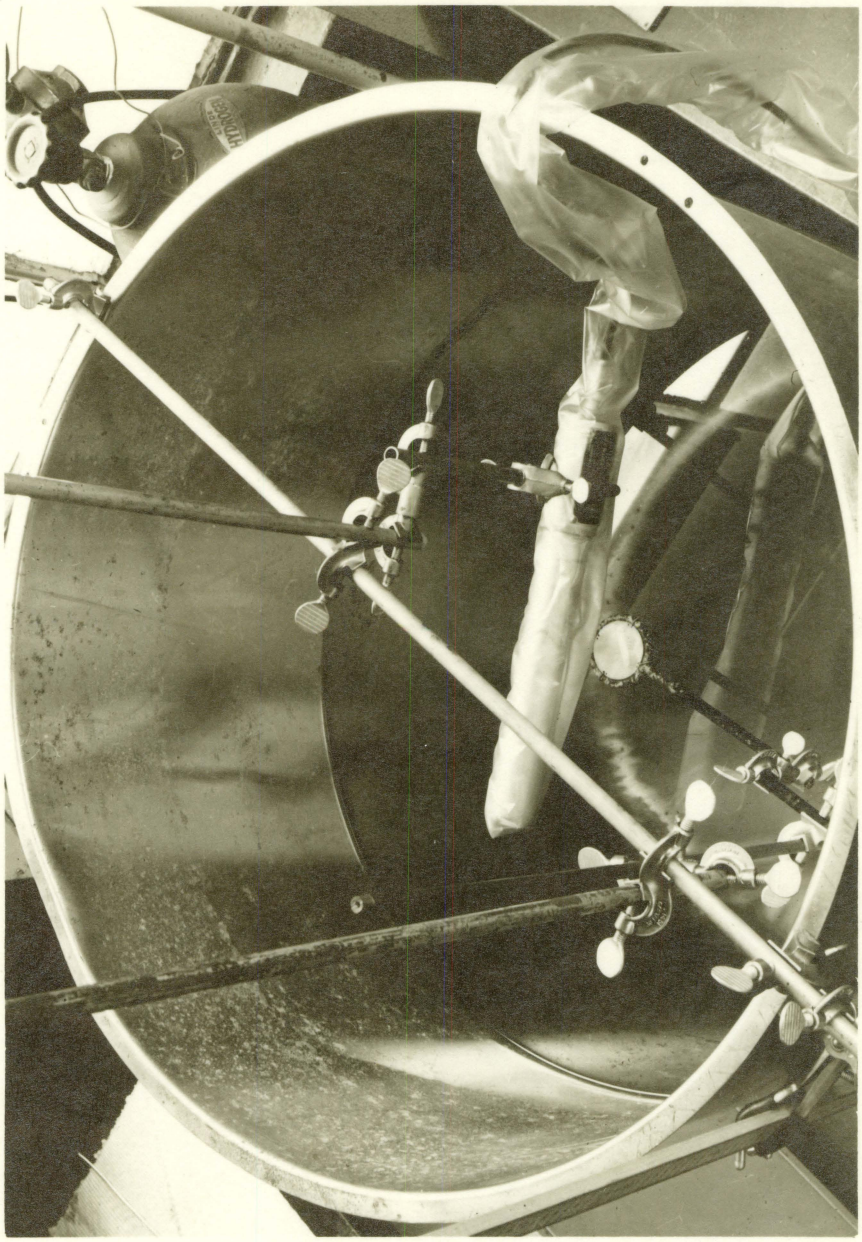
B. Equipment

The major pieces of apparatus are shown in Figure 3. The moderating material was held by a 120 gallon bottom drain stainless steel tank. The tank was cylindrical in shape with a diameter of 29.75 inches and a height of 38 inches. This large size was considered necessary to avoid any wall effects.

The counting circuit was composed of a boron-10 lined proportional counter connected directly to the amplifying circuit of an electronic scaler. The proportional counter was manufactured by General Electric and has a cylindrical active volume of 1.25 inches in diameter by 8.0 inches in length. The B^{10} lining gives the tube a neutron counting efficiency of 7%. A 75 volt plateau of 2.7% slope led the operating voltage to be fixed at 600 volts.

The scaler was a Nuclear Instrument and Chemical Corporation type, model 162. This model has a built-in amplifying circuit so that the scaler may be used with a proportional counter as well as a Geiger tube. It is equipped with a pulse height selector and an attenuation

Figure 4. Arrangement of counting tube and source holder in tank.



switch so that a wide range of activities may be measured. No voltage regulator is needed on the line voltage since the operation of this scaler is independant of line voltage variations between 95 and 130 volts.

Other items of equipment involved were a stop watch for timing counts, a counting register for recording elapsed counts, a meter stick for determining relative source positions, and several clamps and bars for positioning the source and counter.

C. Procedure

The tank was set up with the counting tube fixed in position inside the tank. The source holder (see Figure 4) was constructed so that its position in relation to the counter could be varied and regulated by the meter stick. Since it was necessary for the source and counter to be submerged in the solutions, plastic bags were made which thoroughly waterproofed both.

The tank was filled with water to a height of 25 inches, the source was placed in the source holder, and the counting rate was observed to at least 1% accuracy at small intervals from 0 to 40 centimeters from the source. The migration area was then determined by

plotting the data as described in the section on theory and integrating the curves graphically with a planimeter.

Sixty-one pounds of sugar were added and dissolved and the above procedure repeated. The tank was then drained to its original level and sixty eight more pounds of sugar added with following procedure remaining the same. One-hundred seventy-one pounds were then put in to bring the concentration to its maximum value of 32.2%.

D. Radiation Protection

Due to the high gamma and fast neutron fluxes emanating from the source, it was necessary to practice strict health physics procedures when the source was outside its shielded box. For this purpose several types of instruments were utilized. The gamma flux was monitored by two pocket dosimeters. Fast and slow neutron fluxes were monitored with a film badge. Both of the above could not be read by the individual but were collected and read once per week by members of the Ames Laboratory health physics section.

In order to have an indication of the radiation level at all times in the laboratory, several additional instruments were used. Two self-reading Landsverk pocket

dosimeters were obtained and one was placed on the body and one on the wrist. These gave the wearer an indication of the total milliroentgens of gamma radiation received during the period worn, and could be checked from time to time against a tolerance value of 7.5 milliroentgens per hour. A Tracerlab "cutie pie" rate meter gave direct readings in milliroentgens per hour and was used constantly to monitor the source.

The largest single exposure was received to the hands when the source was placed in the plastic bag and sealed. Twenty milliroentgens were acquired in less than a minute at that time. From then on the source could be manipulated with a fishing pole when moving it from box to tank, giving the laboratory about 50 milliroentgens per hour for a brief time. The level was quite low with the source in the tank, being about 2.5 milliroentgens per hour at the scaler which was four feet from the tank. The radiation at the tank was of the order of 35 to 50 milliroentgens per hour depending on surface location. During the working time the average exposure received was 15 milliroentgens in four hours, much below tolerance, most of which was received when making source adjustments at the tank.

V. RESULTS

A. Water

Graphical integration of the curves shown in Figure 5 gave a value for the migration area of water of 54 square centimeters. The curves were plotted from the data given in Table 1.

Table 1

Neutron Counts--Water

Distance From Source (Cm.)	Counting Time (Minutes)	Counts	Counting Rate (R) $R = \frac{\text{Counts} \pm \sqrt{\text{Counts}}}{\text{Time in Minutes}}$
2.5	1	219000	219000 ± 468
4	1	218000	218000 ± 466
5	1	196000	196000 ± 442
6	1	173400	173400 ± 416
7	1	150400	150400 ± 387
8	1	126700	126700 ± 356
9	1	106000	106000 ± 325
10	1	90200	90200 ± 300
11	1	74200	74200 ± 272
12	1	62400	62400 ± 250
13	1	51150	51150 ± 226
14	1	42250	42250 ± 205
15	1	34350	34350 ± 185
16	1	28450	28450 ± 169
17	1	22950	22950 ± 151
18	1	19000	19000 ± 138
19	1	16000	16000 ± 126
20	1	13050	13050 ± 114
21	1	10850	10850 ± 104
22	2	18120	9060 ± 67
23	2	15320	7660 ± 62
25	2	10900	5450 ± 52
28	4	12880	3220 ± 28
33	8	11488	1436 ± 11

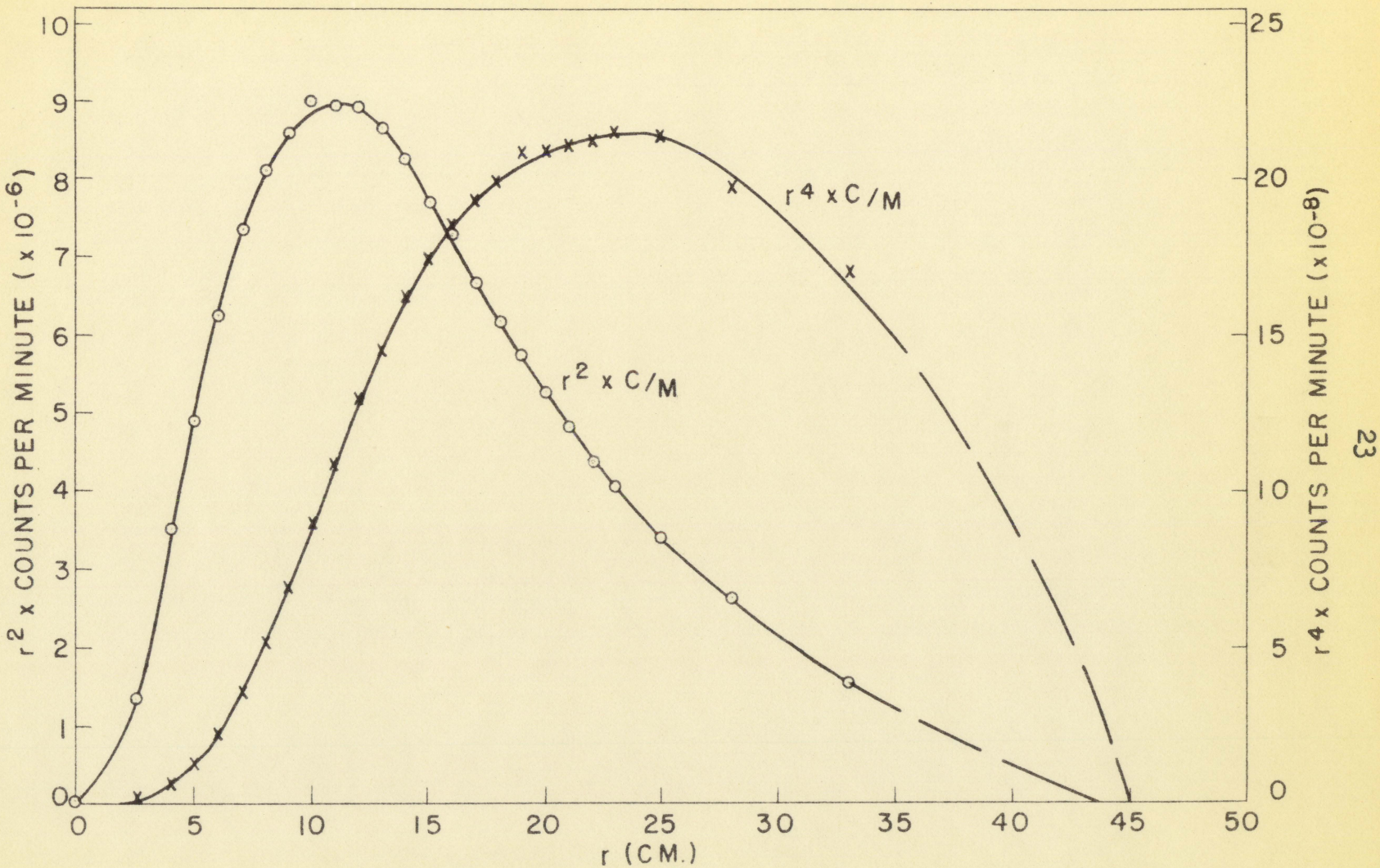


Figure 5. Migration area determination curves for water

B. 8.15% Sugar

With 60.7 pounds of sugar in the water, the migration area increased to 55.8 square centimeters. Data from Table 2 were used to plot the curves of Figure 6.

Table 2

Neutron Counts--8.15% Sugar

Distance From Source (Cm.)	Counting Time (Minutes)	Counts	Counting Rate (R) $R=\frac{Counts \pm \sqrt{Counts}}{Time\ in\ Minutes}$
2.7	1	44400	44400 ± 211
4	1	46900	46900 ± 216
6	1	43250	43250 ± 208
8	1	34250	34250 ± 185
9	1	28600	28600 ± 169
10	1	23600	23600 ± 153
11	1	19960	19960 ± 141
12	1	16580	16580 ± 129
13	1	13780	13780 ± 117
15	2	18820	9410 ± 69
17	2	12350	6175 ± 56
18	3	14970	4990 ± 41
19	3	12420	4140 ± 37
20	4	13560	3390 ± 29
21	5	14025	2805 ± 24
22	5	11650	2330 ± 22
24	7	11410	1630 ± 15
26	9	10539	1171 ± 11
30	20	11480	574 ± 5
35	45	10940	243 ± 2
40	90	10445	116 ± 1

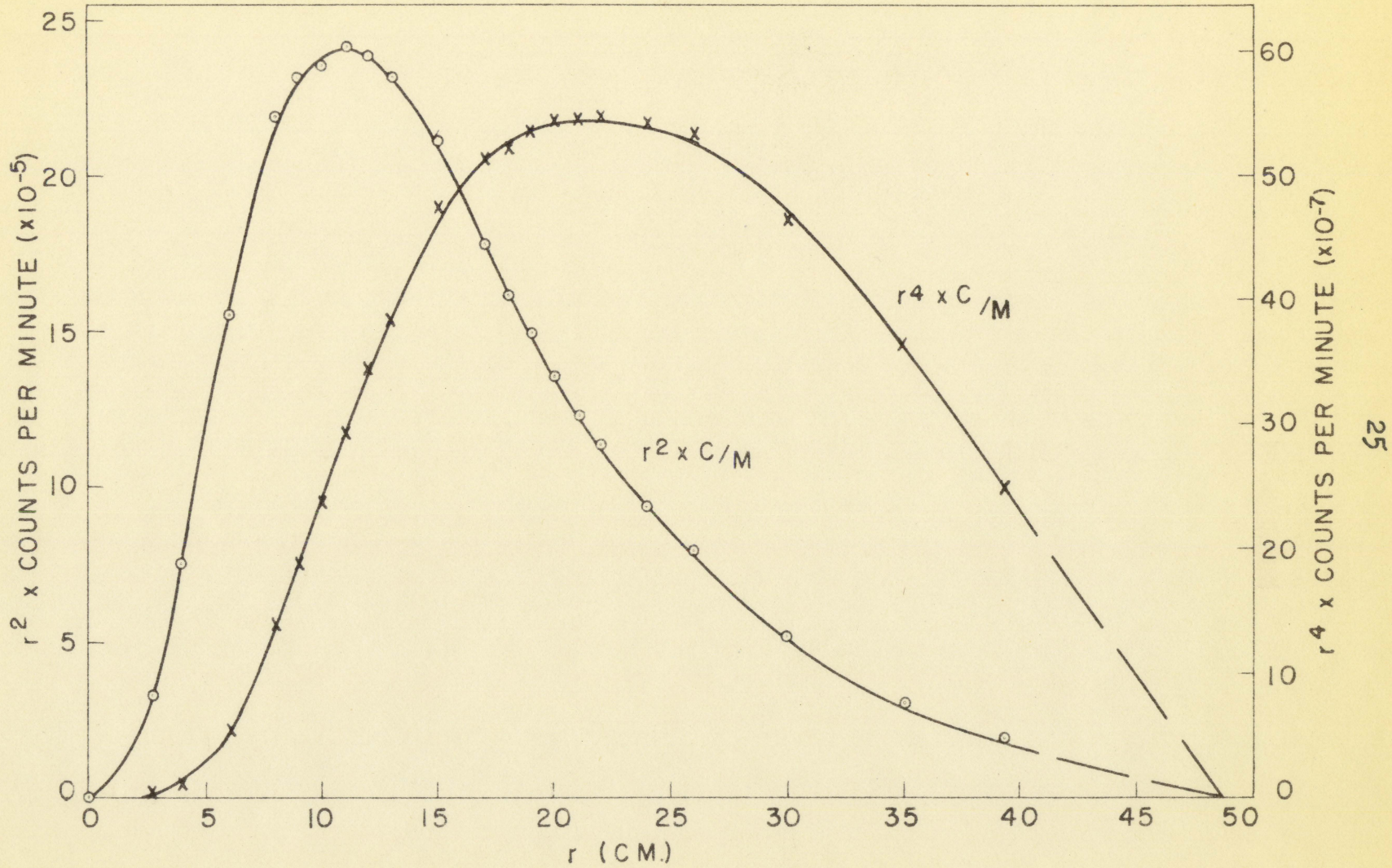


Figure 6. Migration area determination curves for 8.15% sugar

C. 16.22% Sugar

Adding 67.8 pounds of sugar brought the sugar concentration up to 16.22% and the observed migration area up to 58 square centimeters. Data from Table 3 are plotted in Figure 7.

Table 3

Neutron Counts--16.22% Sugar

Distance From Source (Cm.)	Counting Time (Minutes)	Counts	Counting Rate (R) $R = \frac{\text{Counts} \pm \sqrt{\text{Counts}}}{\text{Time in Minutes}}$
3	1	53750	53750 ± 232
4	1	56250	56250 ± 237
6	1	54300	54300 ± 233
8	1	43372	43372 ± 208
9	1	36750	36750 ± 192
10	1	31250	31250 ± 176
11	1	26600	26600 ± 163
12	1	22800	22800 ± 151
13	1	19320	19320 ± 139
14	1	16120	16120 ± 123
16	1	11130	11130 ± 105
18	2	14850	7425 ± 61
20	2	10005	5002 ± 50
22	3	10338	3445 ± 34
24	5	10027	2405 ± 20
26	7	11820	1688 ± 15
30	12	10240	853 ± 8
35	28	10390	371 ± 4
40	65	10664	164 ± 2

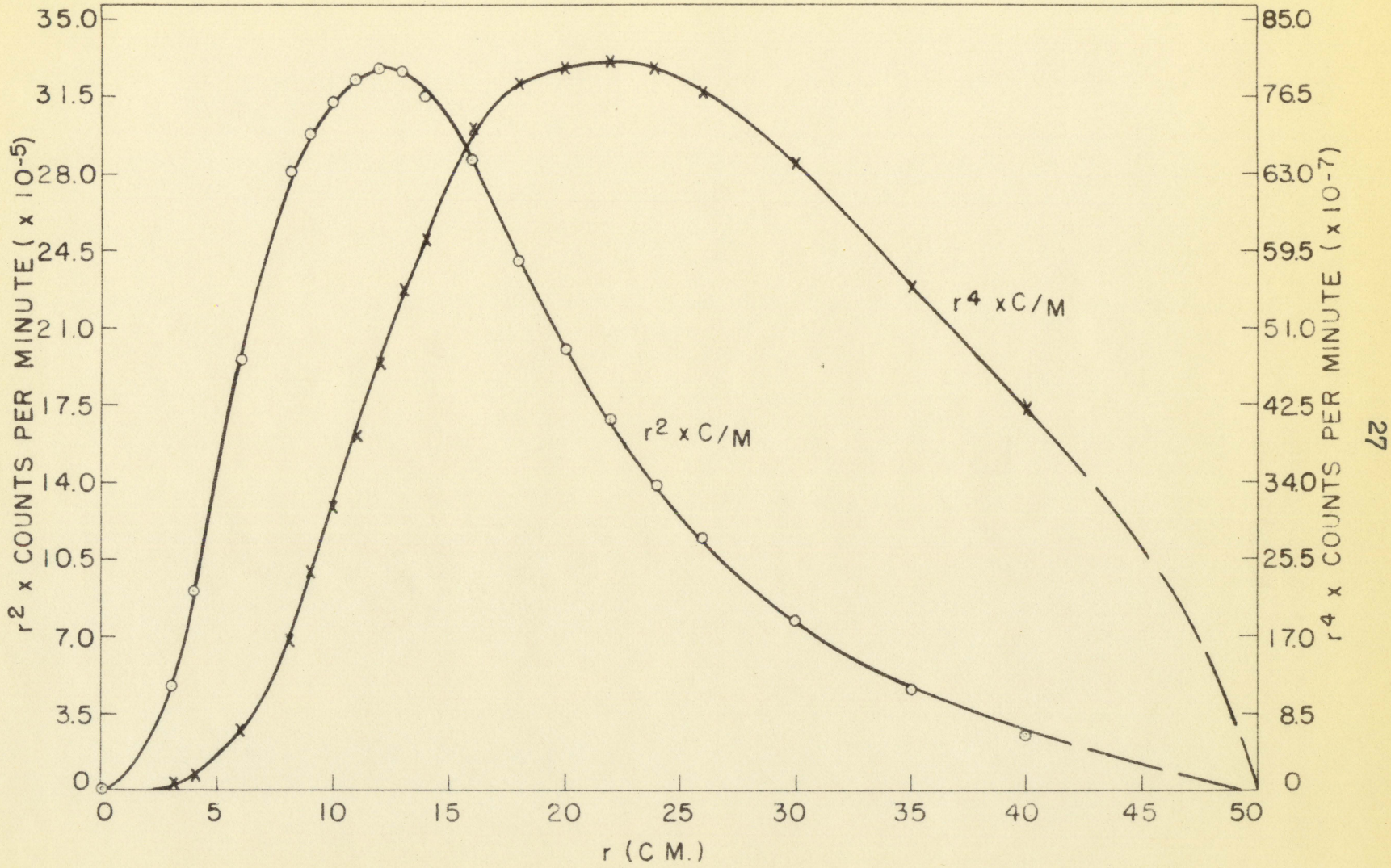


Figure 7. Migration area determination curves for 16.22% sugar

D. 32.2% Sugar

When the final addition of 171.5 pounds of sugar gave the maximum experimental concentration, the migration area again increased to 60.5 square centimeters. Table 4 data are plotted in Figure 8.

Table 4

Neutron Counts--32.2% Sugar

Distance From Source (Cm.)	Counting Time (Minutes)	Counts	Counting Rate (R) $R = \frac{\text{Counts} \pm \sqrt{\text{Counts}}}{\text{Time in Minutes}}$
3	1	53750	53750 ± 232
4	1	55400	55400 ± 235
6	1	51600	51600 ± 227
7	1	47900	47900 ± 218
8	1	41000	41000 ± 202
9	1	35850	35850 ± 189
10	1	30200	30200 ± 174
11	1	25480	25480 ± 159
12	1	21250	21250 ± 146
13	1	17920	17920 ± 134
14	1	14850	14850 ± 122
16	1	10050	10050 ± 100
18	2	13520	6760 ± 58
20	3	13681	4560 ± 39
22	4	12462	3115 ± 28
24	5	10807	2165 ± 21
26	7	10760	1537 ± 15
30	14	10755	768 ± 7
35	30	10383	346 ± 3
40	75	10728	143 ± 2

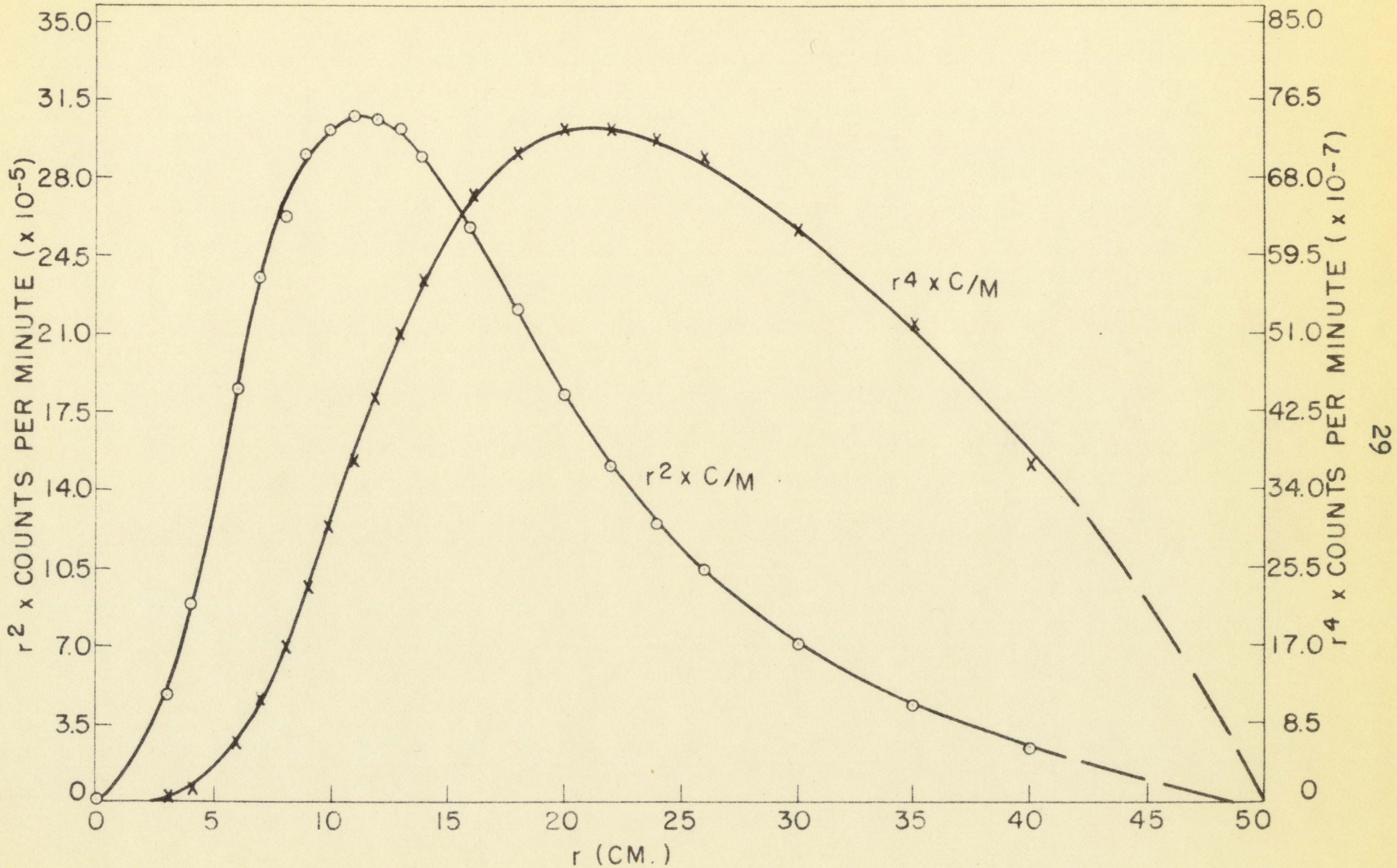


Figure 8. Migration area determination curves for 32.2% sugar

VI. DISCUSSION

A. Analysis of Experimental Method

The first run was made on distilled water in order to standardize the method used by comparing its results to those obtained by other investigators. R. F. Christy (3, p. 128) quotes a value of 54.96 square centimeters for water using a radium-beryllium source. This gives the value obtained of 54 square centimeters an error of 1.8% which shows excellent agreement considering the fact that all radium-beryllium sources have slightly different properties.

Since the experimental method was based on the supposition of a point source and a point detector, it was supposed that the finite source and detector might produce a basis for error since the effective distance between an equivalent point source and detector would not be known. The experimental calculations were based on the assumption that all of the neutrons detected emanated from the top surface of the source. On the basis of the results it appears that no significant error materialized from this assumption.

Due to the fact that the boron lining of the counting tube follows a 1/v cross section ratio there was a definite possibility that fast neutron counts would also be a source of error. The counting tube was wrapped with enough cadmium foil to make the probability of a slow neutron getting through essentially zero, and an additional traverse was run. The ratio of fast neutron counts remained nearly constant throughout the range, varying from two to three per cent. Thus due to the proportionality, no error developed from fast neutron counting.

B. Analysis of Computational Method

The computational method, as described in the sample calculations in the Appendix gave very close agreement on water where a value of 53.5 square centimeters for the migration area was obtained. The empirical correction is made using water data so that for water the method is quite accurate. For computational purposes the analysis of the radium-beryllium source is taken to be (1, p. 56): 69% 5 mev neutrons, 31% 400 kev neutrons.

Although the theory proposed that the method described could be extended to all hydrogenous media, it

appears that this assumption is not well founded. Figure 9 shows experimental and computational results with migration area plotted as a function of per cent of sugar. The computational results agree closely with the experimental results up to ten percent sugar but at that point begin to drop off, until at saturation values the sugar solution shows a lower migration area than that of water. The experimental values do not extend as far as the theoretical and appear to be leveling off somewhat, but there appears little liklihood of their ever becoming as low as water again.

It is believed that the principal source of error in the theoretical method comes from attaching too much importance to the increase of carbon content in the solution. It can be shown that as sugar is added to the water, the number of hydrogen and oxygen atoms per cubic centimeter decrease while the carbon atoms increase. Since hydrogen is mainly responsible for the slowing down of the neutrons, the decrease leads to a greater migration area. This effect is shown by the experimental data plot in Figure 9. However the computational method gives the increasing carbon content credit for making up for the hydrogen decrease, a hypothesis which does not appear to be correct.

C. Analysis of Results

The experimental results shown in Figure 9 form a smooth curve which appears to be leveling off as sugar concentration is increased. Since the water calibration gave results with a 1.8% error, there is no reason to suspect that any of the values are off by more than 3.5%. The individual curves (Figures 5 through 8) cannot be easily compared with one another since it was necessary to move the source holder in relation to the counting tube between runs. However, the general trends may be compared. The counts increased as sugar was added since the number of oxygen atoms per cubic centimeter decreased thus lowering the effect of the absorption resonance of oxygen at 4.5 mev.

The r^2 x counts/minute curves all show the sphere of maximum thermal activity to be between 11 and 12.5 centimeters from the source, where the product of counts/minute and spherical swept area peaks very markedly. The distance of this peak from the source is a function of several physical phenomena. In the first place, the neutron energy spectrum close to the source is centered at an energy level much higher than that to which the detector is sensitive. The detection, therefore, will

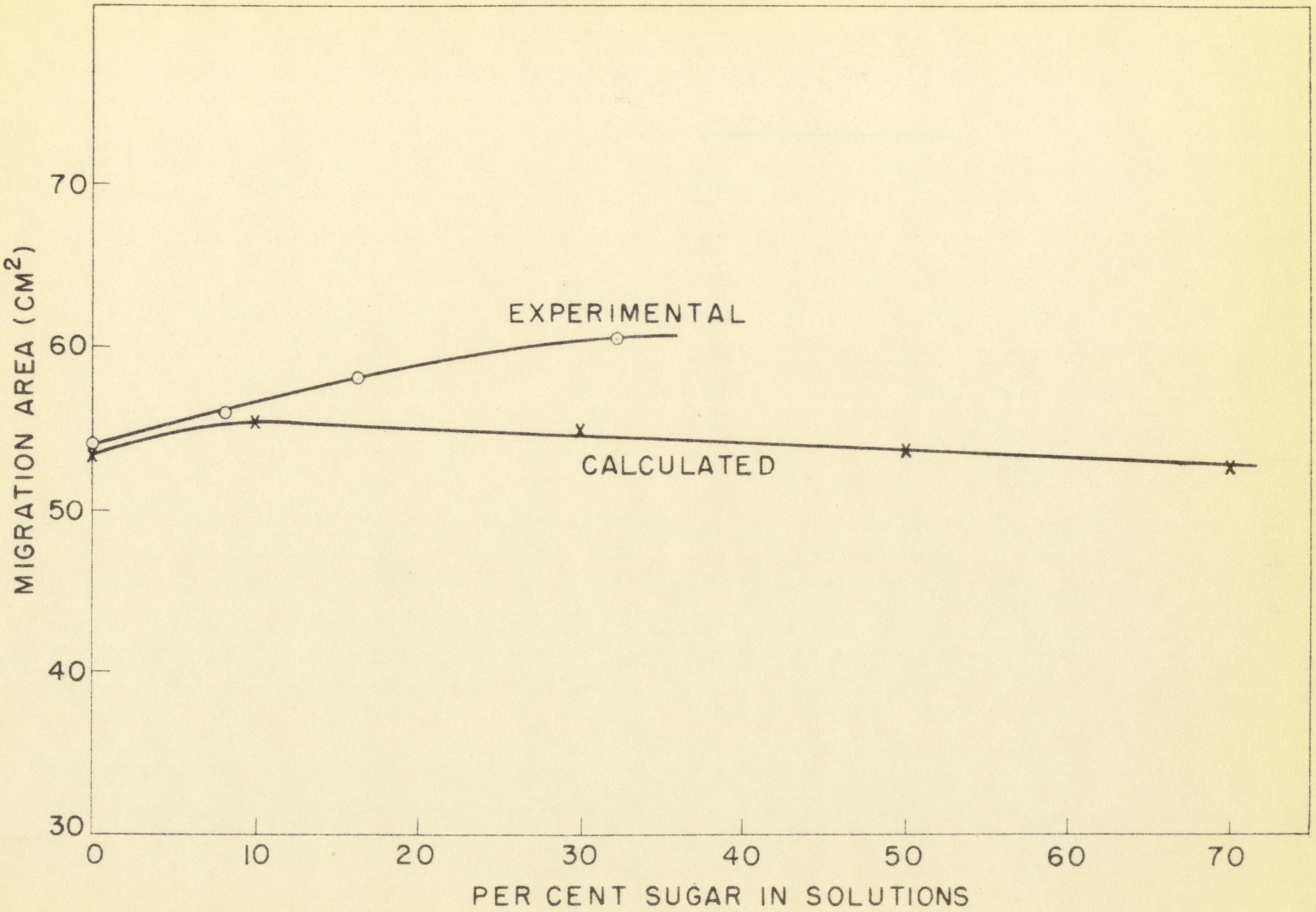


Figure 9. Comparison of experimental and calculated migration areas

not be an effective means of detecting neutrons until the mean effective energy of the neutrons approaches the thermal level. Secondly, the spacial distribution of thermal neutrons is primarily determined by the diffusion of the neutrons while fast. Therefore, the radius of maximum activity is a measure of the mean free path for fast neutrons.

Beyond the sphere of apparent maximum neutron population, the curve decays practically in strict accordance with the exponential law. That this should be so is seen from the fact that after the majority of the neutrons are slowed down to thermal levels, they will diffuse out into the medium following the probability that they will reach a distance x from the origin as $e^{-\Sigma x}$ where Σ is a fictitious scattering and absorption cross section.

VII. CONCLUSIONS

From the foregoing sections the following conclusions seem justified:

1. The experimental method used is a direct method for determining migration areas of solutions, and gave results within 1.8% of published experimental values for water.
2. The computational method used is accurate for water but does not lend itself to other hydrogenous media as exactly.
3. Sugar and water solutions do not have greater moderating abilities than water, since the migration area increases with increasing sugar concentration.
4. The moderating ability of water could be improved by the addition of a soluble substance which contained a large proportion of hydrogen to its total weight and which was more dense than water.

VIII. SUMMARY

The moderating ability of several concentrations of sugar and water was investigated. The migration areas of these solutions were measured experimentally using a radium-beryllium neutron source and computed analytically by an approximate method.

The experimental results indicated a migration area for water of 54 square centimeters. The experimental results for solutions of 8.15%, 16.22%, and 32.2% sugar were 55.8, 58.0, and 60.5 square centimeters respectively.

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XI. APPENDICES

A. Sample Analytical Computation

At a sugar concentration of 10%, the specific gravity of the solution is 1.04. Since the source has an assumed neutron spectrum of 69% 5 mev neutrons and 31% 400 kev neutrons, the slowing down length will be a weighted average of the slowing down lengths from each group.

At 5 mev the microscopic scattering cross sections (5) for carbon, hydrogen, and oxygen are 1.15 barns, 1.65 barns, and 1.3 barns respectively. The total scattering mean free path was found by:

$$\lambda_s = \frac{1}{\sum N_i \sigma_{si}}$$

where

N_i = Number of atoms of the ith kind per cubic centimeter

σ_{s_i} = Microscopic scattering cross section for atoms of the ith kind.

This formula gave a value of 6.41 for λ_s. The scattering mean free paths due to non-hydrogenous components, λ_x, and hydrogenous components, λ_H, were computed from the same formula using only the indicated type of atoms. The ratio of $\lambda_x/\lambda_H = 2.45$. From Figure 2 a

value of 1.87 for λ_o/λ_s was realized, giving λ_o a value of 12.0 centimeters. The initial energy, 5 mev, was divided by e to obtain the geometrical average energy level after the first collision with hydrogen. Thus at 1.84 mev the above process was repeated giving λ_o = 5.24 centimeters. At 677 kev, 249 kev, and 91 kev, the λ_o values were 3.66, 2.22, and 1.37 centimeters respectively.

This process was performed for the second group of neutrons yielding 2.70 centimeters for λ_o at 400 kev and 1.73 centimeters for λ_o at 147 kev.

The Fermi age method was used to determine the slowing down lengths from the 100 kev range to thermal energies, and gave a value for the 5 mev group of 1.7 square centimeters and for the 400 kev group a value of 1.87 square centimeters. The above values were substituted in Equation (5) and the results of each group were weighted. The square of the slowing down length was determined to be 46.8 square centimeters.

The diffusion length squared was determined from the formula given in the theory:

$$L^2 = \frac{D}{\Sigma_a}$$

= 8.2 square centimeters.

The sum of the squares of the slowing down length and diffusion length, or migration area, was thus

determined to be 55.0 square centimeters.

B. Counting Tube Plateau Curve

The plateau curve (Figure 10) for the B^{10} lined neutron counting tube was used to determine the operating voltage of 600 volts. This voltage was approximately the center of the linear portion of the curve.

C. Sugar Solution Densities (6)

The following table of solution densities was used in the analytical computations.

Table 5

Specific Gravity of Sucrose Solutions

Sucrose % By Wt.	Balling - Brix $d\ {}^{17.5^\circ}_{17.5^\circ}$ C.
0	1.00000
5	1.01970
10	1.04014
15	1.06133
20	1.08329
25	1.10607
30	1.12967
35	1.15411
40	1.17943
45	1.20565
50	1.23278
60	1.28989
70	1.35088

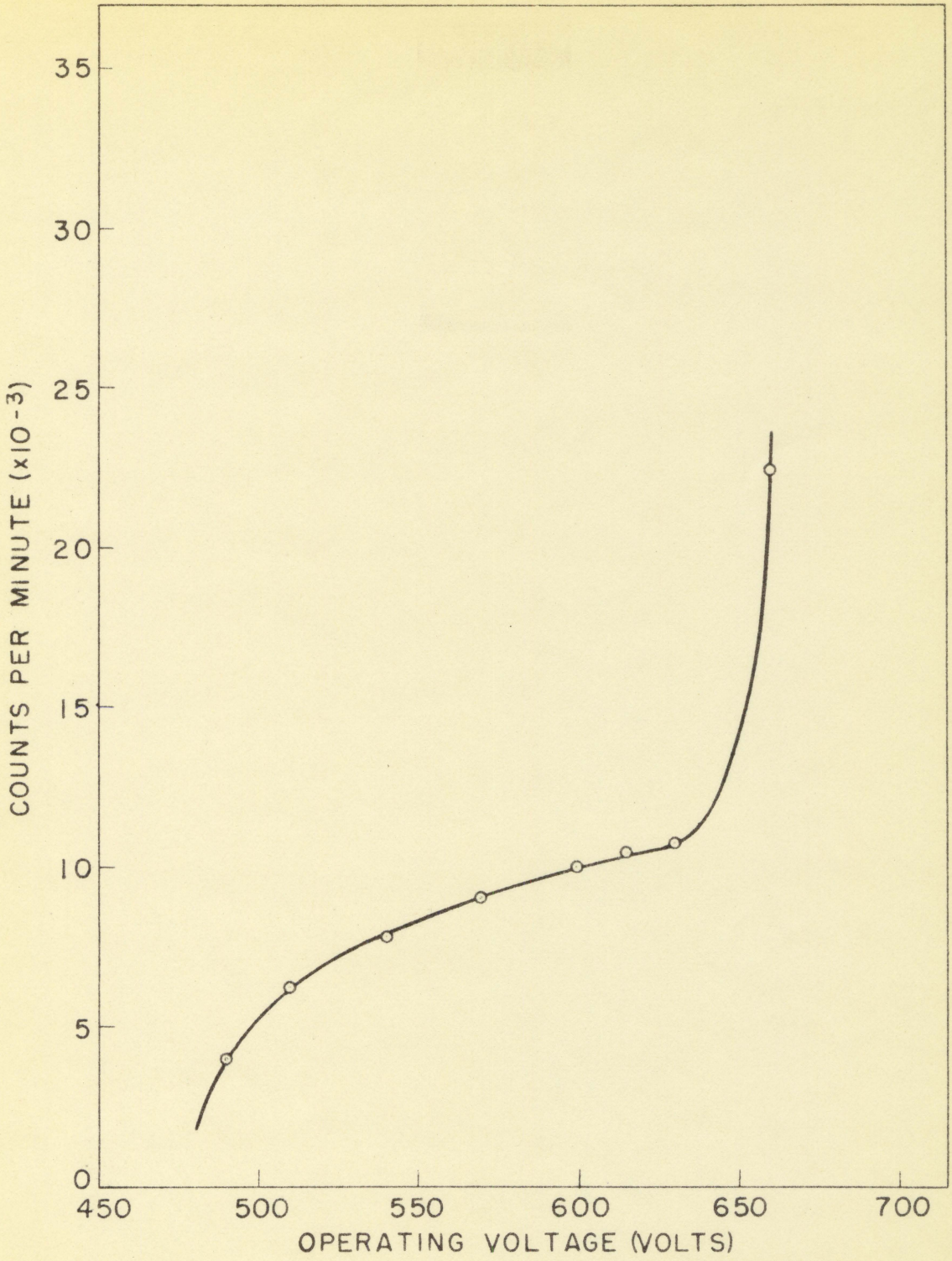

Figure 10. Plateau curve for B^{10} lined neutron counting tube

D. Approximate Sugar Analysis (6, p. 394)

Sucrose	90%
Invert Sugar	5%
Water	2%
Ash	1%
Organic Non-sugar	2%